

# CONTINUOUS STRATEGIES: OLIGOPOLIES

**Oligopolies are markets with a few sellers only.**

**Even if they are not selling perfect substitutes, the actions of each (price, quantity) can affect the profits of the others: a strategic interaction.**

**We can use three strategic game-theoretic models:**

- 1. von Stackelberg leader/follower competition**
- 2. Cournot simultaneous *quantity* competition**
- 3. Bertrand simultaneous *price* competition**

**Plus:**

- 4. Strategic Complements and Substitutes**
- 5. Benchmarking market performance.**

## I. A Von Stackelberg Leader-Follower Model.

e.g., duopoly – two sellers – with a dominant firm (once IBM, now Microsoft, or OPEC, etc.)

**Model:**  **Leader (Spring)** produces quantity  $Q_S$  of bottled water, and

**Follower (Crystal)** produces  $Q_C$  of identical water: a homogeneous good.

- market demand  $\rightarrow$  price  $P$  (i.e. equal for both)

$$P = 10 - (Q_S + Q_C), \quad Q_S + Q_C \leq 10.$$

- common knowledge: Spring sets its production level before Crystal does
- each firm's production level: common knowledge
- order of play & output choices: common knowledge
- no capacity constraints

## Let's put numbers in:

- each firm has identical costs ( $i = C, S$ ):

$$MC_i = AC_i = \$3/\text{unit}$$

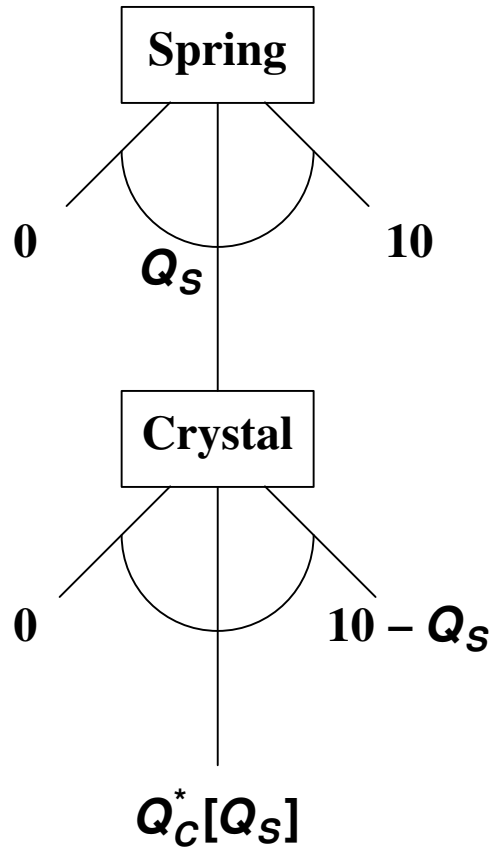
- ∴ payoffs for the leader (Spring) = Revenues less Costs:

$$\pi_S[Q_S, Q_C] = (10 - Q_S - Q_C) \times Q_S - 3Q_S$$

- zero output  $\rightarrow$  zero profit
- The follower (Crystal) has a similar profit  $\pi_C$ .

## Questions:

- What should the Leader (Spring) do?  
Depends on how the Leader thinks the Follower (Crystal) will react.  
Spring should: *Look forward and reason back.*
- for every fixed level of output for Spring  $Q_S$ , what is the best (profit-maximising) level that Crystal can do?  
What is the Follower's *reaction function*?



### ***Cournot Rivalry Game Tree***

**(Quant.  $\Rightarrow$  Cournot,  
Price  $\Rightarrow$  Bertrand.)**

**So: Crystal (the Follower): chooses  $Q_C^*$  to max his profit:**

$$\begin{aligned}\pi_C &= P [Q_S + Q_C] \times Q_C - TC_C [Q_C] \\ &= (10 - (Q_S + Q_C)) \times Q_C - 3 Q_C\end{aligned}$$

- From Crystal's viewpoint, Spring's (the Leader's) output is predetermined — a constant  $Q_S$ , because Spring will already have moved first,

so Crystal sets  $MR_C [Q_S, Q_C] = MC [Q_C] = 3$  to get  $Q_C^*$ :

$$TR_C = (10 - Q_S - Q_C) \times Q_C = 10Q_C - Q_S Q_C - Q_C^2$$

$$\therefore MR_C = 10 - Q_S - 2Q_C^* = MC_C = 3$$

$$\rightarrow Q_C^* = R_C [Q_S] = \frac{7 - Q_S}{2},$$

the profit-maximising output  $Q_C^*$  of Follower (Crystal) is a function of the Leader's choice  $Q_S$

- This function is known as the *reaction function*, since it tells us how the Follower will react to the Leader's choice (of output in this case, but it could be price).

- So:**
- Solve for  $Q_C^* = R_C[Q_S]$ :  $Q_C^* = \frac{7 - Q_S}{2}$ ,  
Crystal's reaction function, contingent on Spring's action  $Q_S$ .

Note: when  $Q_S = 7$ , then  $Q_C^* = 0$ .

- Looking forward & reasoning back, Spring knows this, so Spring's profit:

$$\begin{aligned}\pi_S &= (10 - (Q_S + \frac{7 - Q_S}{2})) \times Q_S - 3Q_S \\ &= 10Q_S - Q_S^2 - 3 \frac{1}{2} Q_S + \frac{1}{2} Q_S^2 - 3Q_S \\ &= 3 \frac{1}{2} Q_S - \frac{1}{2} Q_S^2\end{aligned}$$

- Solving for maximum  $\pi_S$  (i.e.  $\frac{d\pi}{dQ_S} = 0$ ) gives:

$$\frac{d\pi}{dQ_S} = 3 \frac{1}{2} - Q_S^* = 0$$

$\therefore \pi_S$  is max when  $Q_S^* = 3.5$ ,  $Q_C^* = 1.75$ ,  $\therefore Q = 5.25$ ,  
with  $\pi_S^* = \$6.125$  and  $P = \$4.75/\text{unit}$   
and  $\pi_C^* = \$3.06$

## First-Mover Advantage:

- So Spring's *first-mover advantage* is the difference between Spring's profit  $\pi_S$  and Crystal's profit  $\pi_C$ :

$$\$6.125 - \$3.06 = \$3.06$$

- leadership — first mover
- leadership — innovator, monopolist, faced with threat of entry
- incumbent erects barriers to entry by new-comer
- long-term contracts reduce incumbent's flexibility and increase the credibility of defence

## What Would A Monopolist's Outcomes Be?

- The monopolist quantity in this case is

$$Q_M = 3.5, \text{ with } P_M = \$6.50/\text{unit}$$

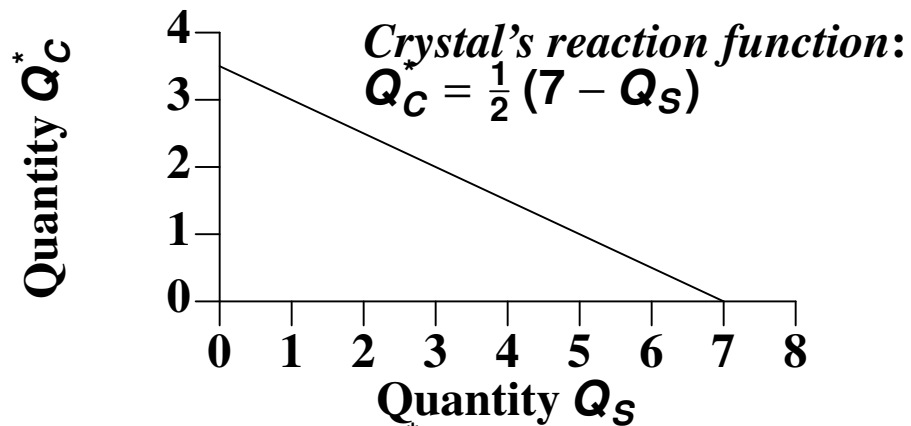
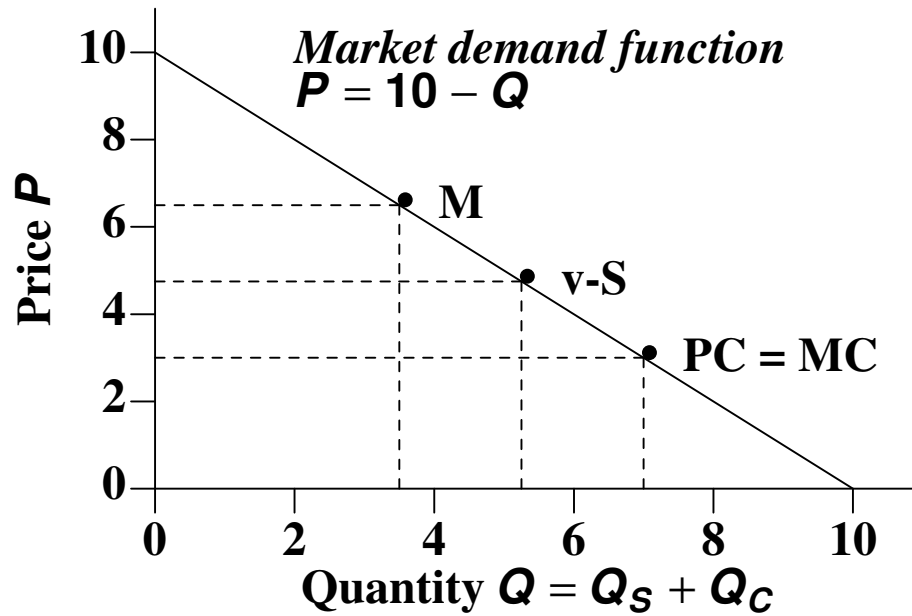
$$\text{and profit } \pi_M = 6.5 \times 3.5 - 3 \times 3.5 = \$12.25$$

(Remember that a monopolist chooses output  $Q_M$  to equate Marginal Revenue with Marginal Cost, or  $10 - 2Q_M = 3$ .)

This means that the Leader could become the Monopolist by paying the Follower not to enter the market, and offering him his (Follower's) profit of \$3.06 ( $= \pi_C$ ) not to, and still be ahead by:  $\$12.25 - 3.06 - 6.125 = \$3.06$ .

$$\text{i.e., } \pi_M - \pi_C - \pi_S = \$3.06$$





i.e. as  $Q_S$  rises,  $Q_C$  falls, & vice versa: *strategic substitutes*.

## 2. Simultaneous Cournot Quantity.

**How much will each firm produce if they move simultaneously?**

**Each needs to:**

1. **make a *conjecture or belief* about how much the other firm will produce – will it be high, with a lower industry price? Or vice versa?**
2. **then determine *its own quantity to produce* – balancing the gain from selling more units against the sacrifice of a lower price.**

**There will be an industry-wide equilibrium when both firms resolve this balance.**

**From Spring's point of view, what should  $Q_S$  be?**

- “If Crystal produces  $Q_C$ , then I, Spring, should max my profit  $\pi_S$ :

$$\pi_S = (10 - Q_C - Q_S) \times Q_S - 3Q_S$$

- which is maximised at

$$Q_S^* = \frac{1}{2} (10 - 3 - Q_C) = \frac{1}{2} (7 - Q_C) ”$$

So  $Q_S^*$  is Spring's *best response* to Crystal's quantity choice of  $Q_C$ , or  $Q_S^* = R_S[Q_C]$ .

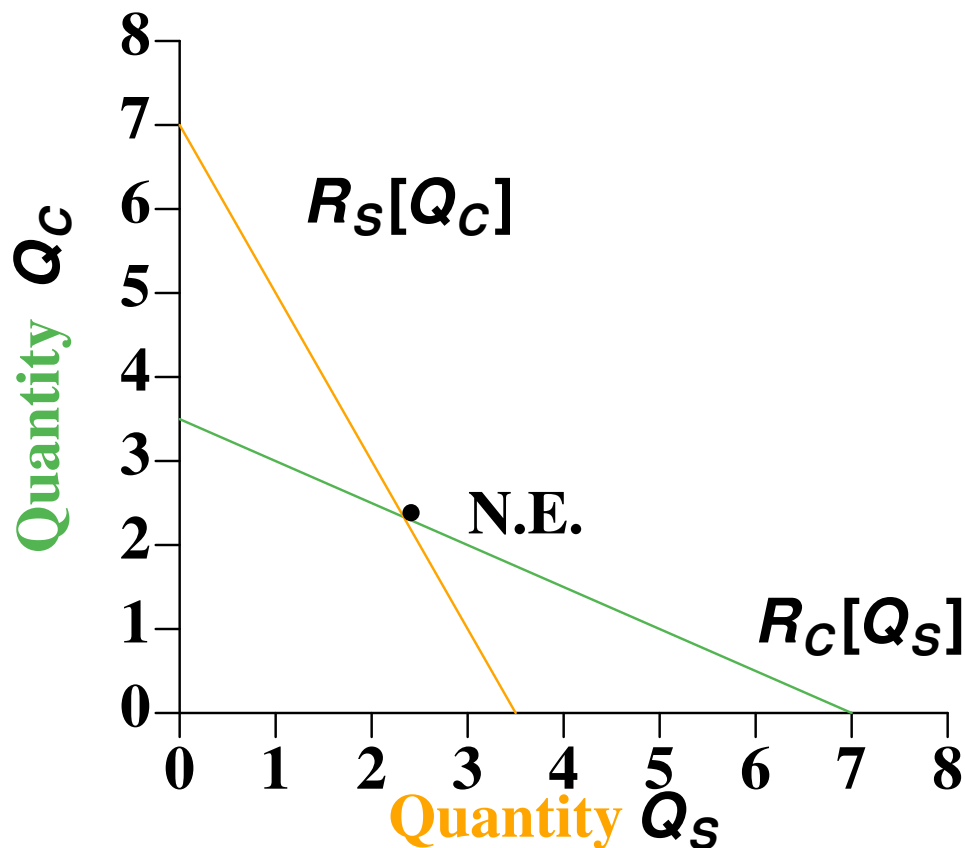
- If  $Q_C$  equals 7 units, then Spring should produce zero.

**Symmetrically, Crystal's *best response*  $R_C[Q_S]$  to a conjectured production level of  $Q_S$  from Spring should be:**

$$Q_C^* = \frac{1}{2} (7 - Q_S)$$

## Reaction Functions:

Plotting both best responses, or *reaction functions*:



*The Two Best-Response Curves: Cournot*  
*Strategic substitutes: higher  $Q_1$   $\rightarrow$  lower  $Q_2$*

## **(Nash) Equilibrium.**

**There is a unique pair of quantities at which the two reaction functions cross:  $(Q_S^*, Q_C^*)$ .**

**Hence at this point:**

$$R_C[Q_S^*] = Q_C^* = 2\frac{1}{3} \text{ and}$$
$$R_S[Q_C^*] = Q_S^* = 2\frac{1}{3}$$

**This is a so-called *Cournot N.E.*, where each player's conjecture is consistent with the other's actual production, and neither has any incentive to alter production. Their beliefs are fulfilled.**

**Price/unit =  $\$5\frac{1}{3}$ , profit of each =  $\$5.44$**

### 3. Simultaneous Price (Bertrand) Competition, Imperfect Substitutes.

Two pizzerias — Donna's Deep Dish and Perce's Pizza Pies — simultaneously compete in a small town.

When Donna's price is  $P_d$  and Perce's price is  $P_p$ , then their sales  $Q_d$  and  $Q_p$  (hundreds per week) are:

$$Q_d = 24 - P_d + \frac{1}{2} P_p$$
$$Q_p = 24 - P_p + \frac{1}{2} P_d$$

The two brands of pizza are (*imperfect*) *substitutes*:  
if the price of one rises, sales of the other go up:  
half the discouraged buyers switch to the other pizza and  
half move to some other kind of food (Big Macs?)

It costs \$6 to make each pizza.

**Perce sets his price  $P_p$  to maximise his profits:**

$$\pi_p = (P_p - 6) \times Q_p = (P_p - 6) \times (24 - P_p + \frac{1}{2} P_d)$$

**Perce's best price  $P_p$  for each level of his conjecture about Donna's price  $P_d$  will give his best response.**

**Expanding Perce's profit function:**

$$\pi_p = -144 - 3P_d + (30 + \frac{1}{2} P_d) \times P_p - P_p^2$$

**Diff. w. r. to  $P_p$  (holding Donna's price  $P_d$  constant):**

$$\frac{d\pi_p}{dP_p} = 30 + \frac{1}{2} P_d - 2P_p.$$

**The first-order condition for  $P_p$  to max  $\pi_p$ :  $\frac{d\pi_p}{dP_p} = 0$ .**

**$\therefore$  Perce's best-response curve  $R_p[P_d]$  is:**

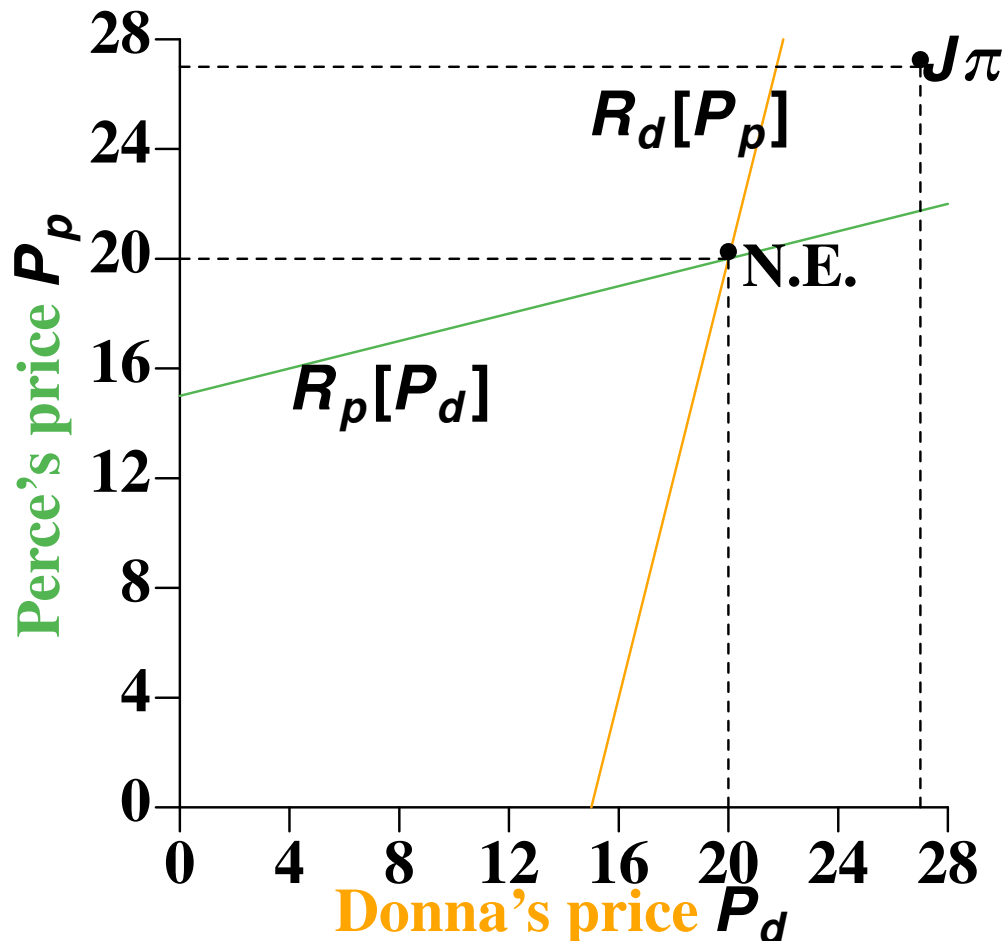
$$P_p = 15 + \frac{1}{4} P_d$$

**$\therefore$  if Donna's price is \$32/unit, then Perce's best price is \$23/unit.**

**Symmetrically, Donna's best-response curve  $R_d[P_p]$  is:**

$$P_d = 15 + \frac{1}{4} P_p$$

## Plotting the Response Curves:



*The Two Best-Response Curves (Reaction Functions): Bertrand*

*(Strategic complements: , as  $P_p$  rises, so does  $P_d$ .)*



**The two reaction curves intersect at the N.E. of the shops' pricing game, at  $P_d = P_p = \$20/\text{unit}$ .**

**Then each shop sells 1400 pizzas a week, at a profit per shop of \$19,600 per week.**

**The best-response reaction curves slope up: when one shop raises its price by \$1, then its rival should increase its price by 25¢: when the first shop raised its price, then some of its customers switched to its rival, which could then best profit from the new customers by raising its price somewhat.**

**∴ a shop that increases its price is helping increase the profits of its rival, but this side-effect is uncaptured (and so ignored) by each shop independently.**

**But, as in the PD, they could collude and increase their profits: if they each charged, say, \$24/unit, then each would sell 1200 pizzas, for a profit of \$21,600.**

**But if  $P_d = \$24/\text{unit}$ , then Perce's best response would be \$21/unit, to which Donna would respond with \$20.25/unit, etc., etc, until they converge to the N.E. prices of \$20/unit.**

***What is best for them jointly?***

**Let  $P_d = P_p = P$  (they both charge  $P$ , because the market demands are symmetric) and  $\pi$  is maximised when  $P = \$27/\text{unit}$ ; at which price each sells 1050 pizzas, at a profit of \$22,050 per shop.**

**Shown as  $J\pi$  on the figure above.**

**Note: if the market demands were *not* symmetric, then it would be wrong to charge the same price  $P$  for both pizzas. Need to choose the two prices to  $\max \pi = \pi_d + \pi_p$ .**

## ***Which form of competition: Cournot (quantity) or Bertrand (price)?***

**What sort of industries compete using quantity and what sort by price? Can they choose?**

***Cournot:* Changing prices can be costly, but so can changing the flow of production to increase or cut back the quantity of good flowing to the market.**

***Cournot* applies to markets in which firms must make production decisions in advance and face high costs for holding inventories: price will adjust more quickly than quantity, and each firm will set a price that sells all that it produces.**

**Such a firm will not expect to steal customers by lowering its price, since its rivals will immediately match any price change, so that their sales equal their planned production volumes. Hence there is less competition than Bertrand.**

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**When firms choose capacity first, and then compete as price setters given the capacities chosen earlier, then the result is identical to Cournot quantity competition.**

**Examples?**

***Bertrand*: When capacity is sufficiently flexible to allow them to adjust production to meet all the demand for their product at any price they announce, then *Bertrand* price competition results. With perfect substitutes, firms will attempt to steal customers by price cutting down to marginal cost.**

**Examples?**

## 4. Strategic Complements and Substitutes

**Compare the reaction functions in the Cournot (quantity) game between the water sellers and those in the Bertrand (price) game between the pizzerias.**

**In the Cournot model the reaction functions were downwards sloping, and in the Bertrand model they were upwards sloping.**

### ***Strategic complements:***

**in general, when reaction functions (best-response curves) are *upwards sloping*, then we say that the firms' actions (here, pizza prices) are *strategic complements*.**

### ***Strategic substitutes:***

**in general, when reaction functions are *downwards sloping*, then we say that the firms' actions (here, water quantities) are *strategic substitutes*.**

## ***Meaning?***

**When actions are *strategic complements*, then an increase by one firm will elicit an increase by the other, & v.v.**

**In the Bertrand (price) model, prices are strategic complements: a competitor's price cut will best be responded to by a price cut. → price wars.**

**When actions are *strategic substitutes*, then an increase by one firm will elicit a reduction by the other, & v.v.**

**In the Cournot (quantity) model, quantities are strategic substitutes: a competitor's quantity increase will best be responded to by a quantity cut.**

***A rule of thumb*: quantities and capacity decisions almost always strategic substitutes, whereas prices almost always strategic complements.**

**So what?**

## **So What?**

**How will a firm expect its rivals to react to its tactical manoeuvres?**

- **When actions are strategic complements, then increased aggression will elicit increased aggression in the rival: (Bertrand)**  
e.g. pizza prices: a competitor's price cut (an aggressive move) will best be responded to by a price cut (also an aggressive move), since the reaction functions are upwards sloping.
- **When actions are strategic substitutes, then increased aggression will result in lessened aggression in the rival: (Cournot)**  
e.g. water quantities: a competitor's quantity increase (aggressive) will best be responded to by a quantity cut (a soft response), since the reaction functions are downwards sloping.

**Compare the Cournot and Bertrand duopoly profits below.**

## 5. Benchmarking Oligopoly Behaviour

**Two companies produce homogeneous output.**

**Linear industry demand curve of  $P = 10 - Q$ ,  
where  $Q$  is the sum of the two companies' outputs,  
 $Q = y_1 + y_2$ .**

**Both companies have identical costs,  $AC = MC = \$1/\text{unit}$ .**

**Five possibilities for the equilibrium levels of prices, outputs,  
and profits:**

- 1. Competitive Price Taking**
- 2. Monopolistic Cartel**
- 3. Cournot Oligopolists**
- 4. von Stackelberg Quantity Leadership**
- 5. Bertrand Simultaneous Price Setting.**
- 6. Comparison Tables and Figures.**



## I Competitive Price Taking

**Each setting price equal to marginal cost.**

**So: Price  $P_{PC} = \$1/\text{unit}$ ,  
the total quantity  $Q = 9$  units between them,  
and each produces output  $y_1 = y_2 = 4.5$  units.**

**Since  $P_{PC} = AC$ , their profits are zero:  $\pi_1 = \pi_2 = 0$ .**

## 2 Monopolistic Cartel

**Collude and act as a monopolistic *cartel*.**

**Each produces half of the monopolist's output and receives half the monopolist's profit.**

**Output  $Q_M$  such that  $MR[Q_M] = MC = \$1/\text{unit}$ .**

**The  $MR$  curve is given by  $MR = 10 - 2Q$ , which results in:**

$$Q_M = 4.5 \text{ units,}$$

$$P_M = \$5.5/\text{unit, and}$$

$$\pi_M = (5.5 - 1) \times 4.5 = \$20.25.$$

**Each produces output  $y_1 = y_2 = 2.25$  units, and earns  $\pi_1 = \pi_2 = \$10.125$  profit.**

### 3 Cournot Oligopolists

Each chooses its output to maximise its profit, assuming that the other is doing likewise: not colluding, but competing. They choose simultaneously.

Cournot N.E. occurs where their *reaction curves* intersect and the expectations of each are fulfilled.

Firm 1 determines Firm 2's reaction function: "If I were Firm 2, I'd choose my output  $y_2^*$  to maximise my Firm 2 profit conditional on the expectation that Firm 1 produced output of  $y_1^e$ ."

$$\max_{y_2} \pi_2 = (10 - y_2 - y_1^e) \times y_2 - y_2$$

Thus  $y_2 = \frac{1}{2} (9 - y_1^e)$ , which is Firm 2's reaction function, given its conjecture  $y_1^e$  of Firm 1's behaviour.

Since the two firms are identical, Cournot equilibrium occurs where the two reaction curves intersect, at  $y_1^* = y_1^e = y_2^* = y_2^e = 3$  units.

So  $Q_{Co} = 6$  units, price  $P_{Co}$  is then \$4/unit, and the profit of each firm is \$9.

## 4 von Stackelberg Quantity Leadership

**What if Firm 1 gets to choose its output level  $y_1$  first?**

**It realises that Firm 2 will know Firm 1's output level when Firm 2 chooses its level: see Firm 2's reaction function from 3. above, but with the actual, not the expected, level of Firm 1's output,  $y_1$ .**

**So Firm 1 chooses  $y_1^*$  to maximise its profit:**

$$\max_{y_1} \pi_1 = (10 - y_2 - y_1) \times y_1 - y_1,$$

**where Firm 2's output  $y_2$  is given by Firm 2's reaction function:  $y_2 = \frac{1}{2}(9 - y_1)$ .**

**Substituting this into Firm 1's maximisation problem:**

**$y_1^* = 4.5$  units, and so  $y_2^* = 2.25$  units, so that  $Q_{St} = 6.75$  units and  $P_{St} = \$3.25/\text{unit}$ .**

**Profits are  $\pi_1 = \$10.125$  (the same as in the cartel case 2. above) and  $\pi_2 = \$5.063$  (half the cartel profit).**

## 5 Bertrand Simultaneous Price Setting.

**Remember: Equilibrium means there is no incentive for either firm to undercut the other.**

**The only equilibrium when they compete using price is where each is selling at  $P_1 = P_2 = MC_1 = MC_2 = \$1/\text{unit}$ .**

**Identical to the price-taking case above.**

**Note: Were  $MC_1$  greater than  $MC_2$ , then Firm 2 would capture the whole market at a price just below  $MC_1$ , and would make a positive profit; and  $y_1 = 0$ .**

## 6 Comparing the Five Market Types

| Market            | Output<br>$y_1$ | Profit<br>$\pi_1$ | Output<br>$y_2$ | Profit<br>$\pi_2$ | Price<br>$P$ | Quantity<br>$Q = y_1 + y_2$ |
|-------------------|-----------------|-------------------|-----------------|-------------------|--------------|-----------------------------|
| 1 Price-taking    | 4.5             | 0                 | 4.5             | 0                 | 1            | 9                           |
| 2 Cartel          | 2.25            | 10.125            | 2.25            | 10.125            | 5.5          | 4.5                         |
| 3 Cournot         | 3               | 9                 | 3               | 9                 | 4            | 6                           |
| 4 von Stackelberg | 4.5             | 10.125            | 2.25            | 5.063             | 3.25         | 6.75                        |
| 5 Bertrand        | 4.5             | 0                 | 4.5             | 0                 | 1            | 9                           |

*Summary of Outcomes.*

