

## 7. Gaining Insight

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## 7.1 *Deterministic evaluation*

- Sensitivity analysis
- Tornado diagrams

## 7.2 *Probabilistic evaluation*

- Cumulative probability distribution
- Sensitivity to probability

## 7.3 *Expected Value of Information*

- Expected Value of Perfect Information (VPI)
- Expected Value of imperfect information

## 7.1 Deterministic Evaluation

Deterministic evaluation may be closest to the way most analyses are performed outside decision analysis.

Sensitivity analysis provides the ability to determine the most important factors which affect either the decision or the value (“the bottom line”). We can then use the *Tornado diagram* to illustrate the relative sensitivities of each variable.

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Variables for Glix:

|                         | P r o b a b i l i t i e s |      |    |
|-------------------------|---------------------------|------|----|
|                         | 10                        | 50   | 90 |
| Market size (Gigagrams) | 0.2                       | 1    | 2  |
| Market share (%)        | 15                        | 20   | 25 |
| Mfg. Costs (\$/kg)      | 1                         | 1.5  | 2  |
| Mktg. Costs (\$/kg)     | 0.5                       | 0.75 | 1  |



baseline

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- Step 5: Plot a Tornado diagram** using the data.

## Building the value model

### The value model for Glix:

#### Fixed inputs:

**Discount rate = 10% p.a.**

**Tax rate = 40%**

**Glix price/kg = \$5.00**

**Project length = 10 years**

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**NPV of Glix = (Revenue – Cost) × Discount Factor for each year**

**Revenue = Price × Volume**

**Volume = Market Size × Market Share**

**Cost = (Manufacturing Cost + Market Cost) × Volume**

**Base case value for Glix:**

$$\text{Revenue} = \$5.00 \times 1,000,000 \text{ kg} \times 20\%$$

$$\text{Costs} \rightarrow (\$1.50 + \$0.75) \times 1,000,000$$

$$\rightarrow \$1,000,000 - \$450,000$$

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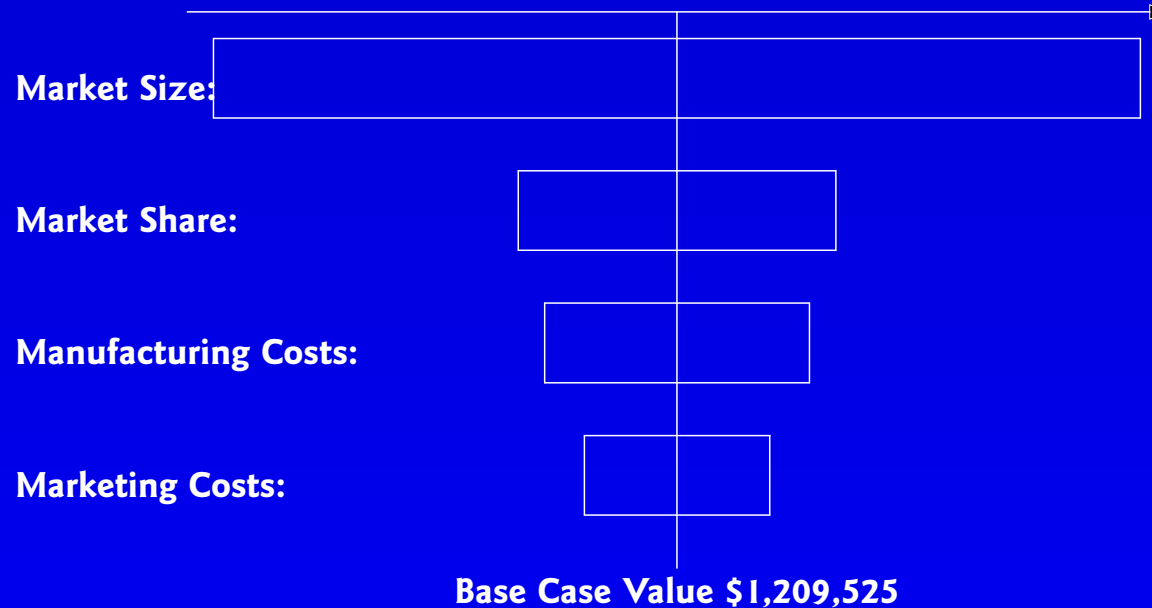
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**Step 5: Draw horizontal bars for each uncertainty relative to their swings in value.**

## The Tornado Diagram.



**See Clemen (Reading 18) and Skinner (Reading 20) for further discussion.**

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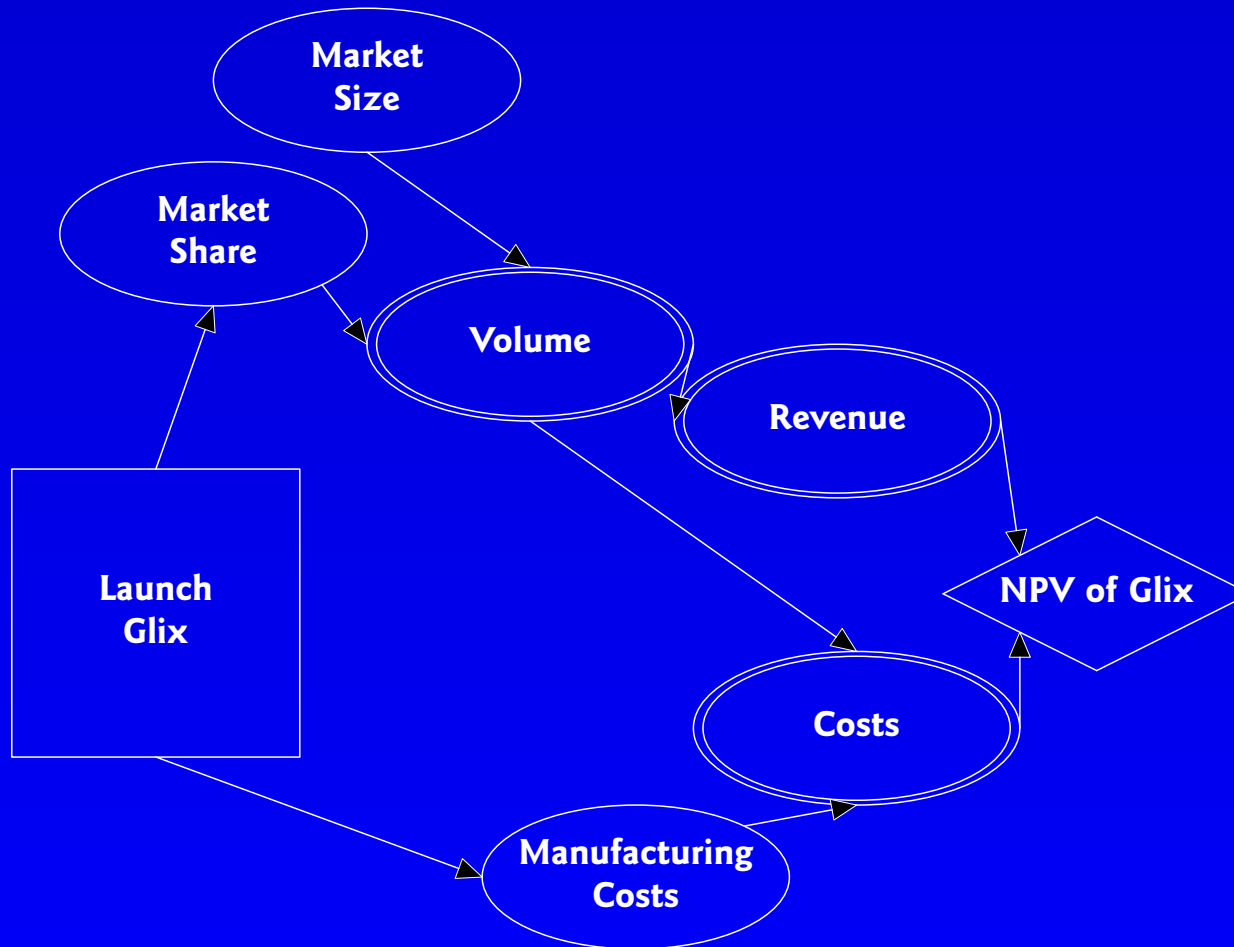
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**With project Glix, the most important uncertainty is Market Size, and the least important is Marketing Costs, from the Tornado Diagram above.**

***Important:* always strive to simplify your Influence Diagrams: use Tornado diagrams and your intuition to reduce the degree of complexity of the ID – they are much more useful when simple!**

## Influence Diagram of Glix Decision



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**The cumulative probability distribution provides a graphical risk profile for the project or each alternative.**

**(This is more technical: see David C. Skinner, *Introduction to Decision Analysis* (Gainesville, Fl., 2nd. ed., 1999), pp. 112–113, 218–220.)**

**But see Laura's decision below.**

## **Another alternative? Selling the Glix project.**

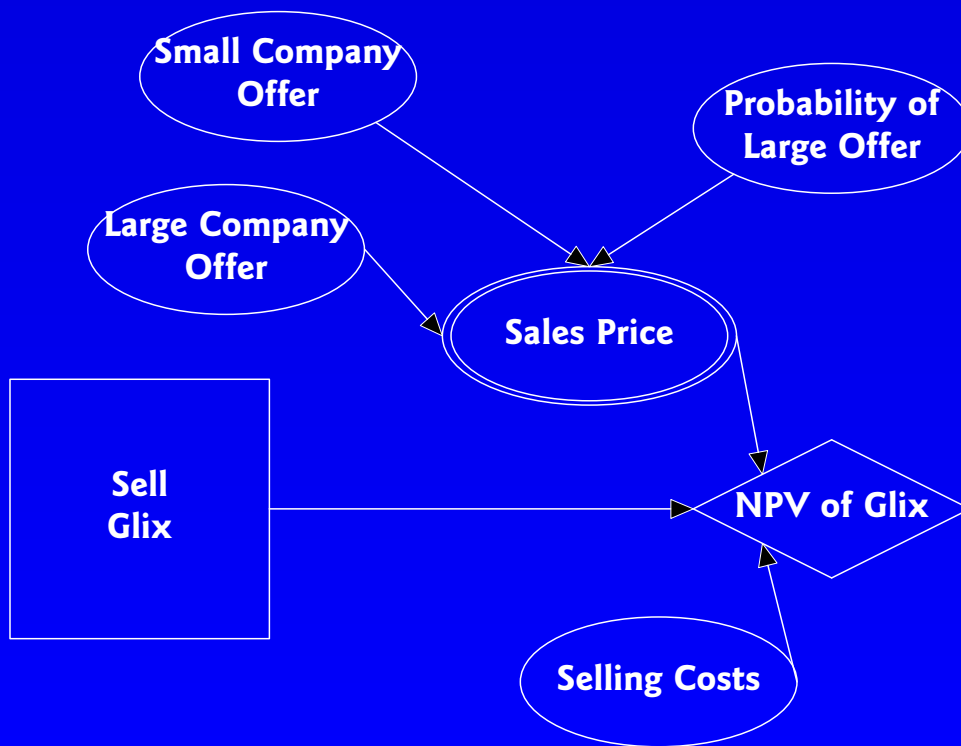
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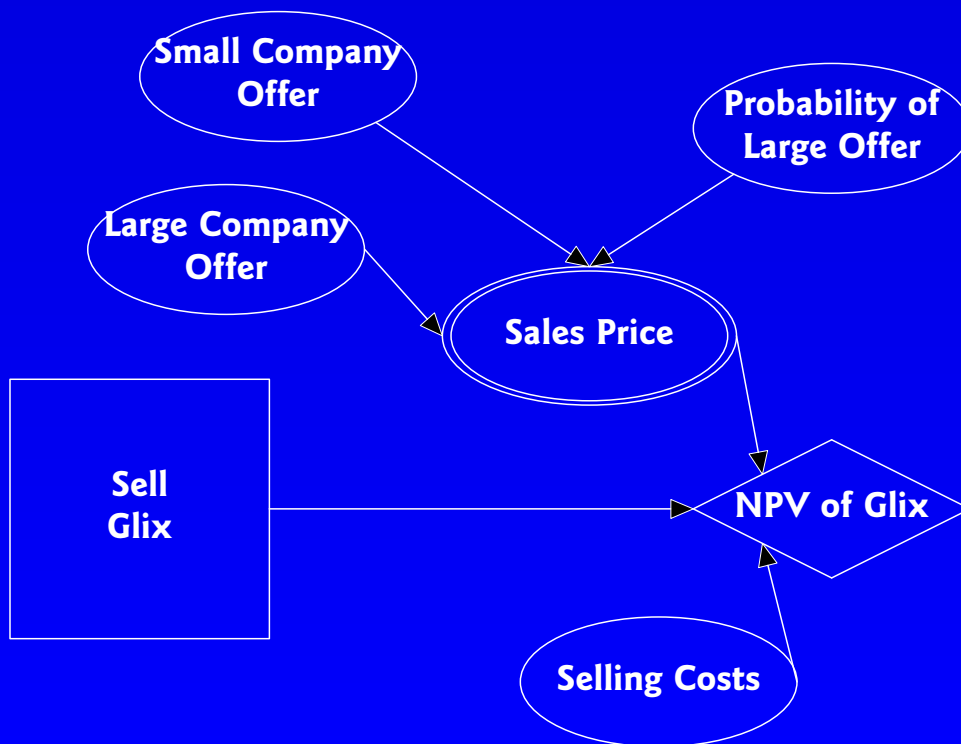
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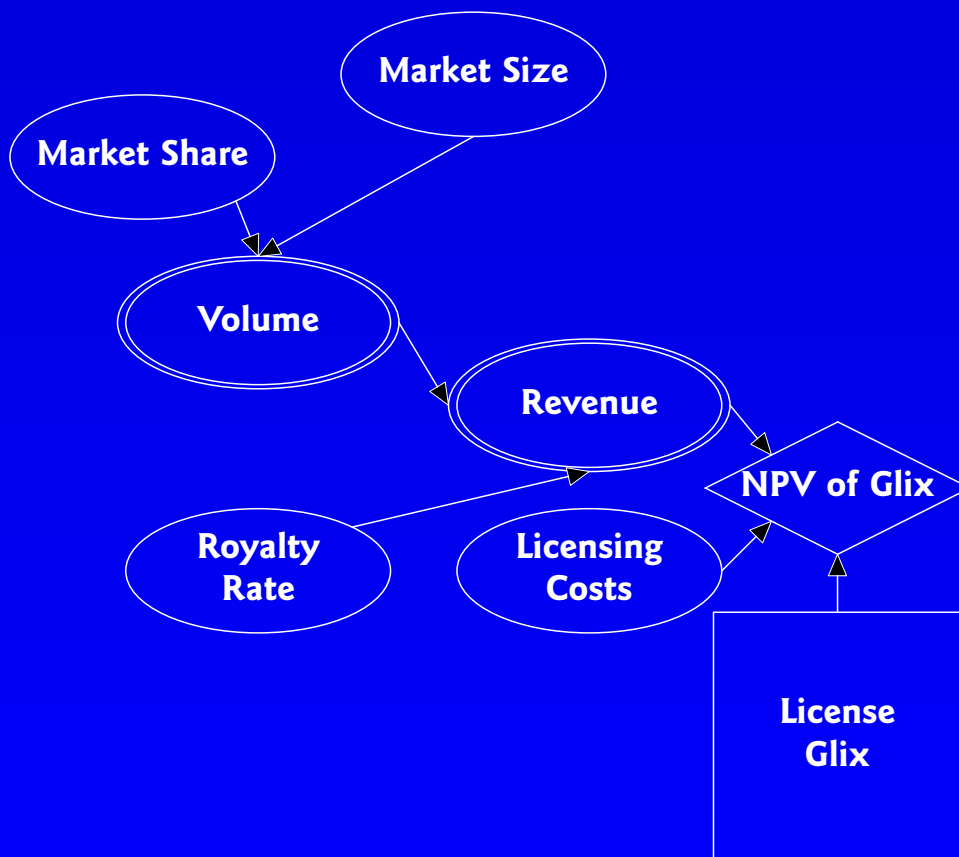
The influence diagram for selling Glix to another company:



The EMV of Selling = \$320,000.

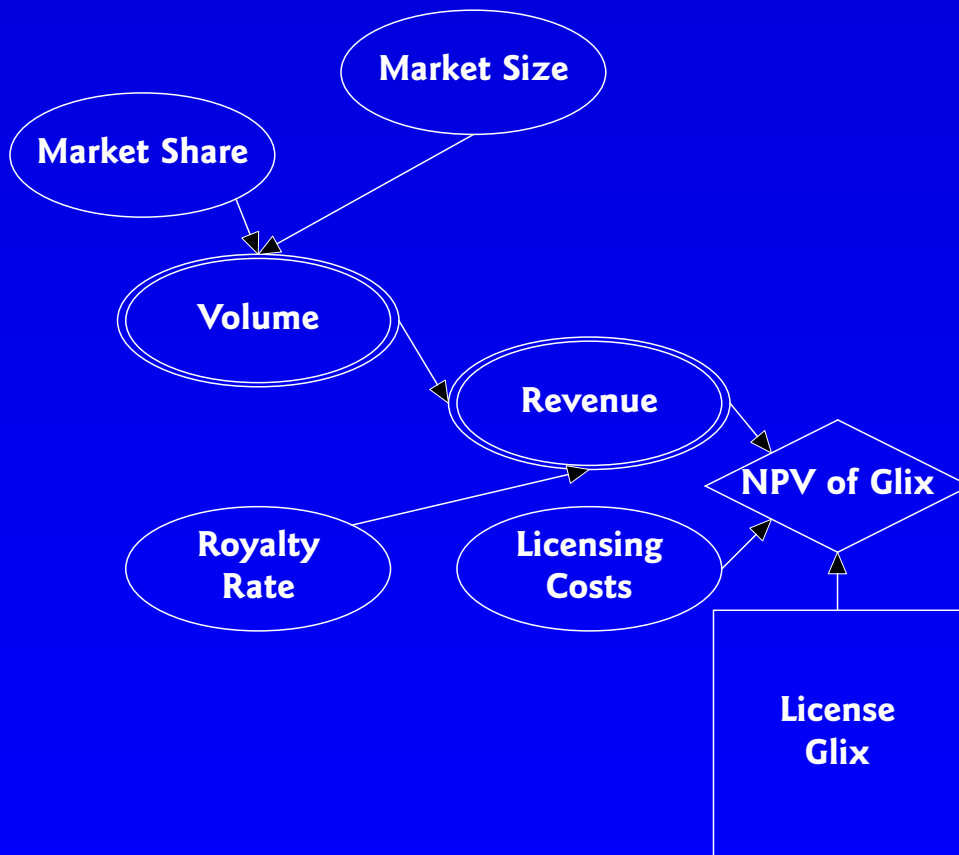
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It turns out, with further analysis, that none of the three alternatives shows complete dominance over the other two.

The “sell” alternative, however, is less attractive, based on an EMV of \$320,455.

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**Having said that any subjective probability which incorporates the expert's available knowledge, beliefs, experiences, and data is correct, we need to know how sensitive the decision is to any particular probability. This will help us choose between launching or licensing Glix.**

**It turns out that we should launch if we are confident that *launching* has a greater than 40% chance of success.**

# Games Against Nature: Gaining Insight — The Value of Information

## Today's topics:

1. **The Value of Perfect Information**
  - a. **For Laura**
  - b. **For Glix**
2. **Probabilistic Sensitivity Analysis**
  - a. **For Laura**
3. **The Value of Imperfect Information**
  - a. **For Laura**
  - b. **For Glix**

# 1. The Value of Information

**We can determine the value of gathering additional information before spending time or money to gather it.**

**The *Value of Perfect Information* is the easiest to calculate, and provides an upper boundary as to the *most we should ever spend on new information.***

***Most companies over-invest in information, spending more than it is worth to them.***

**The Value of Perfect Information (VPI) is the *most that we should spend for new information which is not 100% reliable.***

**We would only value Perfect Information if it changed our decisions, otherwise not.**

## Ia. Laura's Case — The Expected Value of Perfect Information (VPI)

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- If totally reliable (no errors), then**
  - if “Retro is definitely a Goer”, then a return of \$240k, less the price of the Trial**
  - if the Trial indicates Retro is a “Fizzer,” then choose a net return of \$200k with Trad, less the price of the Trial**

---

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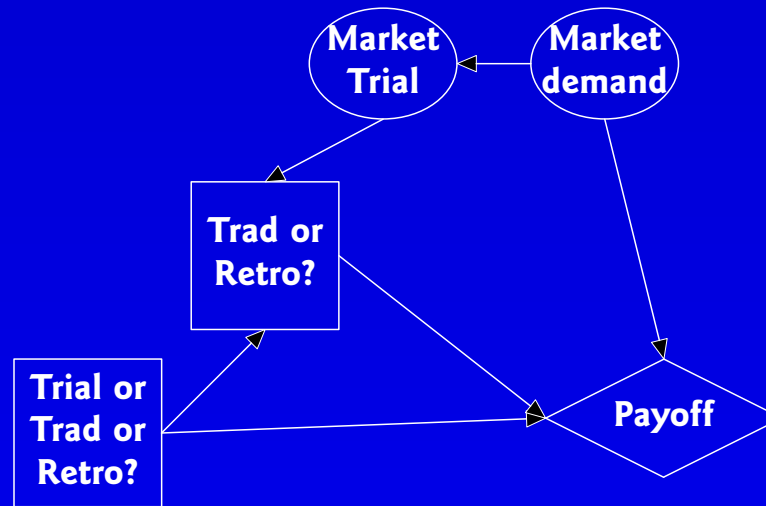
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**The *expected value of information* is the difference between Laura's expected returns *with* the Trial and *without* the Trial.**

## Laura's Influence Diagram: To Trial or Not?

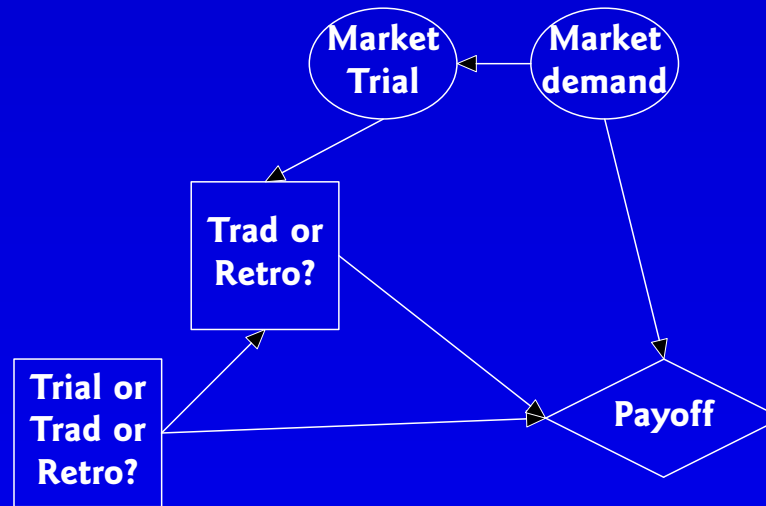


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**That information in turn is influenced (perfectly or not) by the actual Market Demand.**



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**That information in turn is influenced (perfectly or not) by the actual Market Demand.**

**If the Trial is 100% reliable, then there is no uncertainty after the Trial, and hence no arrow from the Market Demand uncertainty to the Payoff: all uncertainty is resolved before the Trad/Retro decision is made.**

## Laura: the VPI

**What Laura would like to know is what a specific piece of information implies for the eventual market demand for Retro, that is:**

**Probability (Retro is a Goer, given that Trial says “Goer” )**

**With perfect information, this probability is 1.**

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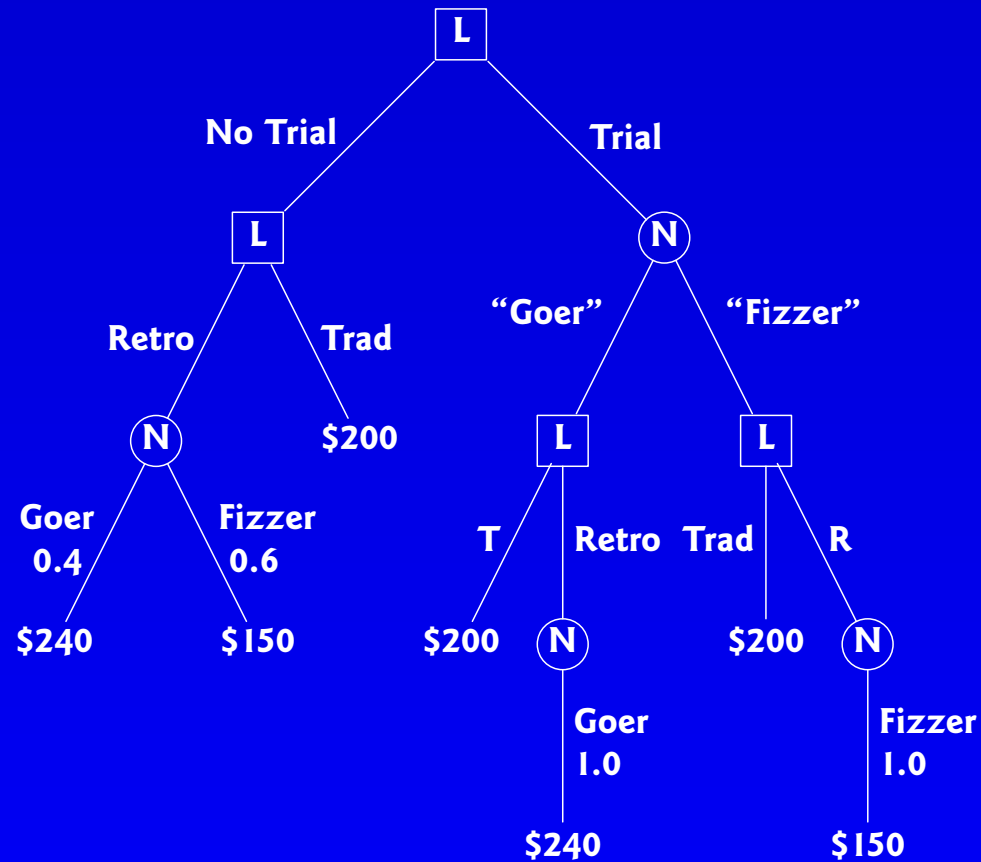
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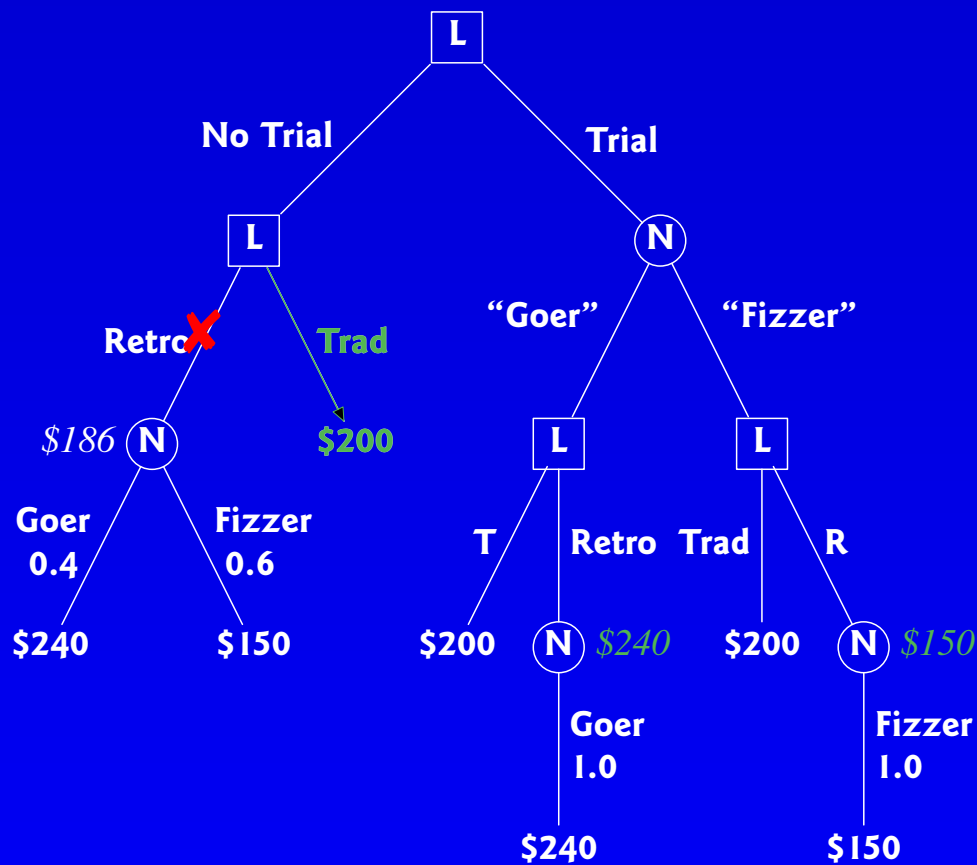
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Q: What is Laura’s estimate of the probability that the Clairvoyant will say: “Goer”?

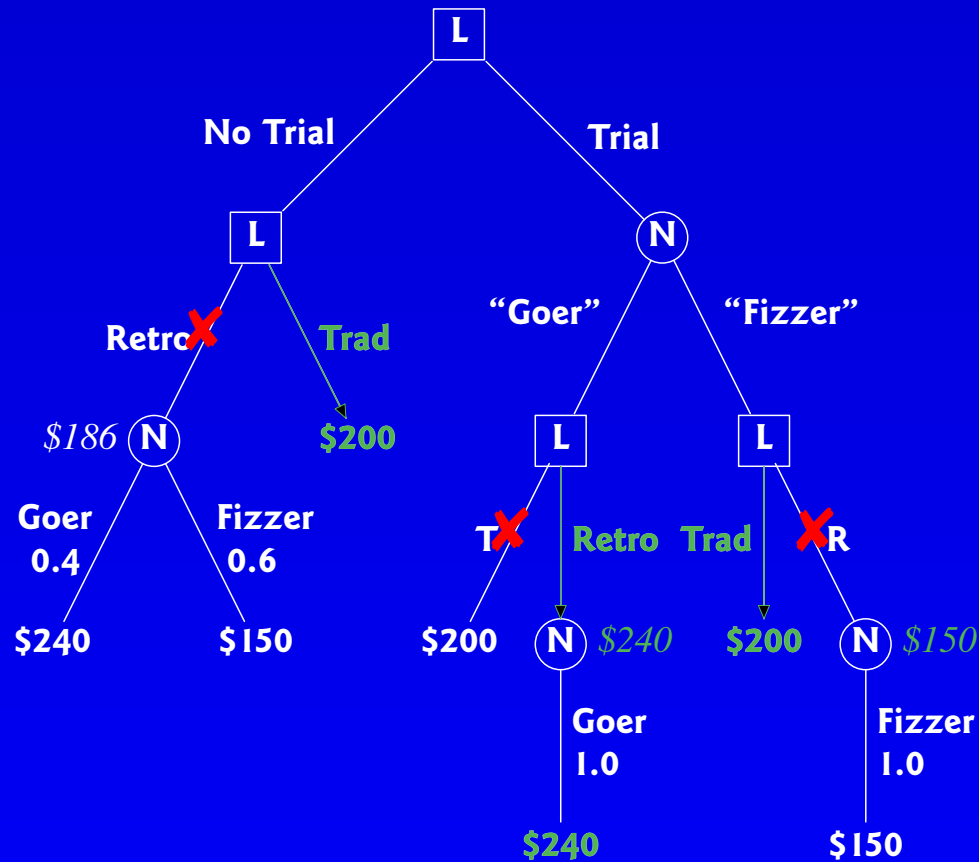
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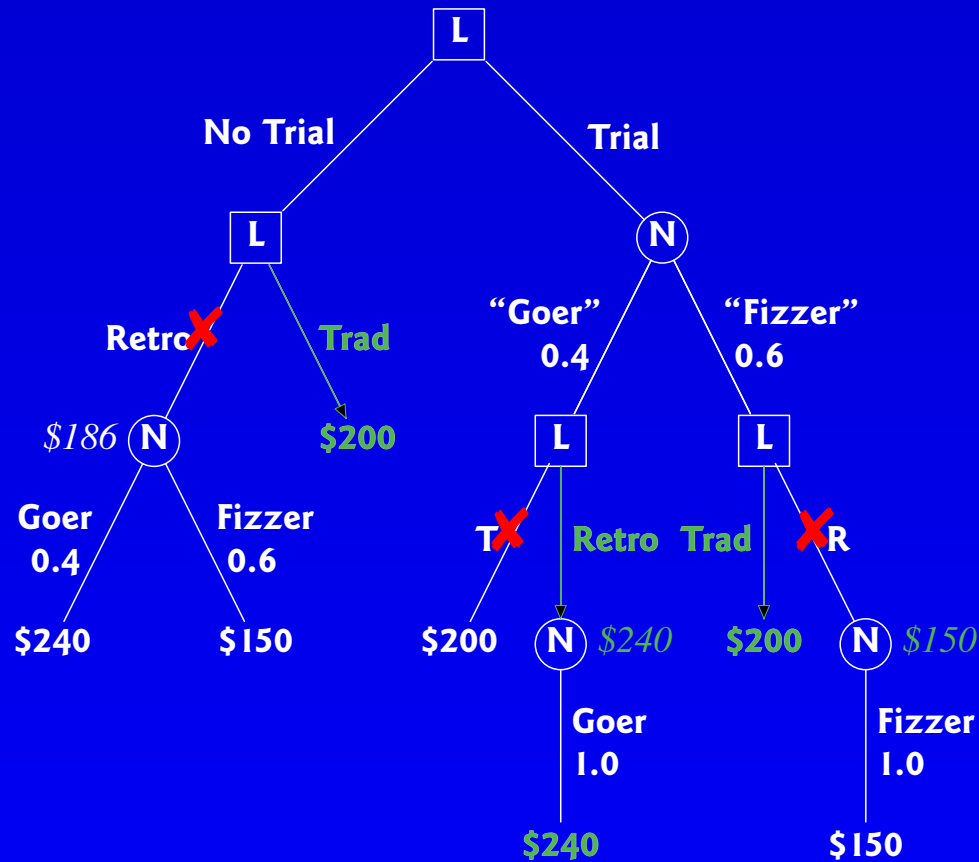
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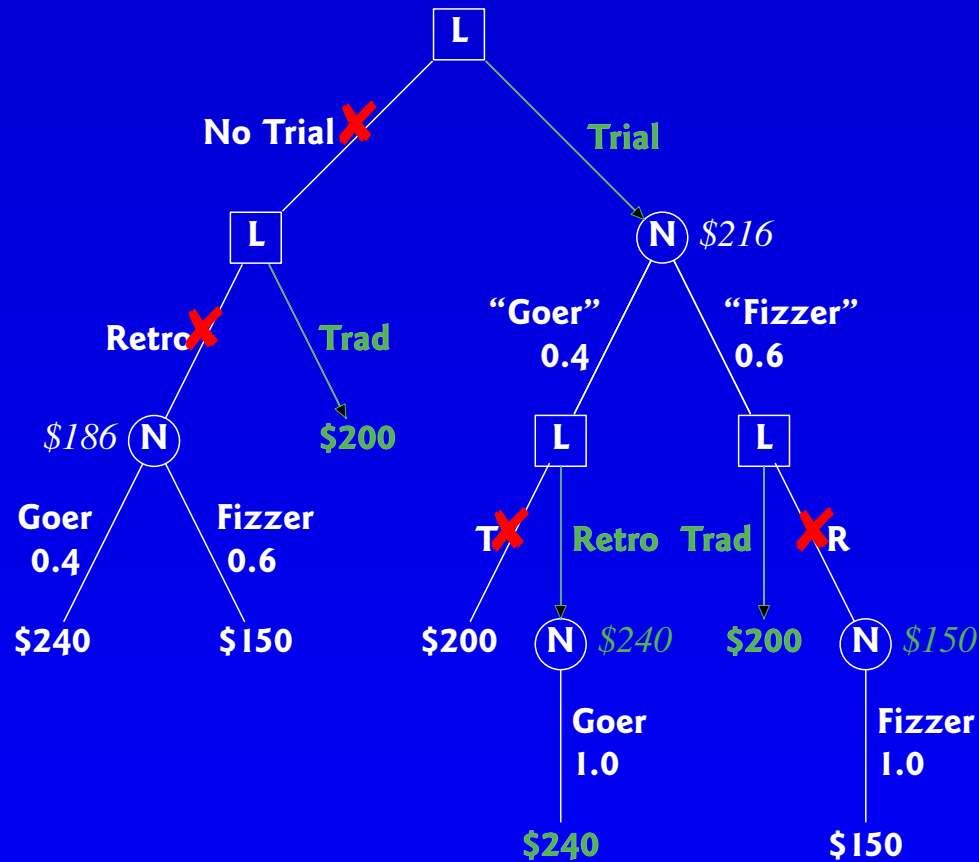
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*The Shoe Decision with Perfect Information*  
 (Remember: the Trial is 100% accurate.)



**The new decision tree has four decision nodes □:**

- 1. whether or not to test,**
- 2. which range to choose without testing,**
- 3. which range to choose if the test says “Goer,” and**
- 4. which range to choose if the test says “Fizzer.”**

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1. whether or not to test,
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**The second, third, and fourth decision nodes are trivial:**

2. Choose Trad if Laura chooses not to test. (Without testing, Trad pays \$200k, which is better than the \$186k expected from Retro.)
3. choose Retro if the test says it’s a “Goer”,
4. otherwise choose Trad,

## How many chance nodes ○?

- **Possibly three: the outcome of the test, and the 2 outcomes if she chooses Retro.**
- **But if the test is 100% reliable, it would rule out any uncertainty about Retro, one way or the other, and so the second and third chance nodes disappear.**

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Well, her "prior" that Retro will be a Goer is probability = 0.4.

And *consistency* dictates that this is also her belief that testing will give the result that Retro is a "Goer".

➤ Laura's expected return from the Trial:

$$\$240\text{k} \times 0.4$$

(the Trial indicated that Retro is a "Goer" and Laura chooses Retro)

$$+ \$200\text{k} \times 0.6$$

(the Trial indicated that Retro will "Fizz" and Laura chooses Trad)

$$= \$216\text{k}$$

## Laura's Expected VPI

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This is the *Expected Value of Perfect Information* in this decision;

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**For an on-line applet for simple calculations of EVPI, see**

[http://www.cs.usask.ca/content/resources/tutorials/csconcepts/1999\\_6/Tutorial/Java/EVPIApp/evpi.html](http://www.cs.usask.ca/content/resources/tutorials/csconcepts/1999_6/Tutorial/Java/EVPIApp/evpi.html)

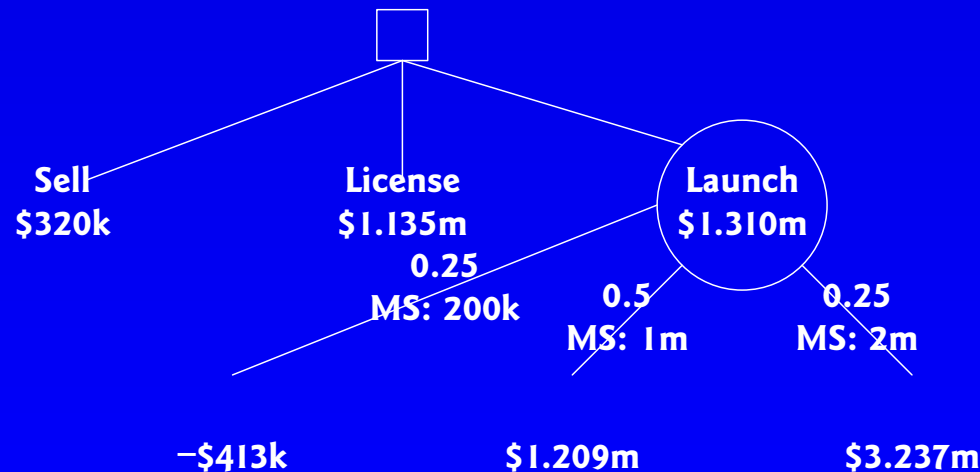
## 1b. Calculating the VPI of the Glix case

VPI is calculated by placing the uncertainty you want to evaluate before the decision. Then, recalculate the expected value.

Focus on Market Size (MS) uncertainty:

L = 200k, M = 1m, H = 2m.

Original tree: EMV = \$1,310,910



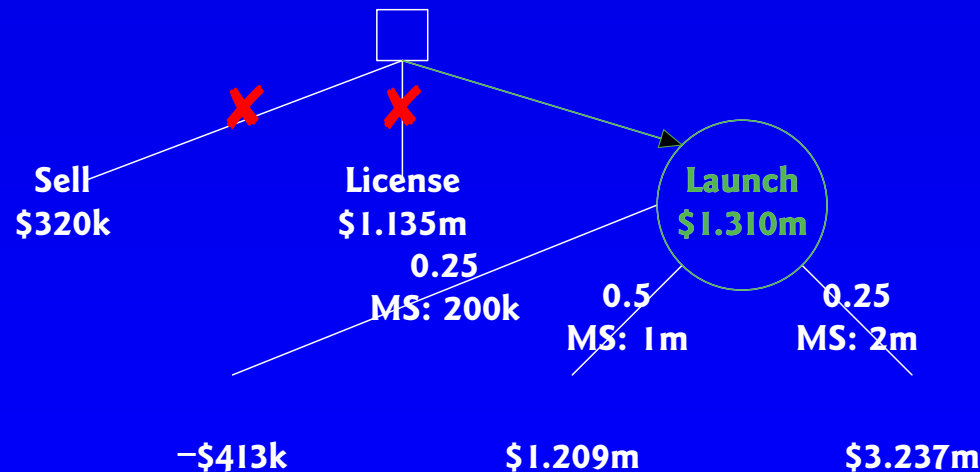
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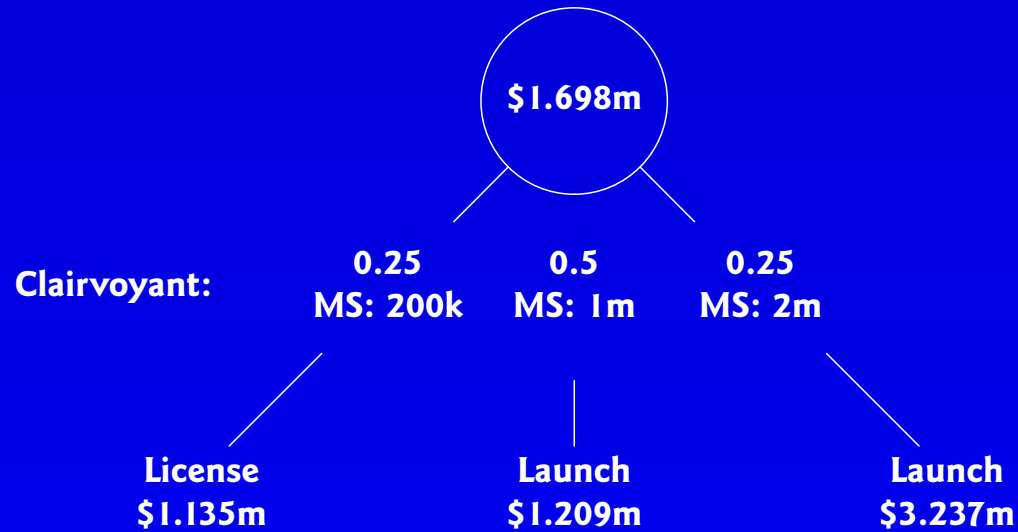
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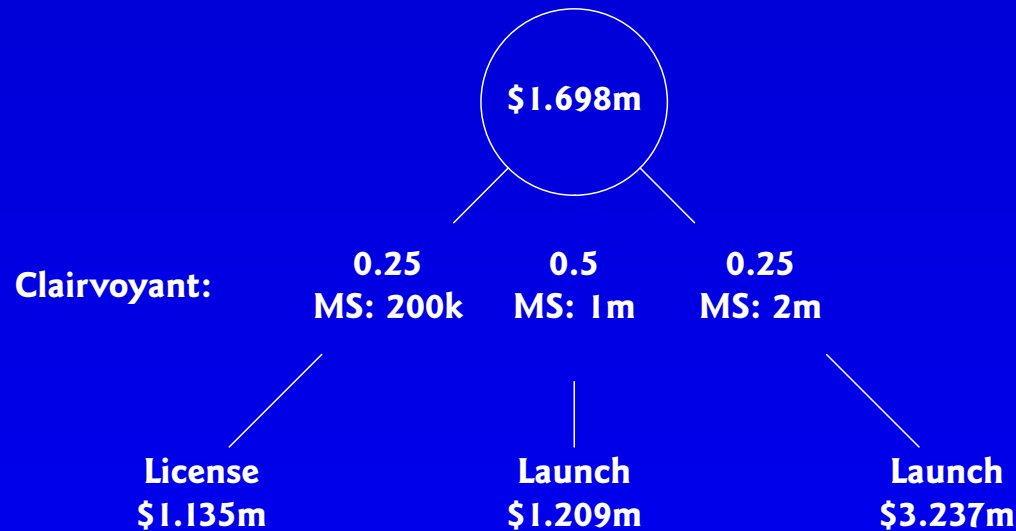
## Plot the Tree with Perfect Information

Tree with perfect information: EMV = \$1,697,866



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$$\therefore \text{VPI} = \$1,697,866 - \$1,310,910 = \$386,956$$

## 2. Probabilistic Sensitivity Analysis — Laura

Laura's belief in the probability  $p$  of Retro's being a Goer = 0.4

At what (cross-over) prior probability  $\hat{p}$  would Laura choose Retro with No Trial?



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What would the expected value of a completely accurate Trial be then?

If Laura's probability that Retro is a Goer is  $p$ , then that must be her best guess as to the probability of the event that the Trial says Retro is a "Goer".

To be consistent, what else could she believe?

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Laura's belief in the probability  $p$  of Retro's being a Goer = 0.4

At what (cross-over) prior probability  $\hat{p}$  would Laura choose Retro with No Trial?

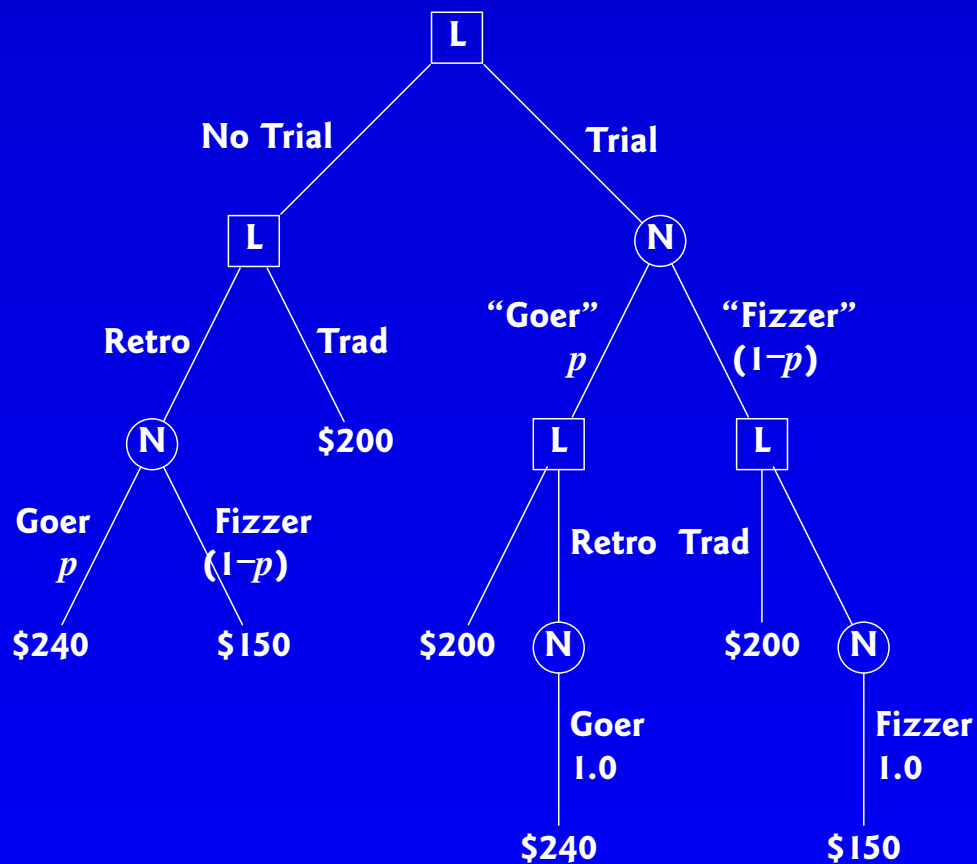
What would the expected value of a completely accurate Trial be then?

If Laura's probability that Retro is a Goer is  $p$ , then that must be her best guess as to the probability of the event that the Trial says Retro is a "Goer".

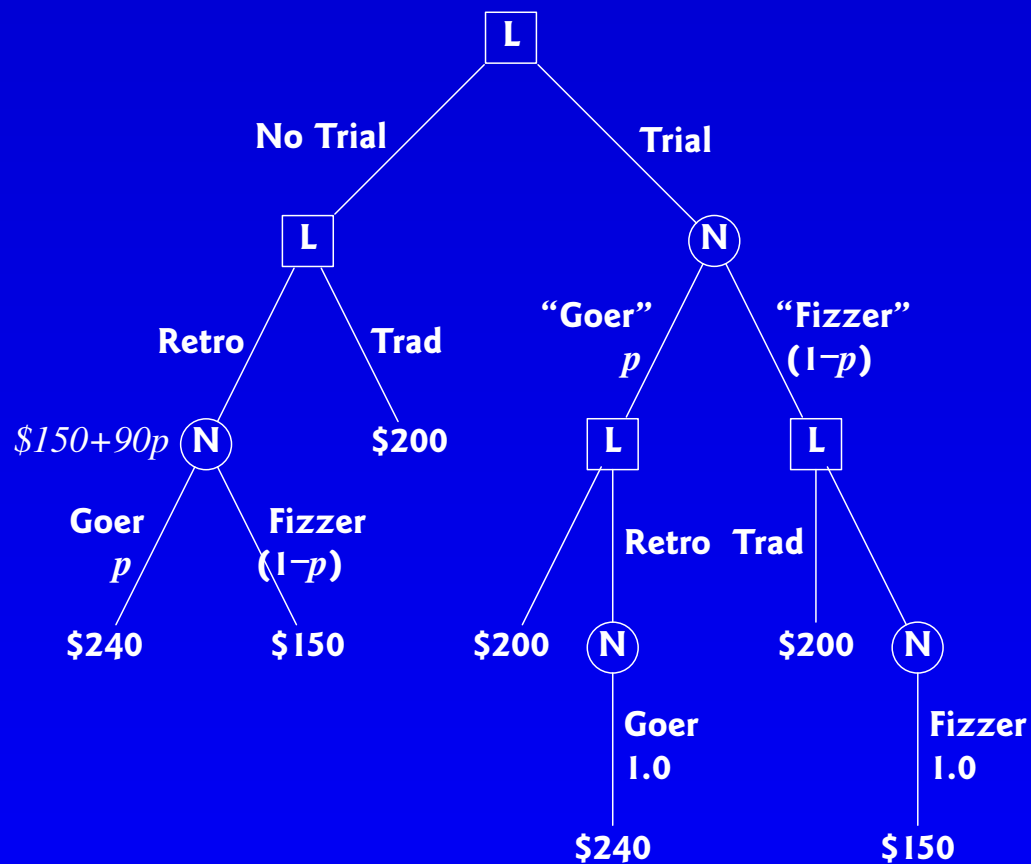
To be consistent, what else could she believe?

If she's uncertain about Retro's success, then she cannot be certain that a 100%-reliable Trial would say that Retro would, or would not, be a "success".

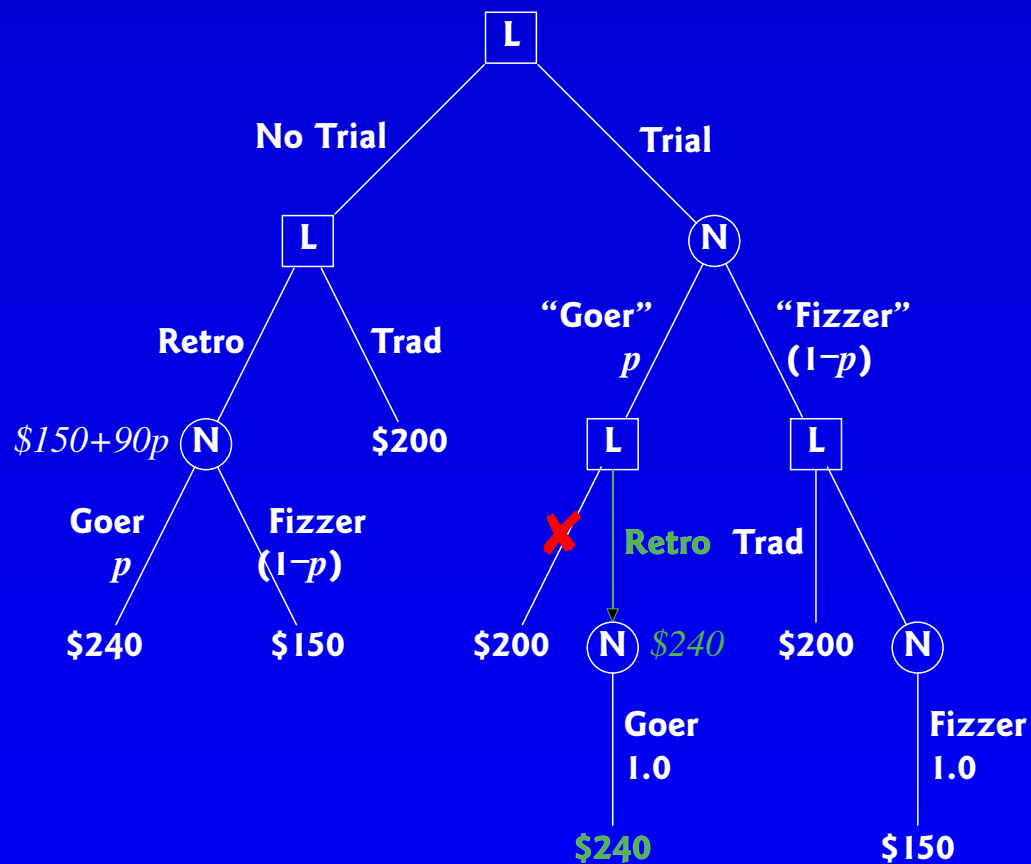
## Laura's Decision Tree with Perfect Information:



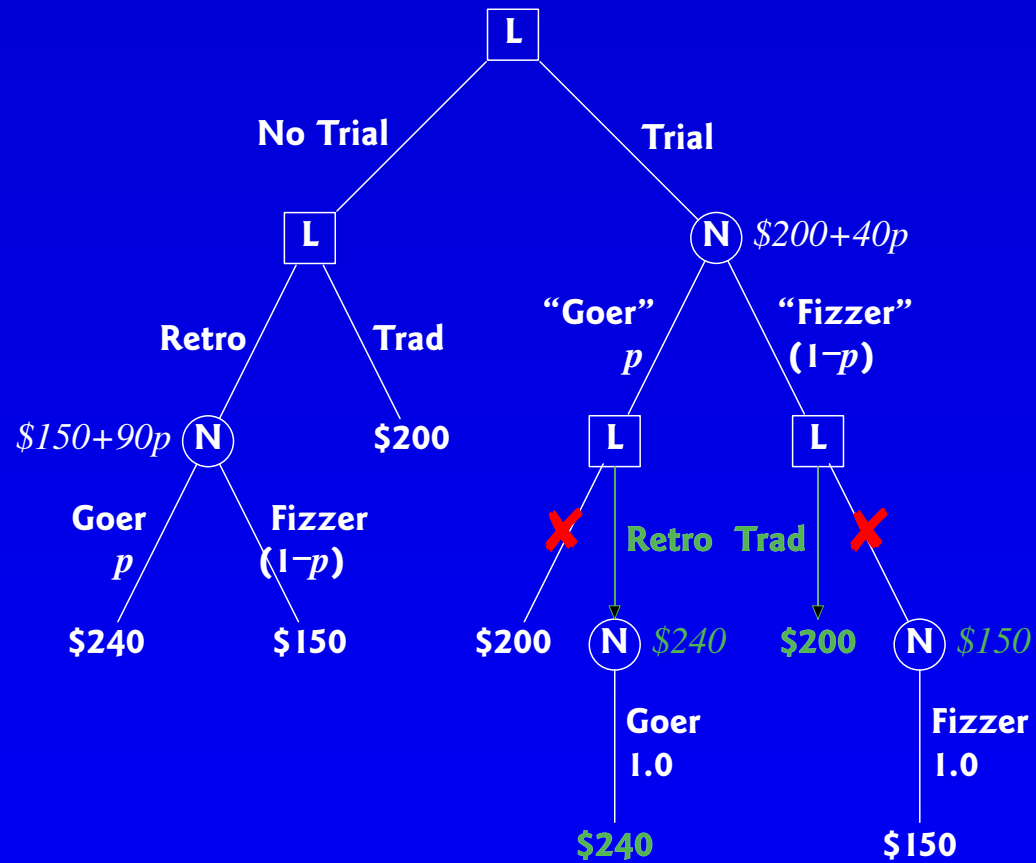
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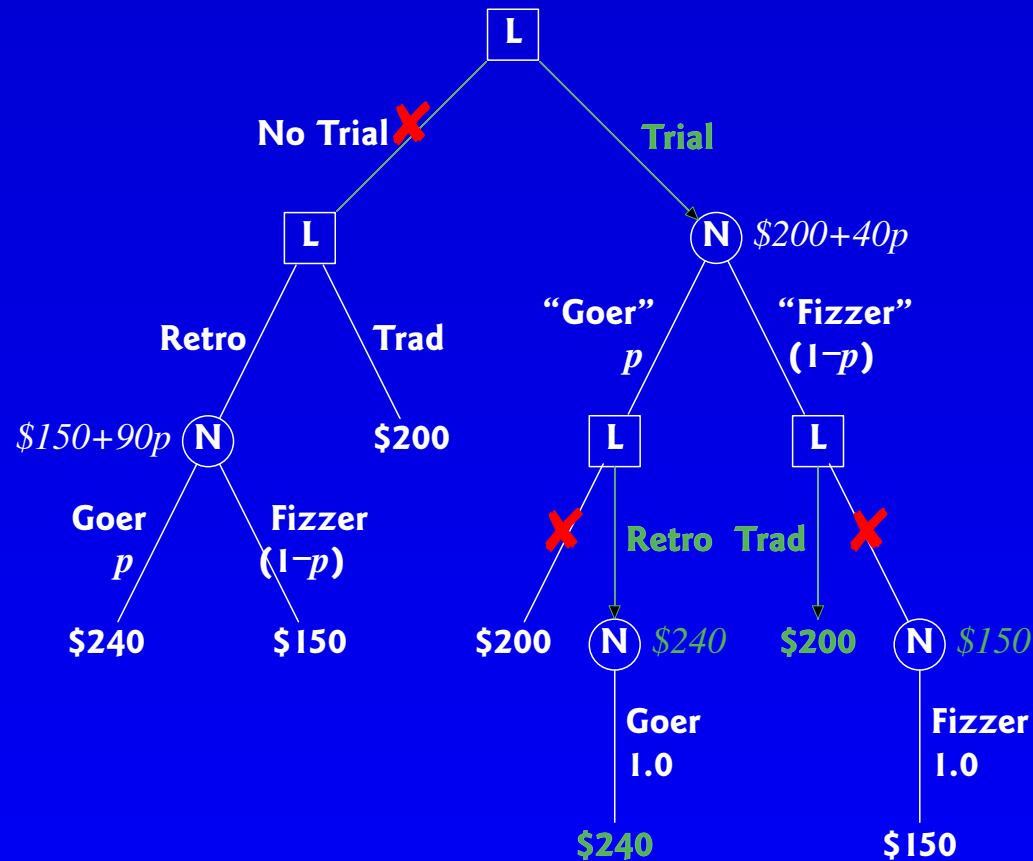
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## Laura's Decision Tree with Perfect Information:



*The Shoe Decision with Perfect Information*  
(i.e. a 100%-reliable test)



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$$\text{\$150} + 90p,$$

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**$\therefore$  the cross-over probability  $\hat{p}$  is 0.556.**

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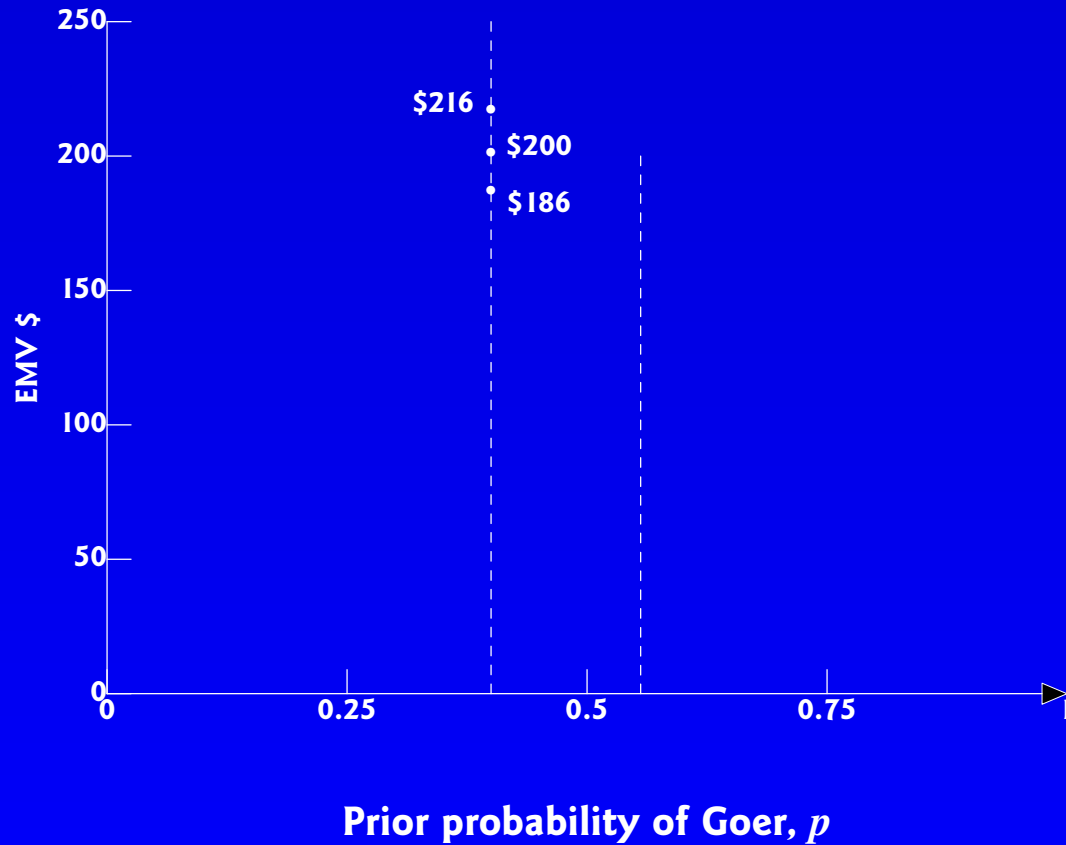
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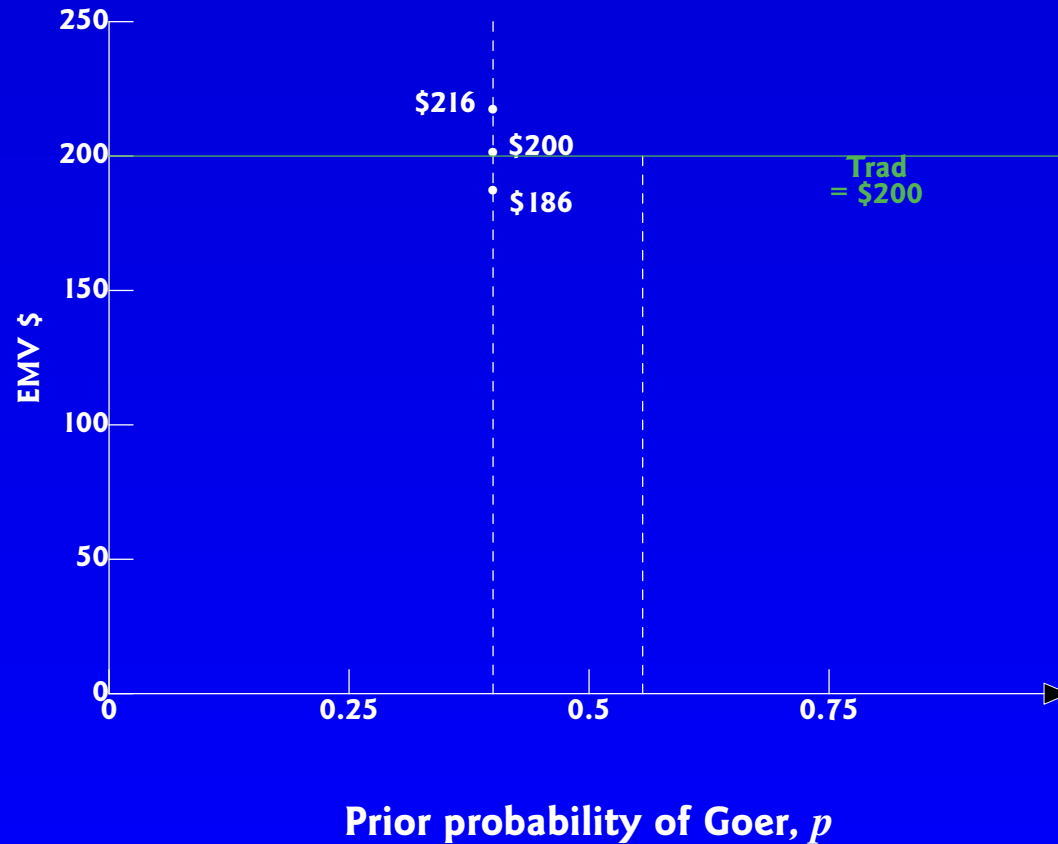
**On the Trial side of the tree, to be consistent the probability of the test indicating that Retro will be a “Goer” must be  $p$ , and a “Fizzer”  $1 - p$ .**

**The expected value of choosing Trial =**  
 **$200 + 40p$ .**

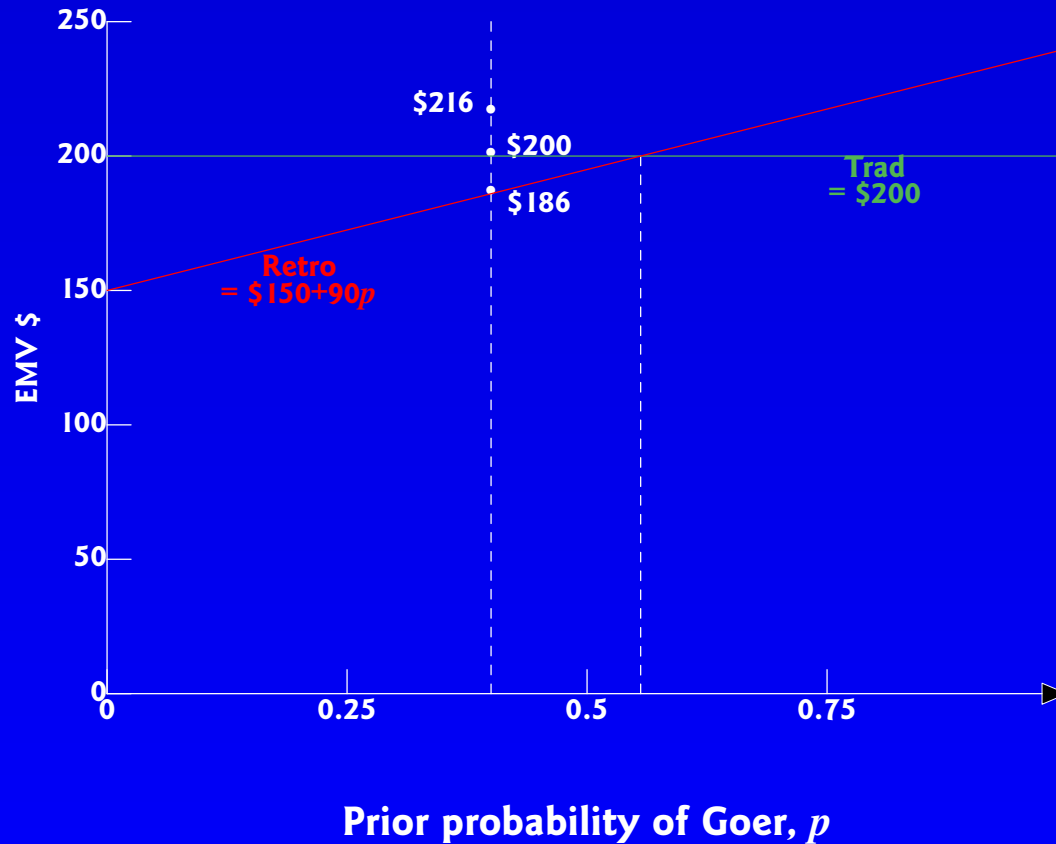
**Plotting these expected values as a function of the prior probability  $p$  of Goer:**



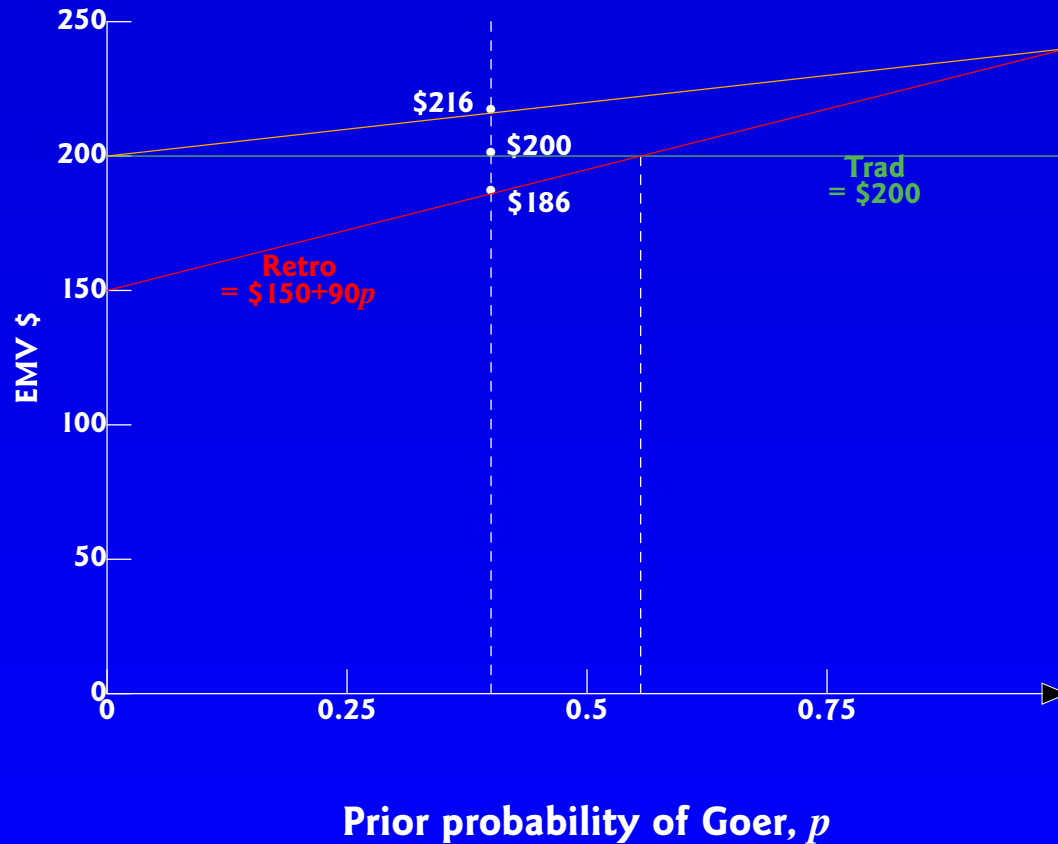
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**Plotting these expected values as a function of the prior probability  $p$  of Goer:**



**Sensitivity Diagram: Expected Value against Prior Probability  $p$  of Goer**

## The Expected Value of Perfect Information (VPI) — Laura

The **orange line**: the expected value with the Trial.

∴ the minimum vertical distance down from the **orange line** to **green** or **red** line is the Expected Value of Perfect Information at any probability of Goer  $p$ .



## The Expected Value of Perfect Information (VPI) — Laura

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At  $p = 0.4$  ➤ the expected value of Retro is \$186,  
➤ the value of Trad is \$200,  
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## The Expected Value of Perfect Information (VPI) — Laura

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At  $p = 0.4$  ➤ the expected value of Retro is \$186,  
➤ the value of Trad is \$200,  
➤ and the value with Trial is \$216,

The Expected VPI = the improvement in expected value with the Trial,

= the difference between the **orange line** and the next highest value, whether **Trad** or **Retro**.

= \$216k - \$200k = \$16,000 when  $p = 0.4$

## **Certainty and the Expected VPI**

**When Laura is certain about the outcome with Retro, the value of reducing uncertainty is zero.**

## Certainty and the Expected VPI

When Laura is certain about the outcome with Retro, the value of reducing uncertainty is zero.

She is certain twice: when she

- knows that Retro is a Goer ( $p = 1.0$ ), or
- is certain that Retro is a Fizzer ( $p = 0.0$ ).

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The cross-over probability  $\hat{p}$  at which choosing Retro has a higher expected value than choosing Trad is 0.556.

Probability  $\hat{p}$  corresponds to the highest expected VPI, and occurs when her decision is most sensitive to the probability  $p$  of Goer.

Highest expected VPI (@  $\hat{p} = 0.556$ ) = \$22,240.

## Which variables are most crucial? — Laura

We have considered the decision's sensitivity to a *single variable*: the probability that Retro is a Goer.

But there might be some uncertainty about the payoffs of Trad and Retro under the two possibilities.

*Which is the most critical variable on which to perform a sensitivity analysis?*

## Which variables are most crucial? — Laura

We have considered the decision's sensitivity to a *single variable*: the probability that Retro is a Goer.

But there might be some uncertainty about the payoffs of Trad and Retro under the two possibilities.

*Which is the most critical variable on which to perform a sensitivity analysis?*

Holding all other variables at their most likely values, one by one each variable be taken from its lowest likely value to its highest, and the effect of this on the optimand (the variable being maximised or minimised) be plotted.

A *Tornado plot*, with the variable with the greatest effect on top and that with the least on the bottom.

Those variables which can push the maximand lowest are the ones that should be subject to a sensitivity analysis.



## **3a. Laura's Case: The Expected Value of Imperfect Information**

**But what if the Trial is *not* 100%-reliable?**

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We'd like to know the maximum that risk-neutral Laura should pay for the test.

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## 3a. Laura's Case: The Expected Value of Imperfect Information

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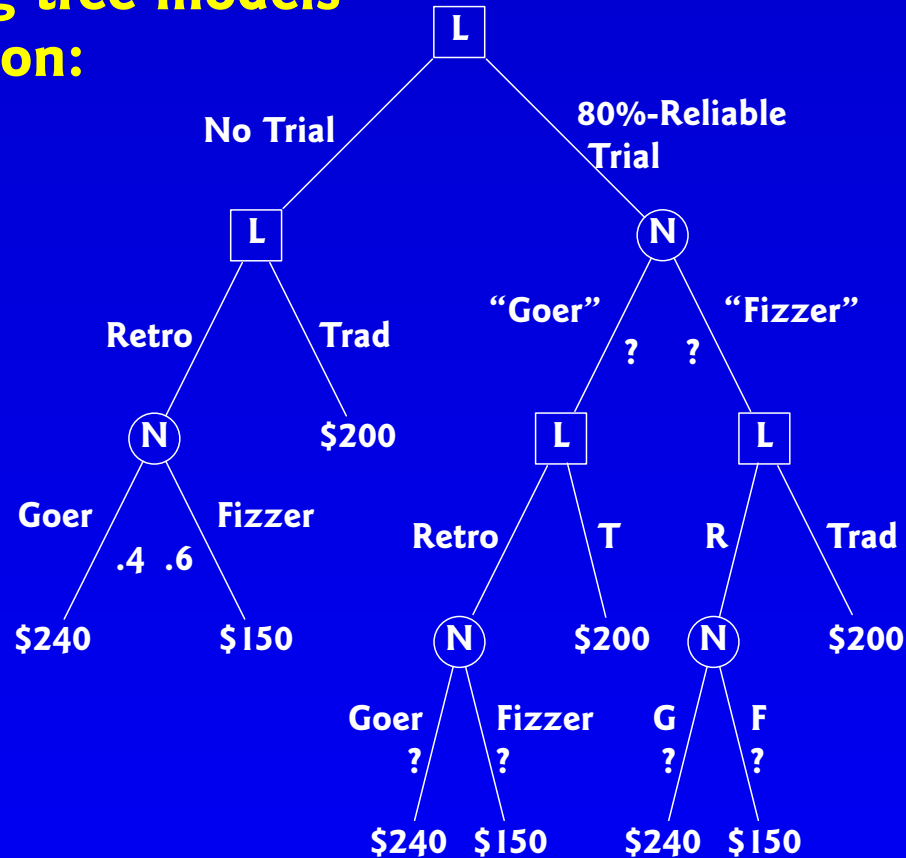
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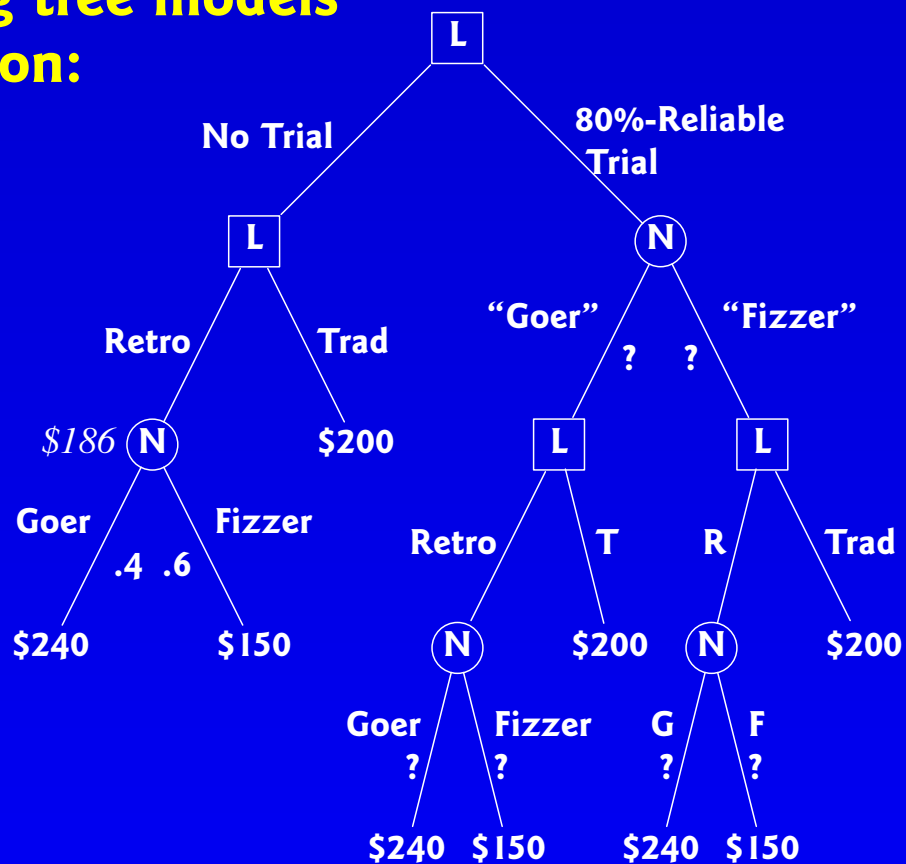
- Laura's probability that the unreliable test will indicate "Goer",
- and the *Conditional Probability* of Retro being a Goer if the test indicates "Goer".

(With a 100%-reliable test, the former probability is 0.4 and the latter is 1.0.)

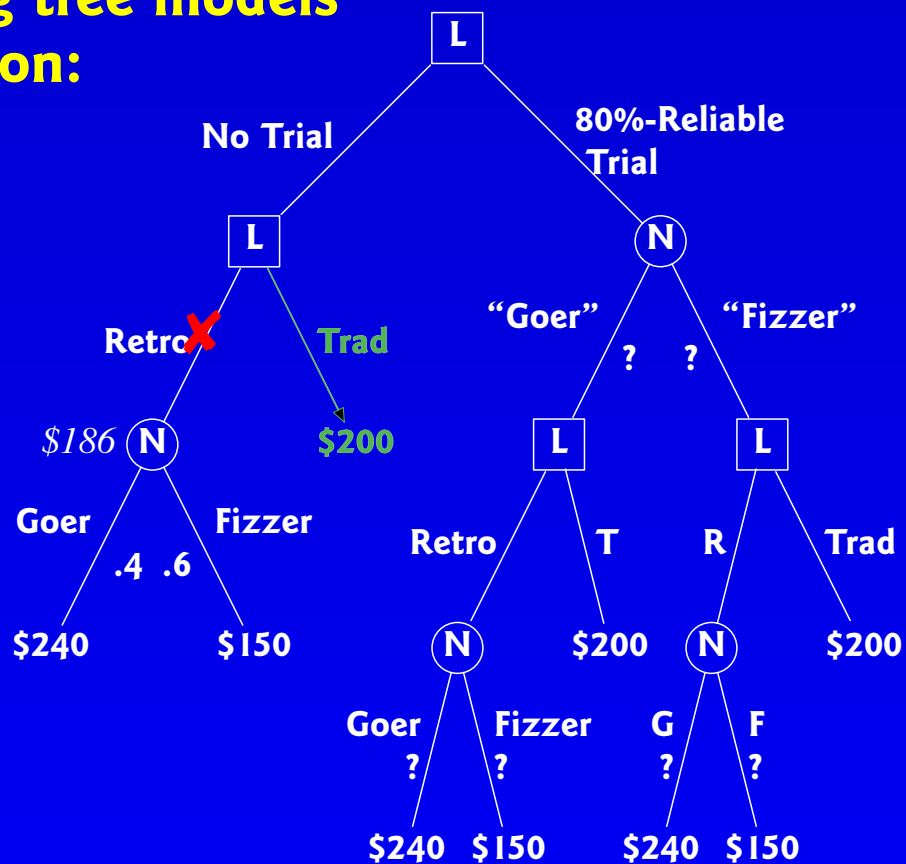
## The following tree models Laura's decision:



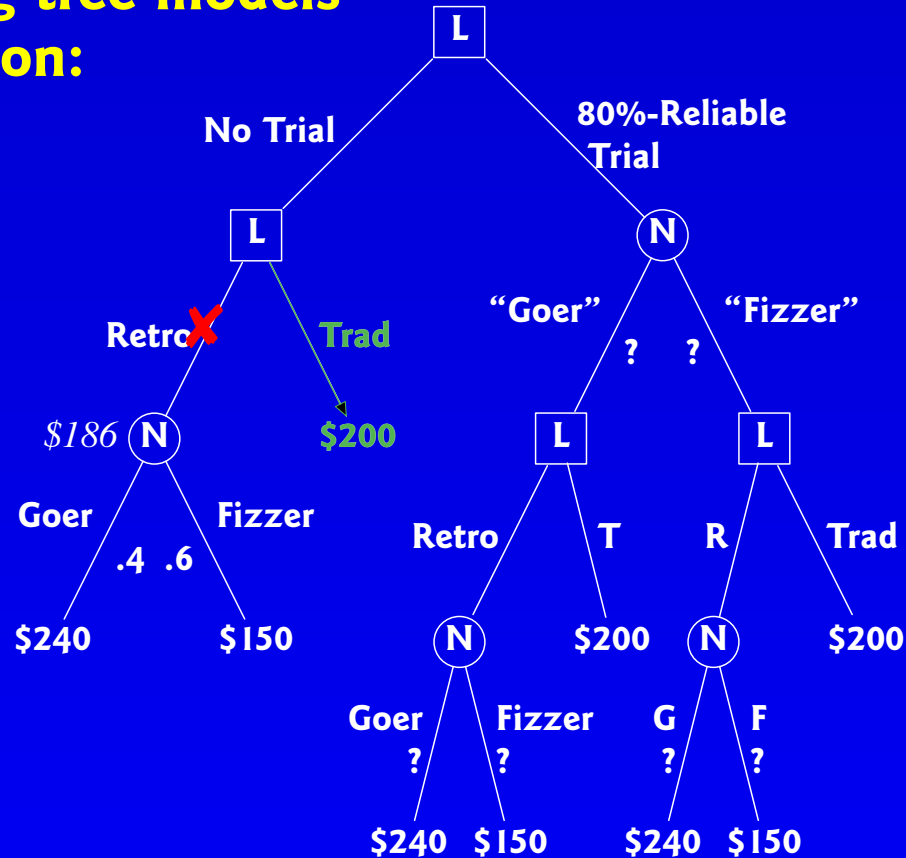
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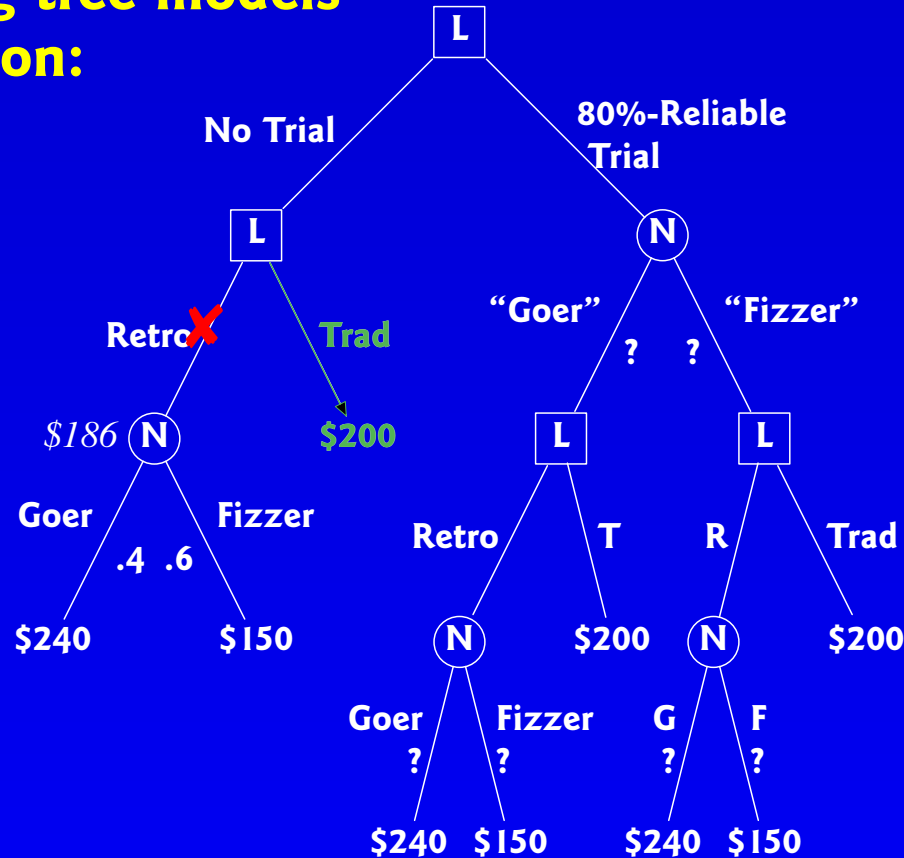
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**What probabilities do the question marks represent?**



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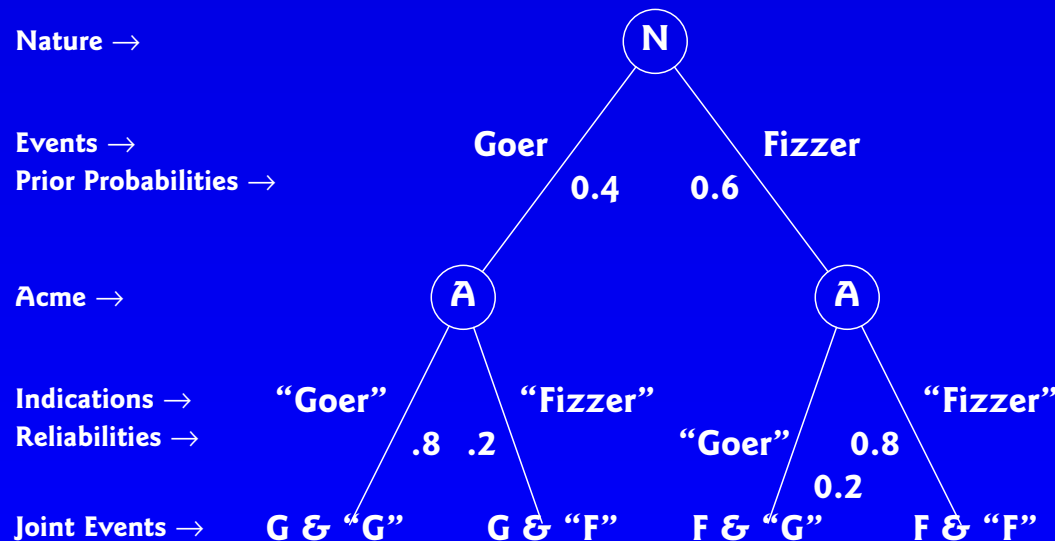
**What probabilities do the question marks represent?**

To answer this question we need to flip a smaller probability tree to calculate the *conditional probabilities*.

## Laura and the Shoe Decision (cont.)

Laura decides to employ the Acme Marketing Company.  
Unfortunately, they are only 80% reliable:

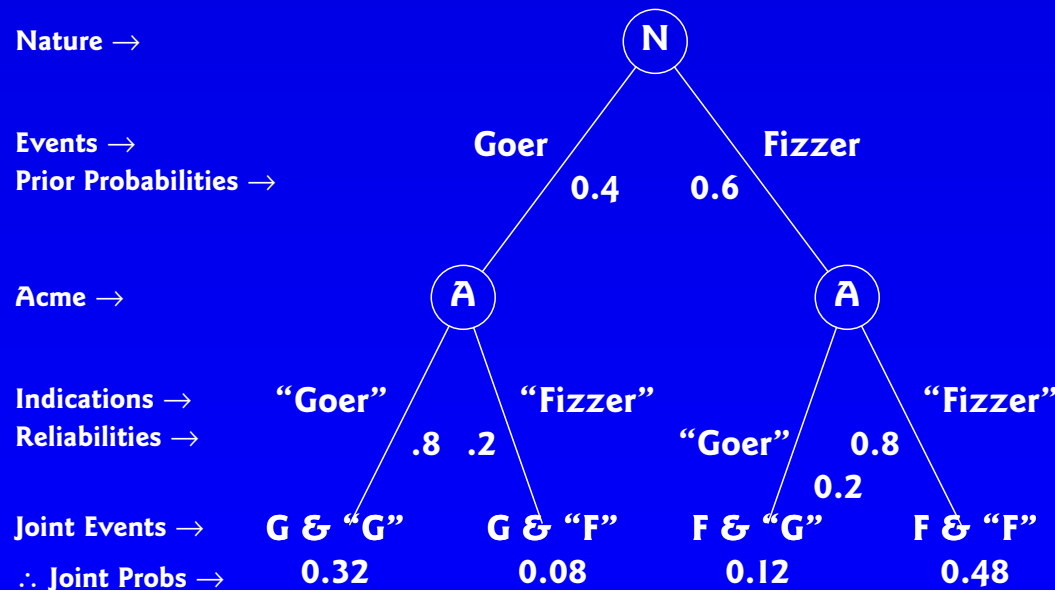
- given that Retro is a Fizzer, Acme will say “Goer” 20% of the time (a false positive), and
- given that Retro is a Goer, Acme will say “Fizzer” 20% of the time (a false negative).



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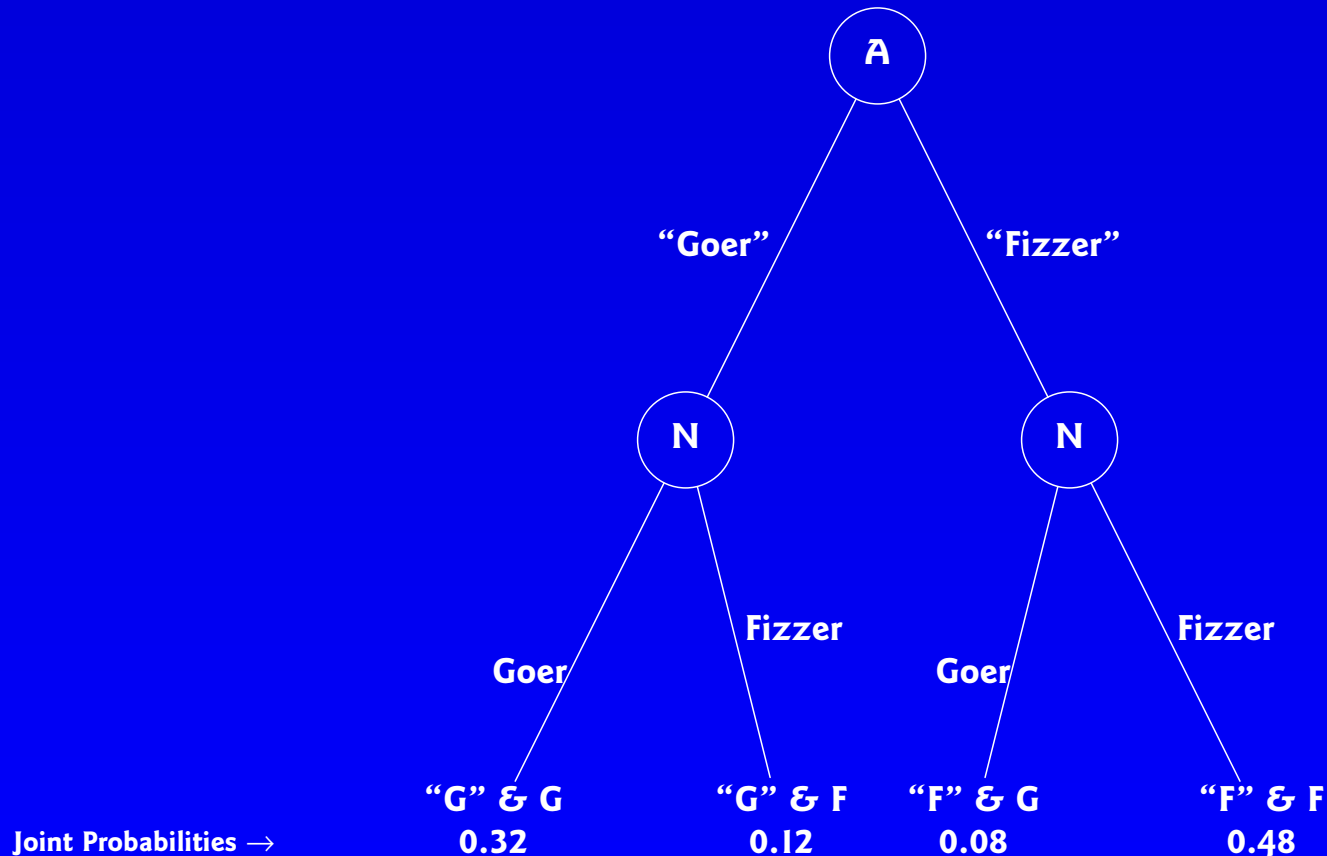
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*Market Testing – a probability tree*

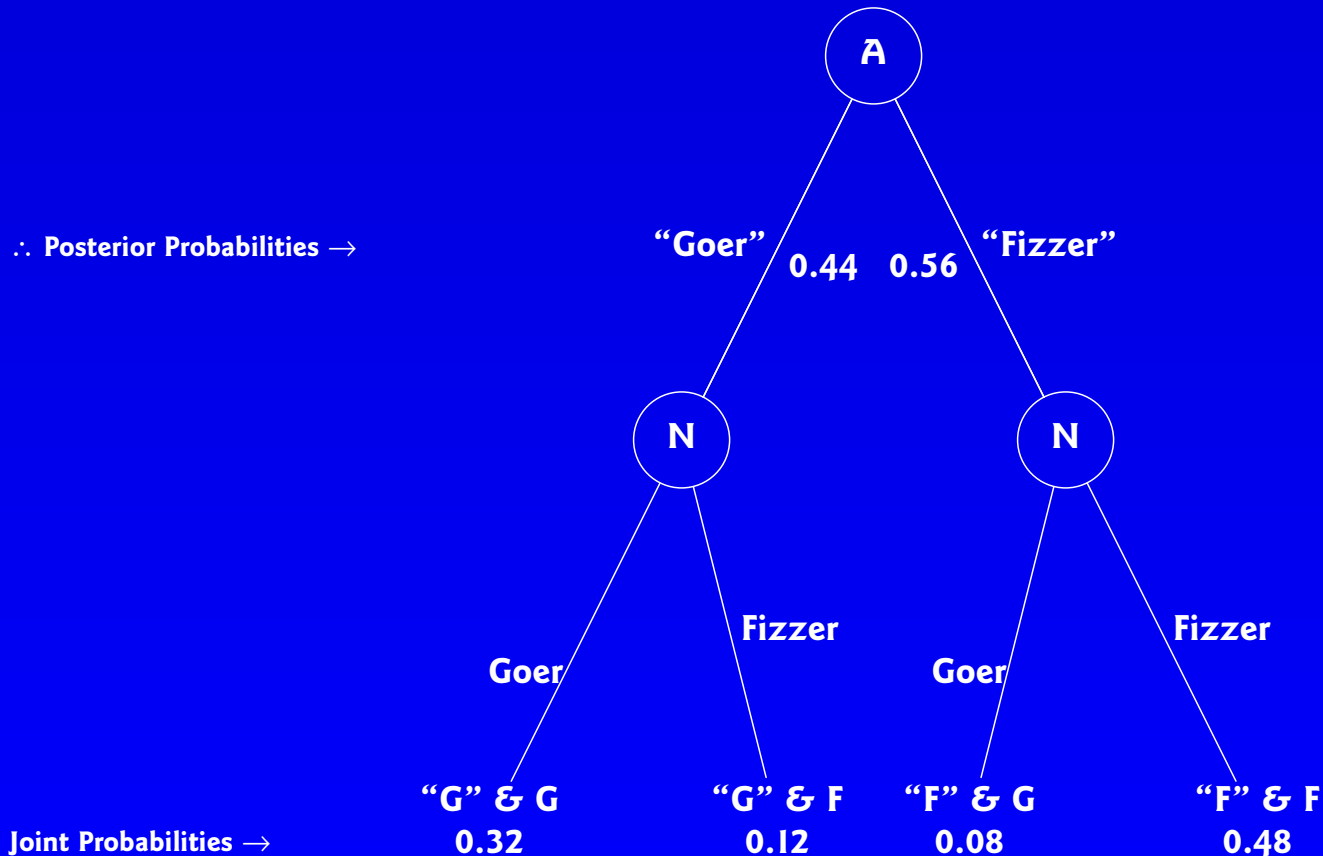
***The flipped probability tree: the Test result comes first.***

**“Flip” the above tree, to determine the chance of Retro being a Fizzer, given that the unreliable test indicates “Fizzer”, and so on.**



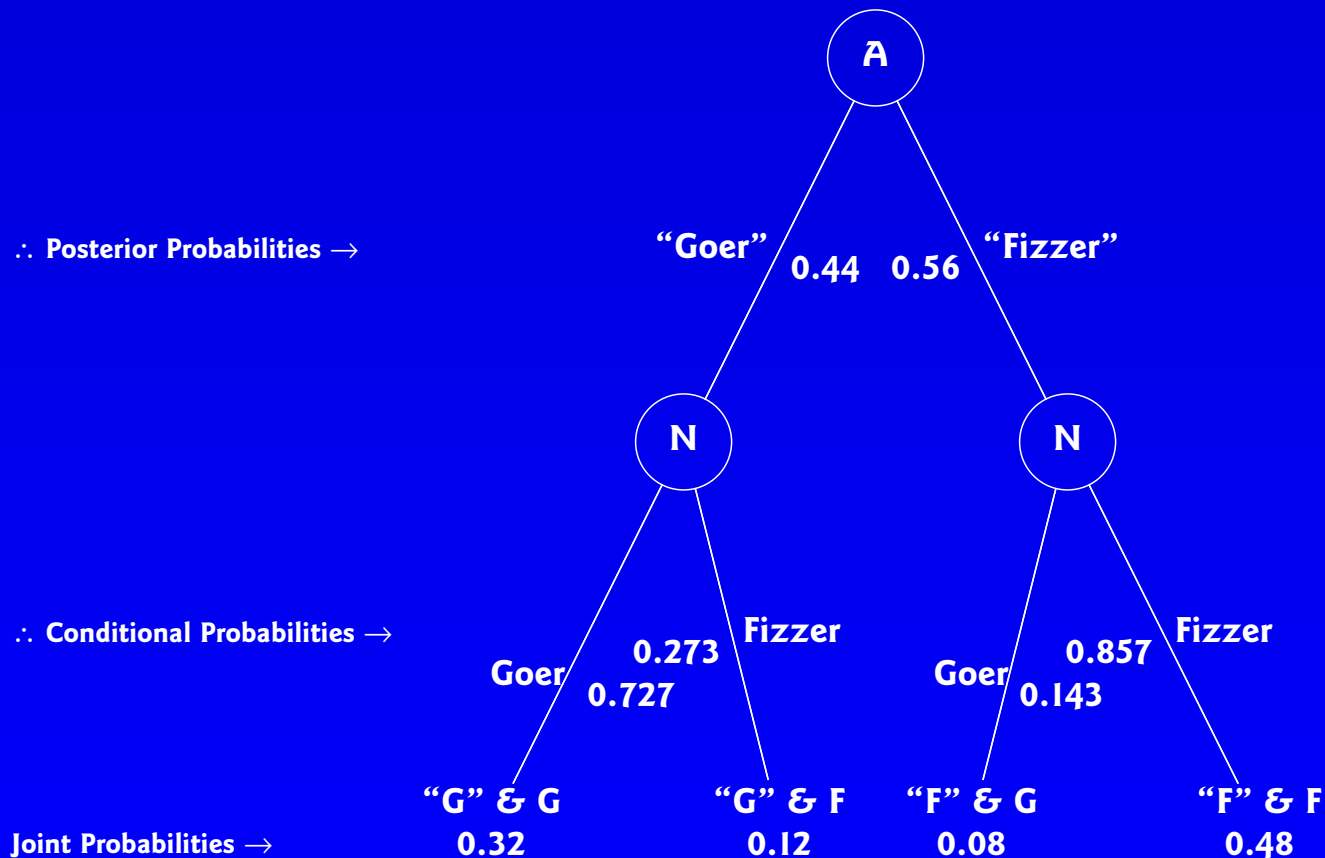
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## ***Market Testing***

**From the flipped probability tree:**



## Market Testing

From the flipped probability tree:

➤ the conditional probability of Retro being a Goer given that Acme says it's a "Fizzer" is  $\frac{0.08}{0.08+0.48} = \frac{1}{7} = 0.143$ ;

➤



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- based upon Laura's prior belief that Retro is a Goer with a probability of 40%, she expects that with probability  $0.32 + 0.12 = 0.44$  Acme will say "Goer".

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We can now replace the question marks in the decision tree above, which allows us to solve the decision problem, with expected values.

(Tree flipping gives the same results for *conditional probability* as using Bayes' Theorem.)

## ***Expected Value of Imperfect Information – Laura***

From the sensitivity graph above,  $p = 0.44$  is less than the the cross-over probability  $\hat{p}$ :

$\therefore$

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∴ If Acme says “Goer”, which Laura expects will happen with probability of 0.44, then she will choose Retro.

∓ Her expected payoff is  $(150 + 90p) \times 1000 = \$215,430$ , with the conditional probability that Retro is a Goer, given that Acme said “Goer”,  $p = \frac{8}{11} = 0.727$ .

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⊗

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- Her expected payoff without this information is \$200k, since she chooses Trad.
- ∴ The expected value to Laura of 80%-reliable information is  $\$206,789 - \$200,000 = \$6,789$ .

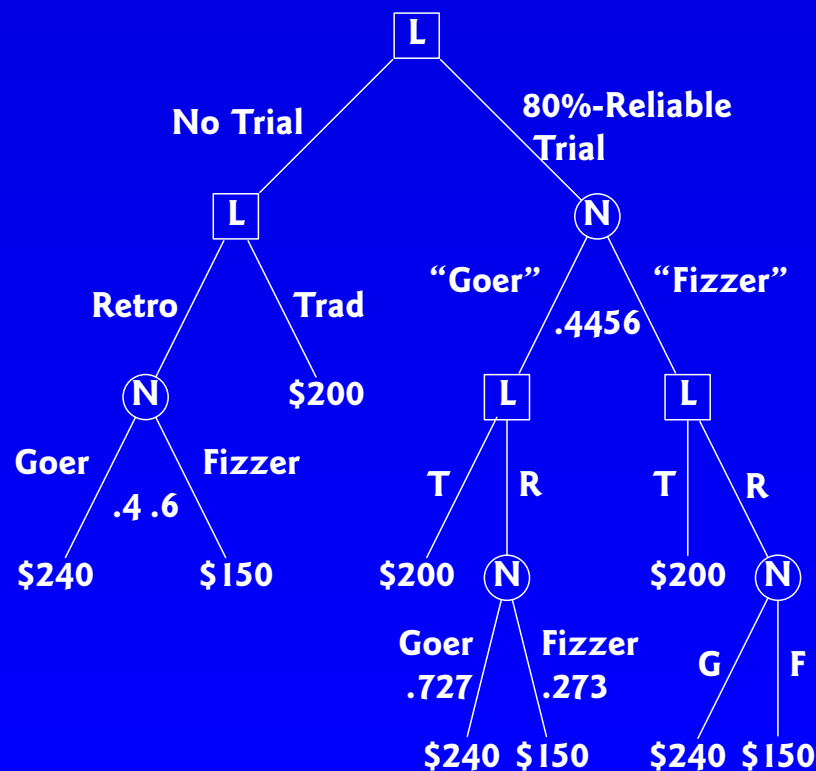
***After tree-flipping:***

- **Laura's conditional probability that Retro is a Goer, given that Acme has states that it will be a "Goer," is  $\frac{8}{11}$  or 0.727.**
-

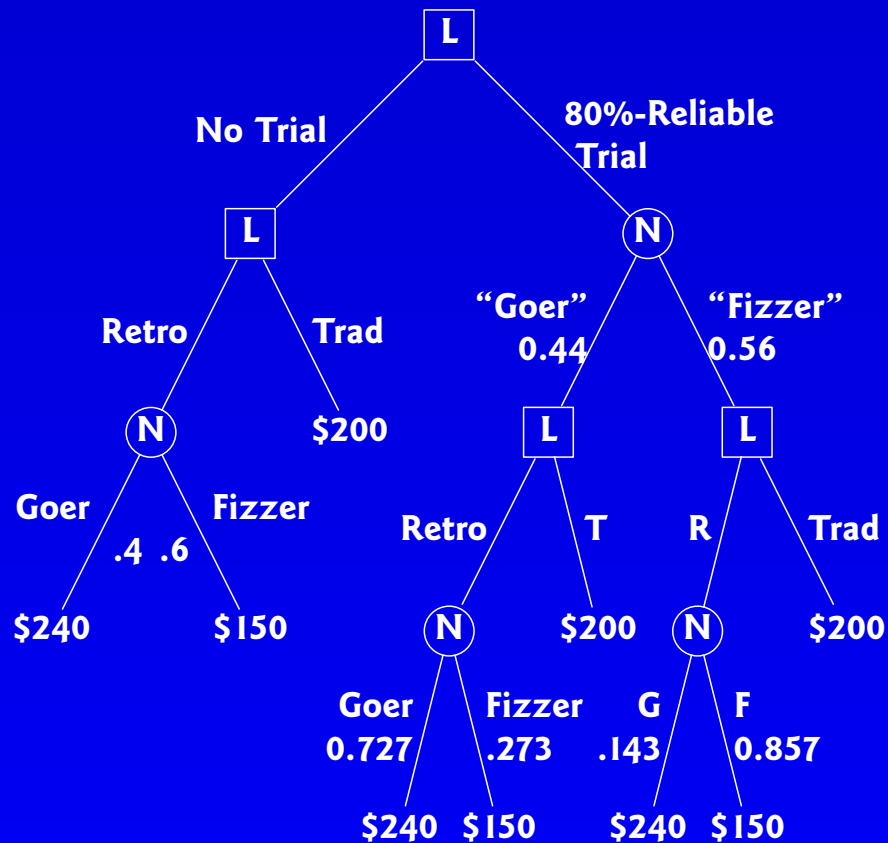
## After tree-flipping:

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- Laura's probability that Acme will state that Retro is a "Goer" is 0.44.

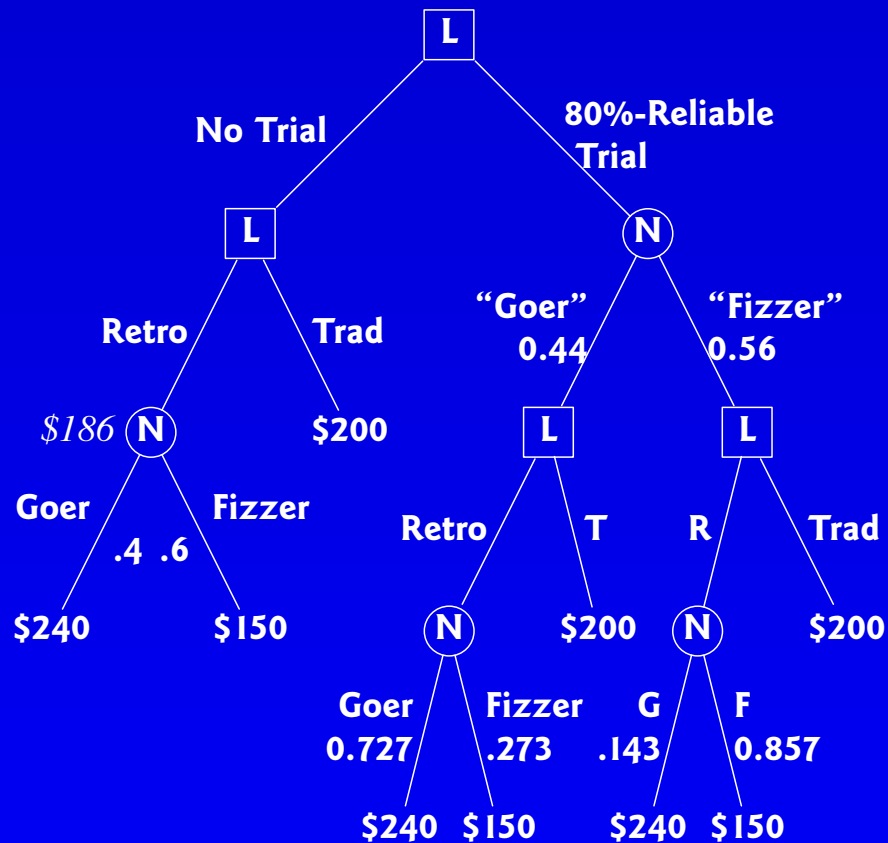
## Laura's full decision tree:



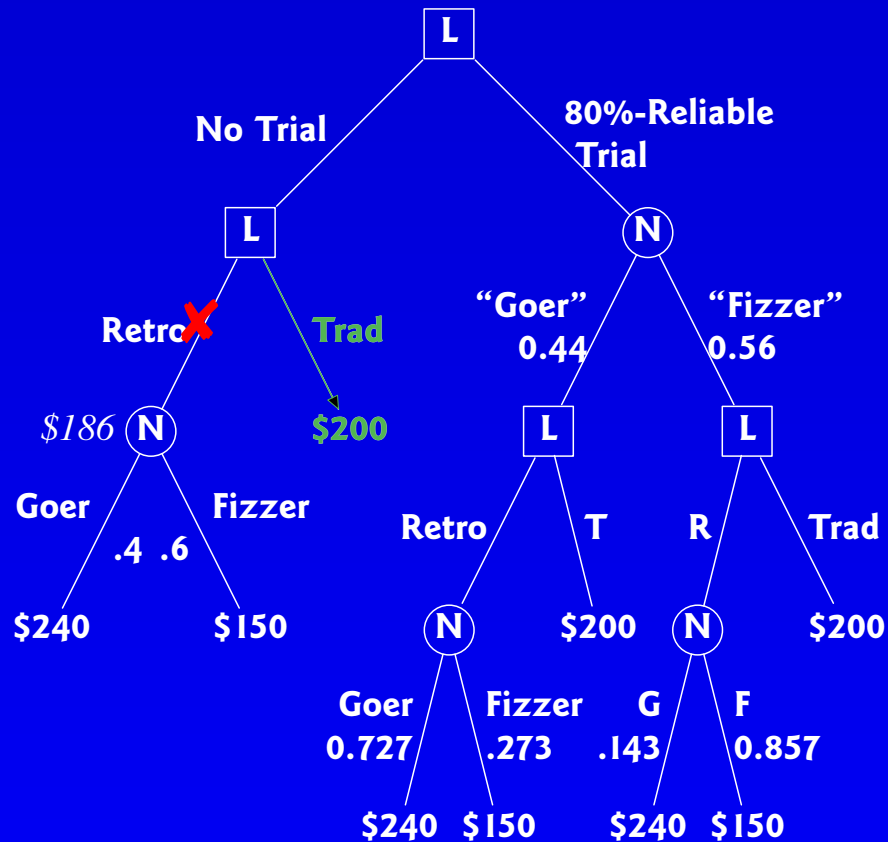
So the following tree models Laura's decision:



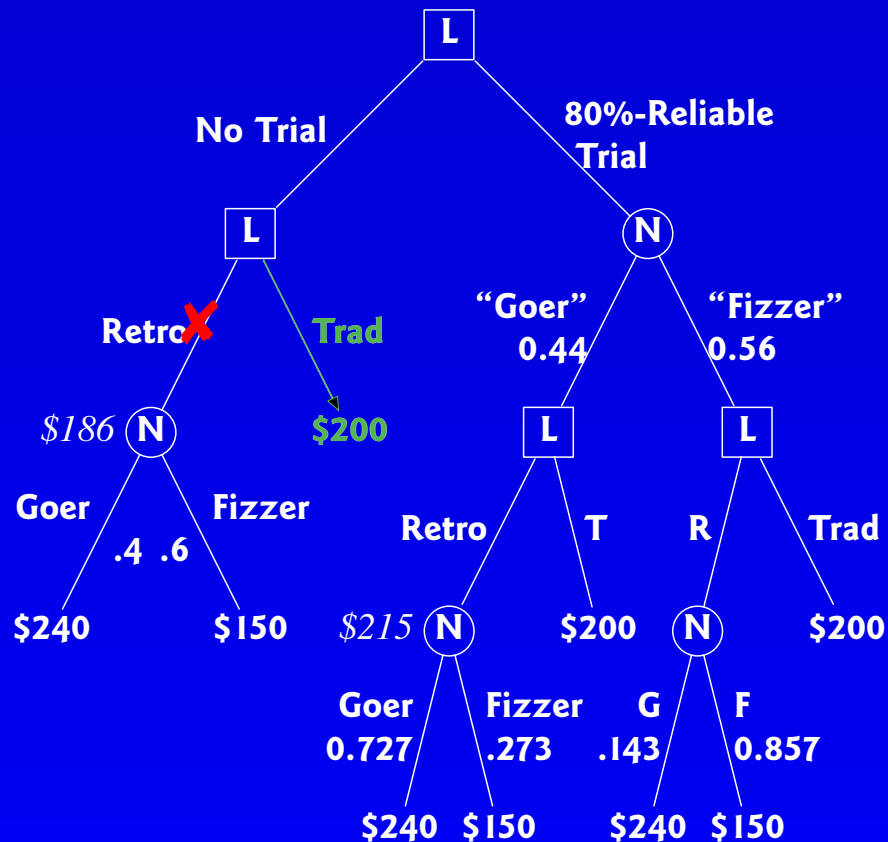
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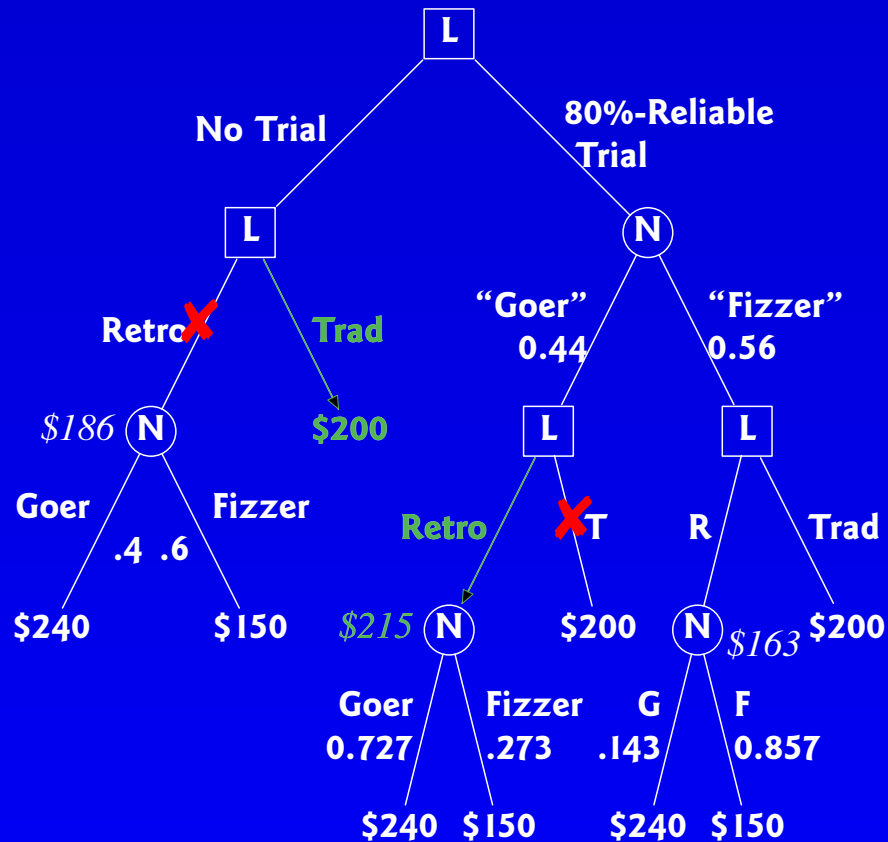


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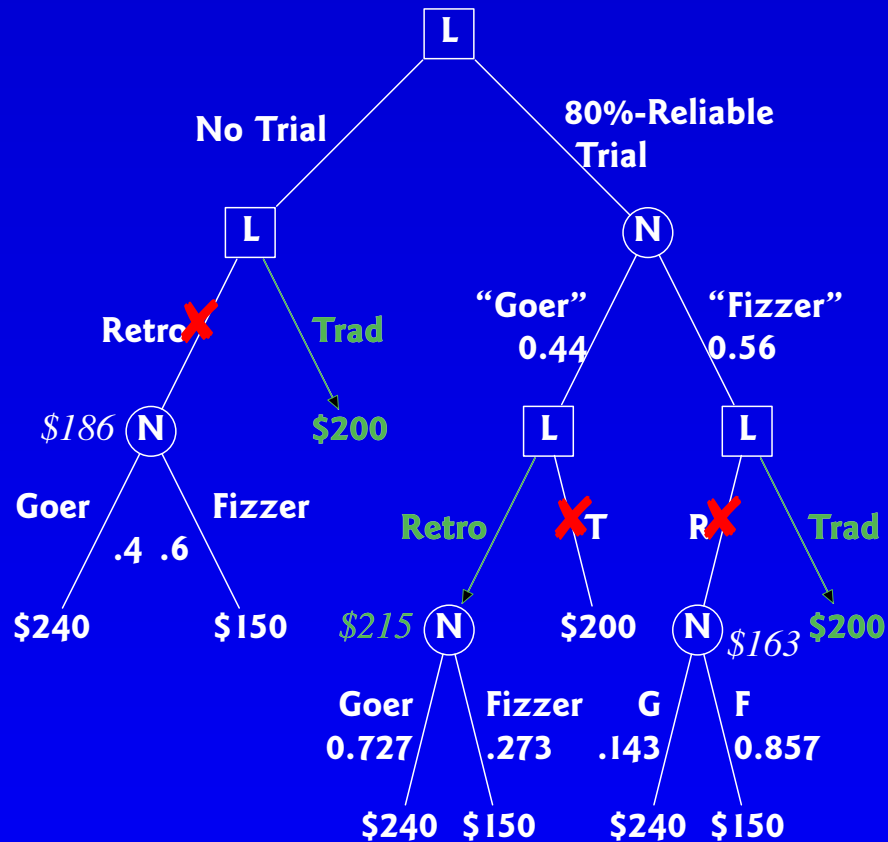




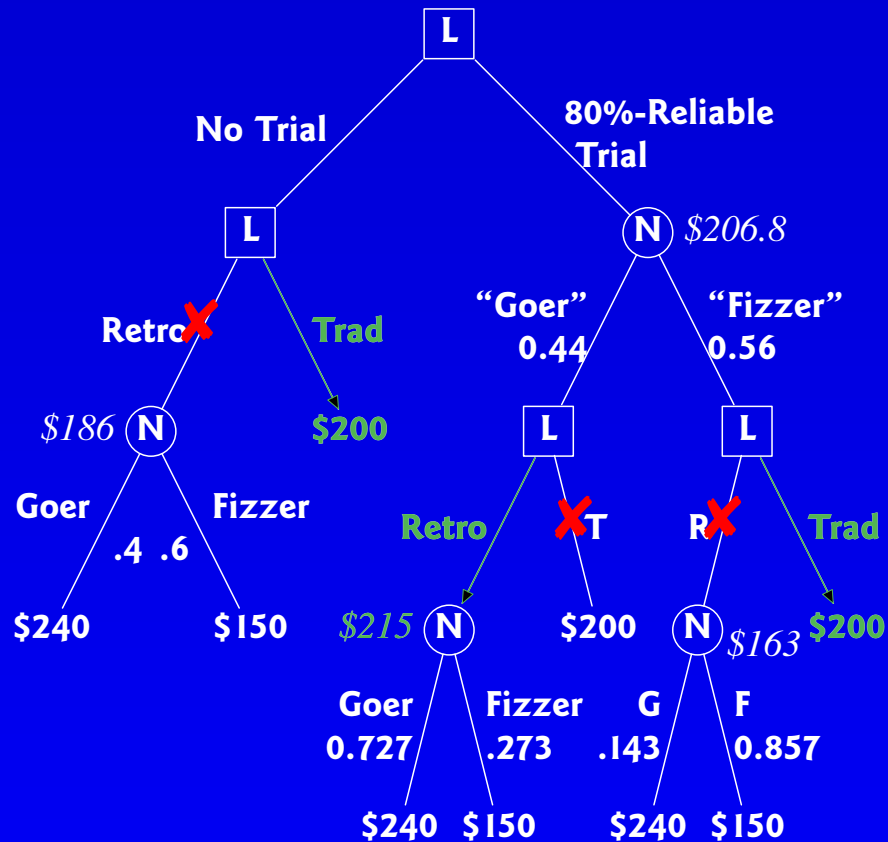
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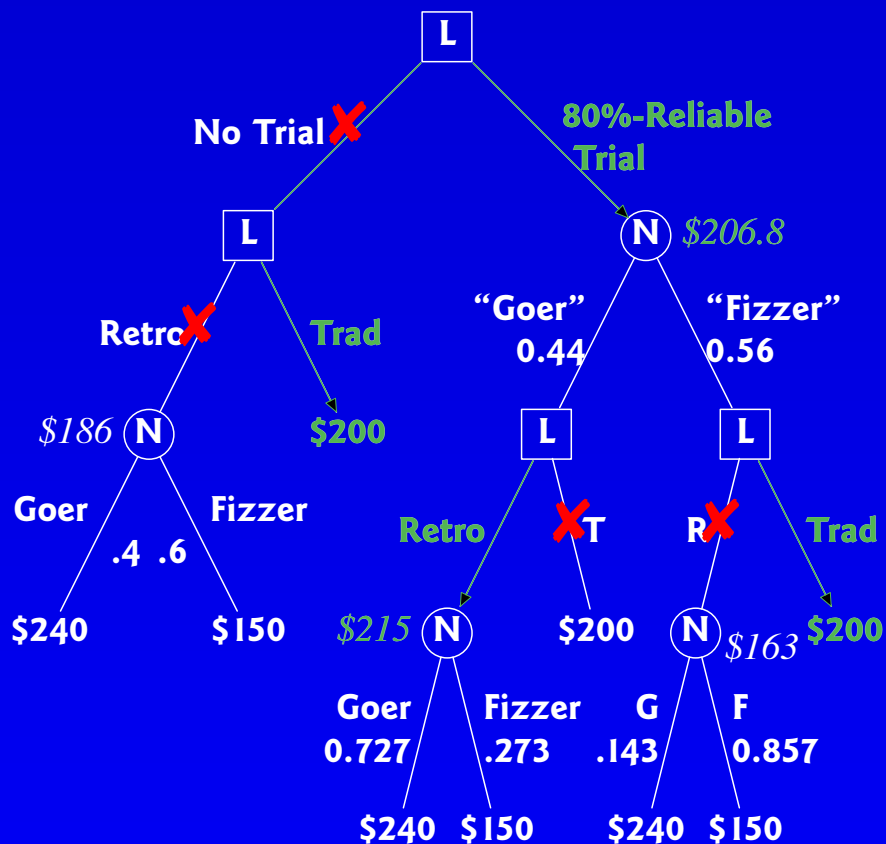
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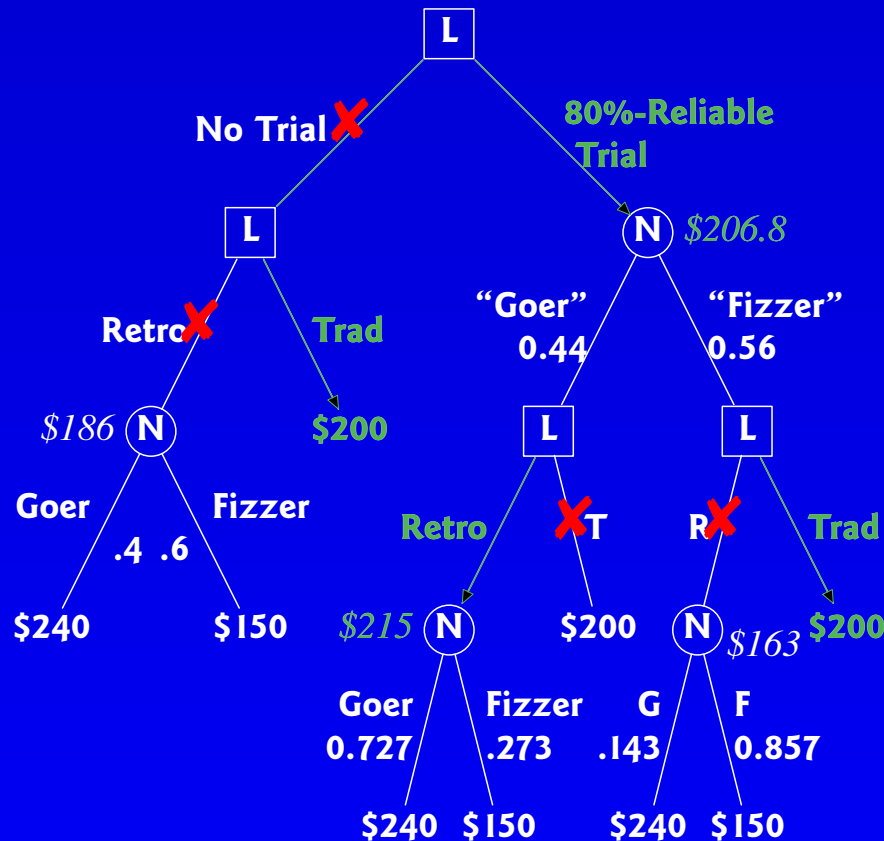
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EV with the 80%-test = \$206,790

EV without the test = \$200k

∴ EV of the 80%-reliable information = \$6,790

### **3b. Value of Imperfect Information of Glix:**

**We know the Value of Perfect Information is \$386,956.**

**What if we could conduct a market survey for \$300,000?**

**Would it be worth the investment?**

**First, we must create a new influence diagram.**

**Notice that the Survey is influenced by Market Size rather than vice versa. This is to preserve the state of nature.**

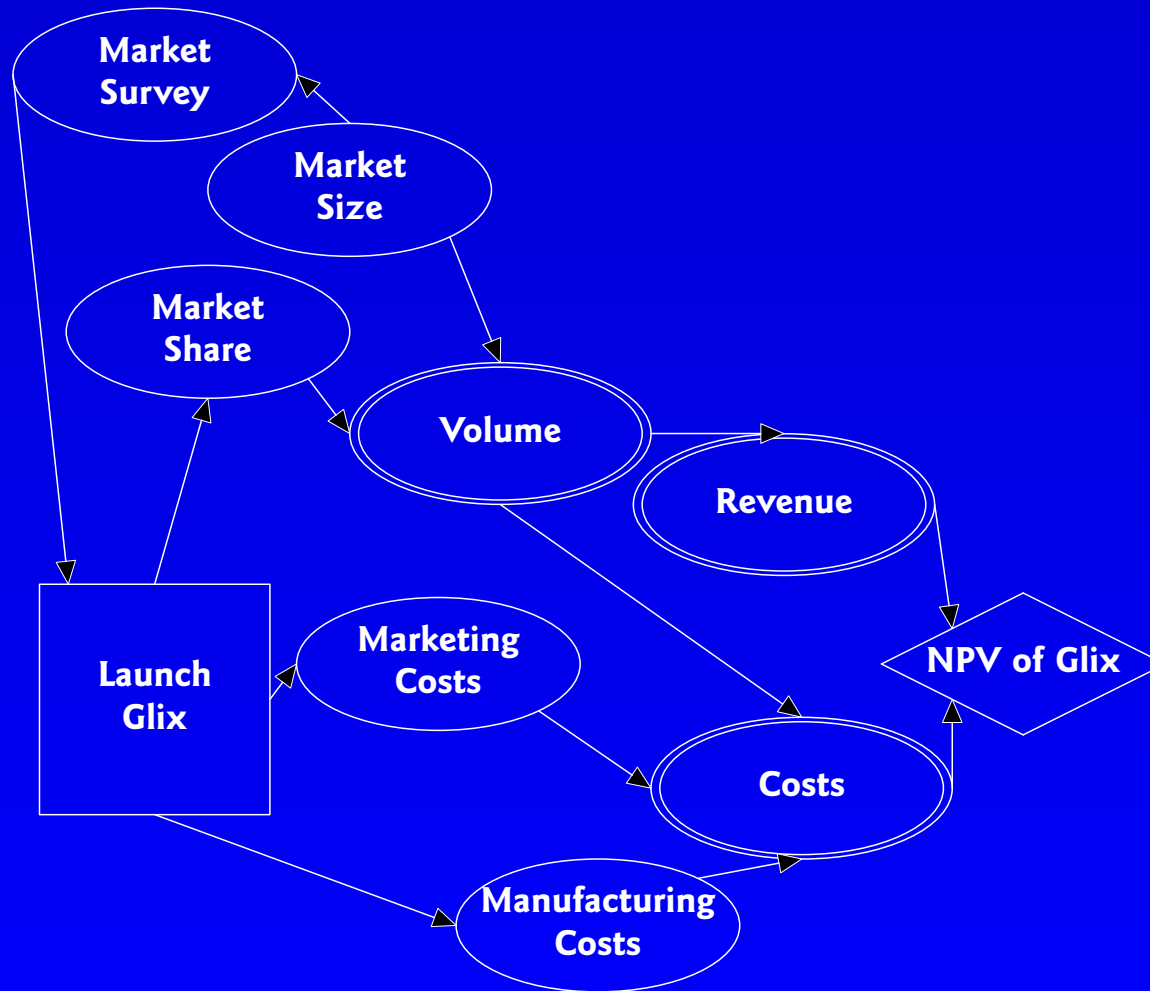
**Recall: there are three possibilities for Market Size:**

**Low = 200,000**

**Medium = 1,000,000**

**High = 2,000,000**

## Glix's Launch? – Survey Influence Diagram.



## **Glix: The Value of Imperfect Information — Reversing or Flipping the Tree**

**In order to calculate the value of imperfect information, we must flip the tree, to obtain the *conditional probabilities*,**



## Glix: The Value of Imperfect Information — Reversing or Flipping the Tree

In order to calculate the value of imperfect information, we must flip the tree, to obtain the *conditional probabilities*, such as  $\text{Prob}(MS = 200k \mid \text{the survey indicates "L"})$ , which is correct.

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*Assume:* if the survey is incorrect, then the two wrong indications are equally likely.

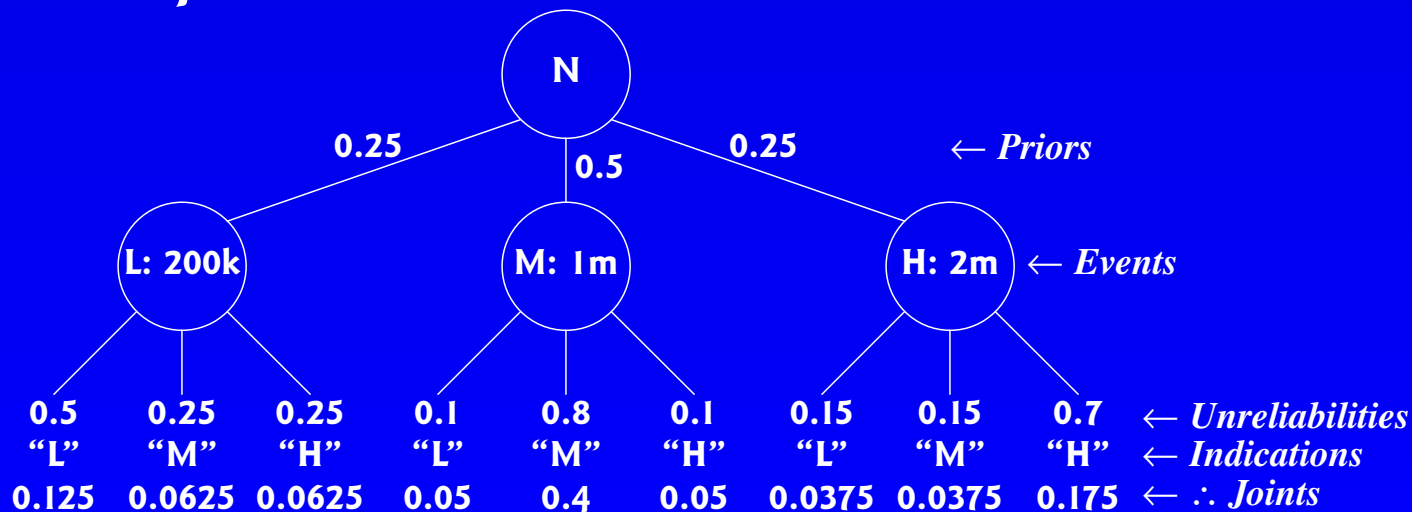
*Prior tree: Glix*

## Glix: The Value of Imperfect Information – Reversing or Flipping the Tree

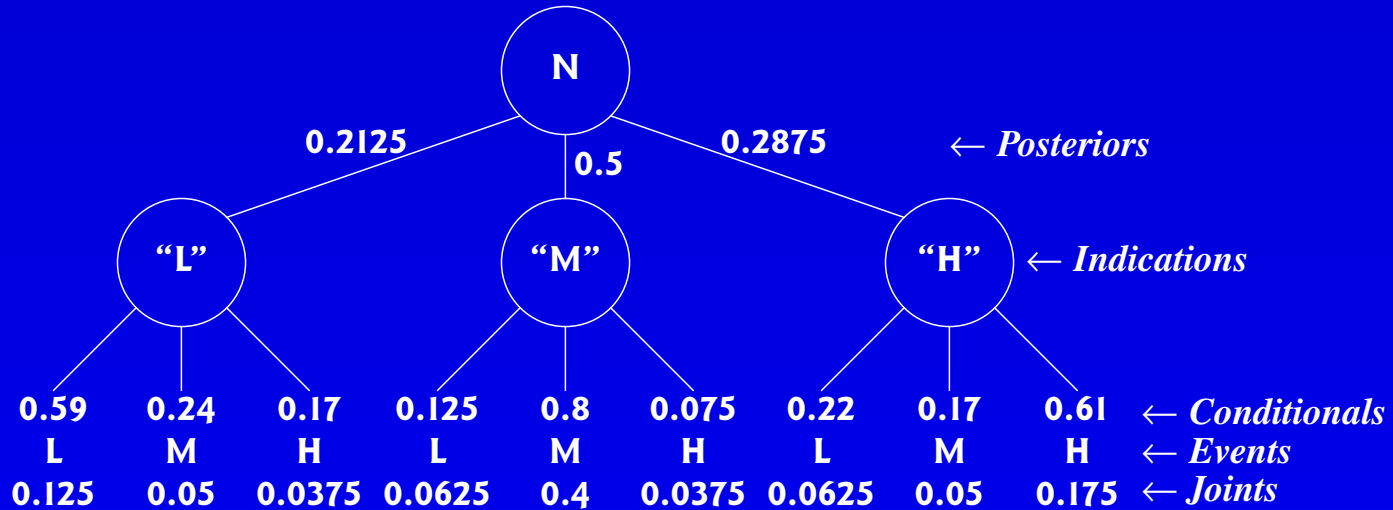
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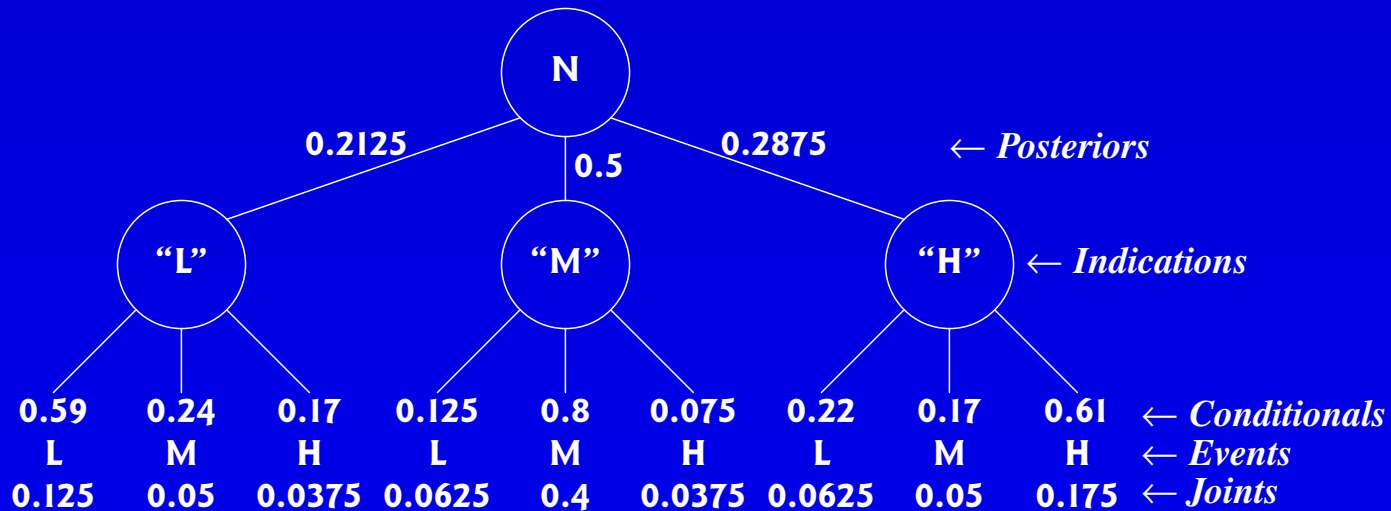
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## The Posterior tree (flipped)

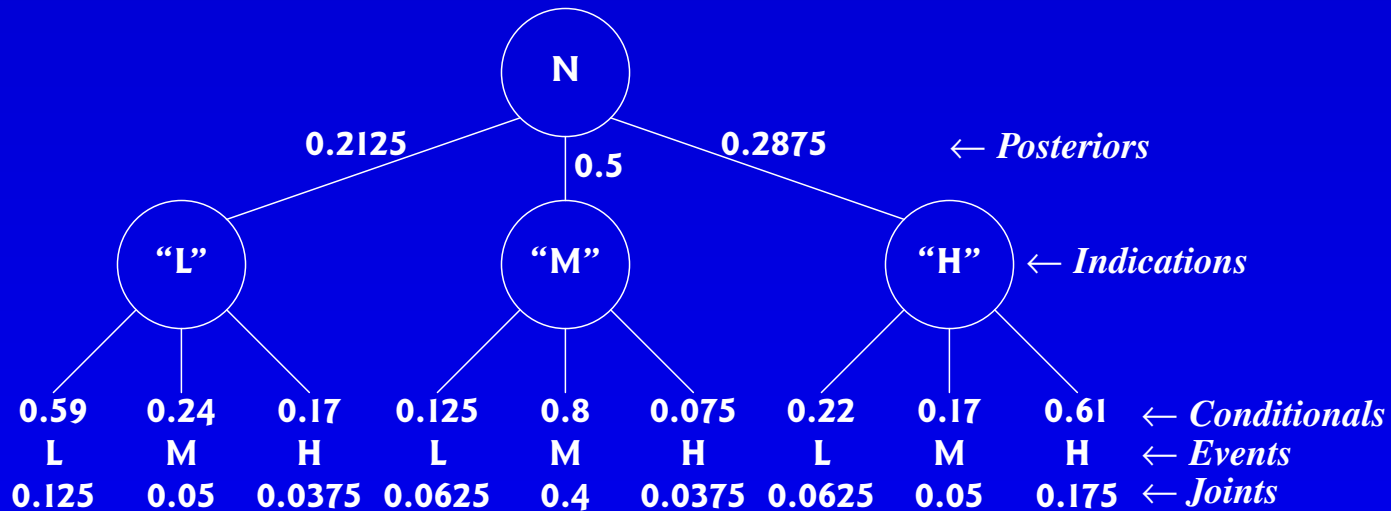


## The Posterior tree (flipped)



The posterior tree indicates that the Survey assessment is correct 70% of the time (the joint probabilities sum to 0.70):  
 $p(L \& \text{“L”}) + p(M \& \text{“M”}) + p(H \& \text{“H”}) = 0.125 + 0.4 + 0.175 = 0.70$

## The Posterior tree (flipped)



The posterior tree indicates that the Survey assessment is correct 70% of the time (the joint probabilities sum to 0.70):  
 $p(L \& \text{"L"}) + p(M \& \text{"M"}) + p(H \& \text{"H"}) = 0.125 + 0.4 + 0.175 = 0.70$

We seek the **Conditional Probabilities**: given an Indication, how likely is the Event?

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## **∴ The Value of the Survey for Glix:**

**The unconditional EMV with the Survey**

$$\begin{aligned} &= 0.2125 \times \$1.135\text{m} + 0.5 \times \$1.1584\text{m} + 0.2875 \times \\ &\$2.089\text{m} = \$1.420\text{m} \end{aligned}$$

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**∴ The value of the Survey = \$1.420m – \$1.310m = \$110k,  
which is the maximum that should be paid for the Survey.**

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The value of gathering additional information can be calculated *before* gathering the information.

Remember to consider the feasibility and reliability of gathering additional information. Just because you can calculate the value *does not mean* that you can either find the information or obtain it.

(Reading: Clemen, Reading 18 in the Package)