Cellular Automata

CA - The Concept Example I History Example II Why CA? Simulation in the Social Sciences Rainer Hegselmann and Andreas Flache (1998), Understanding Complex Social Dynamics: A Plea For Cellular Automata Based Modelling

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Basic features of cellular automata (CA)

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CA - The Concept Example I History Example II Why CA? Critique Discrete Location: Cells are arranged in a regular D-dimensional grid.

Discrete States: Every cell adopts one out of a finite set of states.

Discrete Time: Time is discrete.

Locality: Cells change their states according to local rules.

Rules: The same transition rule applies to all cells.

Updates: In each period cells are updated (simultaneously or sequentially).

2D Grid Without Edges



CA - The Concept

History

. Why CA?

Critique



Grid



Torus

Locality: Different Kinds of Neighbourhoods



Triangular, hexagonal and irregular grids are also possible.

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Opinion Dynamics



CA - The Concept

Example I

History Example II Why CA? Critique



- Cells have Opinions $u_i(t) \in [0, 1]$
- von Neuman neighborhood N_i
- Stepwise Averaging $u_i(t+1) = \frac{1}{\#N_i} \sum_{j \in N_i} u_j(t)$
- Discretisation through step function
- Stewise updating of randomly selected cells

Opinion Dynamics - Typical (stable) Results



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Opinion Dynamics - Interpretation

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CA - The Concept

Example I

History

Example

Critique

- CA reach stable state
- Micro rules lead to macro patterns
- More options more consensus
- Extreme opinions disappear
- Discretisation matters
- Beware of artefacts



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A Brief History of CA

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- 1940's First CA models in natural sciences by John von Neumann and Stanislaw Ulam
 - 1949 James M. Sakoda's Checkerboard Model of Social Interaction (published 1971)
 - 1969 Thomas Schelling's Segregation Model
 - 1975 Peter S. Albin classifies checkerboard models as CA in *The Analysis of Complex Socioeconomic Systems*
- 1990's More frequent use of CA models in social and behavioral sciences

Sakoda's Checkerboard Model

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- Original model is 8x8
- Here: 40x40 extension
- Two groups with positive (1), negative (-1) or neutral (0) attitudes to each other
- Migration is possible in a limited radius and directed through optimization.
- In segregation setting, meeting points acquire attraction.



Schelling's Segregation Model

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- Considering Moore neighbourhood
- Migrate if frequency of kin is below a given minimum level.
- Go to nearest neighbourhood where this requirement is met.
- Displays the unintended consequences of intentional actions.



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Evolution of Support Networks

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- Cells are "rational egoists" and play the *Support Game* in their von Neumann neighbourhood, an extension of the iterated Prisoner's Dilemma.
- Players have different risk classes
 p_i ∈ [0.1, 0.2, ..., 0.9] (i.e. their probability of needing support).
- Migration opportunities are offered randomly with fixed probability *q*.
- All *p_i* and *q* are known to all players, and they calibrate their strategies accordingly, maximising their expectations in a pessimistic scenario.

Well-ordered Support Networks emerge



Support Networks - Results

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Maximum *q* possible for support between risk classes.

- Ordered Support Networks emerge.
- Similar risk classes tend to form clusters.
- Annular arrangement of classes.
- Feasibility of cooperation is a function of p_i, p_i and q.
- Expected utility of support decreases at both extremes for risk classes.

Support Networks - Network Dividend

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- Comparison with unconnected case.
- Intermediate risk classes gain most from support networks.
- Equality increased in all three setups.

Rational egoism, class segregation, increasing equality and increasing wealth go hand in hand.

Why use Cellular Automata?

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- Locality, overlapping neighbourhoods, and repeated interaction are properties of a significant class of dynamic social processes.
- Good examples of unintended consequences in social action. (Schelling, Support Networks)
- Provide quantitative explanations and predictions for artificial worlds. Qualitative understanding for "real" world processes might be derived.
- Explore theoretical assumptions and develop new theories.
- Explore the dynamics of elementary social interactions.

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I LIKE THIS CONCEPT

- Provides means to explore *dynamics* in social interactions, in a controlled environment, on a level of abstraction and simplicity where we might still be able to trace back the developments.
- Spirit of Simplicity: Uses a clear set of assumptions and a supposedly infallible deduction process. Provides solid ground for further argument and analysis.

I LIKE THIS ARTICLE

- Graphical illustrations are chosen well, they are illustrative and helpful.
- Article gives a concise overview of the state of the art.

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PARAMETRISATION

- Seemingly arbitrary parametrisations / implementations
- Any other motivations for mechanisms besides computational simplicity?
- e.g. Opinion Dynamics: Why is an opinion $u_i \in [0, 1]$?
- e.g. Opinion Dynamics: Averaging in neighbourhood

ALTERNATIVE INTERPRETATIONS

- Processes are so general many alternative interpretations might be possible
- e.g. Opinion Dynamics: Study effort

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why CA

Critique

METHODOLOGY

- How much tweaking was necessary to produce results
- e.g. Support Networks: *q* = (0.05, 0.1, 0.15)

INTERPRETATION

- Highly reliant on the description of pictures
- Some analytic explanations only for Support Networks

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ROBUSTNESS

- How robust are findings?
- Highly reliant on graphical, i.e. qualitative means.
- Are there measures for qualitative robustness, given abundant simulation data?
- Possibly, robustness analysis could tell us more about real world phenomena.
- e.g. Sakoda: How about values besides 1, 0, -1
- Nicely done by Schelling provides a minimum level for segregation.

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Original Setup - Segregation

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More on Sakoda

More on Support Networks



Original Setup - Suspicion

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More on Sakoda

More on Support Networks



Support Game - Structure



Support Game - Formulae

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More on Sakoda

More on Support Networks

PD-condition (mutual support is profitable for both)

$$p_i(1-p_j)(S-D) > p_j(1-p_j)(M-H) \ i \neq j.$$
 (1)

• Defection is the dominant strategy in a one shot game

Mutual support is profitable for both players (over time)

COOP-condition (existence of a cooperative game solution)

$$a_i \ge rac{1}{1-p_j(1-p_i)+p_i(1-p_j)rac{S-D}{M-H}} = a_i^+ \ i \ne j$$
 (2)
 $a = (1-q)^2$ (3)

 Cooperative supergame equilibria exist if the probability of being involved in a further iteration of the game is sufficiently high.