

# **SIMULTANEOUS-MOVE GAMES I**

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The table is a 2x2 matrix with 'Roger' on the vertical axis and 'Juan' on the horizontal axis. The rows are labeled 'West (W)' and 'East (E)'. The columns are labeled 'West (W)' and 'East (E)'. The payoffs are: (1, 0) for (W, W), (0, 1) for (W, E), (0, 1) for (E, W), and (1, 0) for (E, E). Red arrows point from the (W, W) cell to the (W, E) cell, and from the (E, W) cell to the (E, E) cell.

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Juan's best shot depends on what Roger anticipates.

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Juan's best shot depends on what Roger anticipates. And Roger's best move depends on Juan's aim.



## **Simultaneous-Move Games —**

**are:**

- **Discrete, “pure” strategies (no dice-throwing)**
  - **Either at the same time, or without knowledge of an action already taken.**
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**e.g. Choice of product design, advertising campaign, features**

**e.g. Goalie v. striker; server v. receiver**

## **Contents of This Lecture**

- 1. The Payoff Matrix**
- 2. Nash Equilibrium (N.E.)**
- 3. The Prisoner's Dilemma**
- 4. Four Methods for Finding the N.E.**
  - Each has a dominant strategy**
  - One has a dominant strategy**
  - Eliminate dominated strategies**
  - Best-response analysis**
- 5. Other Games**
- 6. Four Lessons**

**(Read Rothschild — Reading 10, in Weeks 2–3 — for next class.)**

## **How To Avoid Circularity?**

**There is circularity:**

**I'm deciding what to do, while you are too;**

**what I decide affects you, and**

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— A Nash Equilibrium.**



## **Payoff Matrix (or “normal” or “strategic” form)**

- **Dimensions = number of players, here = 2.**
- **# rows = # strategies of Mr Row = 4.**  
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<i>Row</i>	<b>T</b>	<b>3, 1</b>	<b>2, 3</b>	<b>10, 2</b>
	<b>H</b>	<b>4, 5</b>	<b>3, 0</b>	<b>6, 4</b>
	<b>L</b>	<b>2, 2</b>	<b>5, 4</b>	<b>12, 3</b>
	<b>B</b>	<b>5, 6</b>	<b>4, 5</b>	<b>9, 7</b>

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- *Non-zero sum* (or “positive sum”) game:  
the sum of the payoffs is not constant across cells.  
By convention, the payoffs are: R,C.
- (See solution on page 7 below.)

## A Zero-Sum Payoff Matrix

Gridiron football:

		<i>D e f e n s e</i>		
		Run	Pass	Blitz
<i>Offense</i>	Run	2	5	13
	Short pass	6	5.6	10.5
	Medium pass	6	4.5	1
	Long pass	10	3	-2

- Show the payoffs of one player only (here, Offense).
- Payoffs in yards gained by Offense. (Defense loses that amount.)

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- Payoffs in yards gained by Offense. (Defense loses that amount.)  $\therefore$  Zero-sum game.
- N.E. at {Short pass, Pass}, and Offense gains 5.6 yards.

## Nash Equilibrium

From p.4 above , a N.E. at  $\{L,M\}$ , payoffs (5,4):

		<i>Column</i>		
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<i>Row</i>	T	3, 1	2, 3	10, 2
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Why?



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**Why? Because Ce is Column's *best response* to Row's L, and vice versa.**

**So  $\{L,Ce\}$  is each player's best response to the other's action.**

**$\therefore$  Neither would change unilaterally.**

**$\therefore$  we have an equilibrium (a N.E.).**

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Is this an equilibrium? No: if Column chooses  $Le$ , then R chooses B, because  $5 > 4$ . But  $\{B, Le\}$  is not an equilibrium: Column chooses  $Ri$  ( $7 > 6$ ). Etc.
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2. The N.E. need not be the jointly best combination of strategies: strategies  $\{B, R_i\}$  has payoffs of  $(9, 7)$ , but it's not an equilibrium, absent "cooperative" behaviour.
3. Nor does the N.E. require equilibrium choices to be *strictly* better than other choices: if  $\{B, C_e\}$  had payoffs of  $(5, 5)$ , then  $\{L, C_e\}$  would remain a N.E. because Row has no incentive to change her choice from L to B.
4. Could do *cell-by-cell inspection* to find all N.E., but simpler methods exist.

## **N.E. as Beliefs**

**Players need not have best responses to opponents' action which have not yet happened.**

**Players can think ahead, and form *beliefs* of what opponents will do.**

**Then a N.E. can be defined as a set of strategies (one per player) such that:**

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- 1. each player has correct beliefs about the strategies of the others, and**
- 2. the strategy of each is the best strategy for herself, given her beliefs about the others' strategies.**

## Now: Four Methods to Find N.E.

Say you're the Row player:

1. Look for a *dominant strategy* (a row always preferred, no matter which column the other player chooses), and choose it.
2. Does the other player have a *dominant strategy* (column)? If so, expect that strategy.
3. Look for *dominated actions* (rows never preferred, no matter what the other player would choose), and *eliminate* them.

Successively eliminate each other's dominated strategies (rows, columns).

4. Use arrows for both of you, and identify any cells with no arrows leaving: *best response* or N.E.

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Consider the Prisoner's Dilemma:

Years of prison (Ned, Kelly).

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		Spill (D)	Mum (C)
<i>Ned</i>	Spill (D)	8, 8	0, 20
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- Likewise for Kelly.



## Both Players Have Dominant Strategies

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- (See Lectures 15, 16 later.)

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here {D,D}  $\rightarrow$  (8,8); while {C,C}  $\rightarrow$  (1,1)**

**That is, there is a conflict between collective interest (at C,C) and individual self-interest (at D,D).**



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**How to overcome the {D,D} trap?**

**(See Lectures 15, 16 later.)**

## **Ex: The Advertising Game is a P.D.**

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*Case: Telstra and Optus and advertising.*

*David Ogilvy: Half the money spent on advertising is wasted; the problem is identifying which half.*

**Telstra and Optus independently must decide how heavily to advertise.**

**Advertising is expensive, but if one telco chooses to advertise moderately while the other advertises heavily, then the first loses out while the second does well.**

## **Payoffs in the Telstra/Optus Game:**

**Let's assume if both Advertise Heavily then Telstra nets \$70,000, while Optus nets \$50,000.**

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**But if Telstra Advertises Heavily while Optus Advertises Moderately only, then Telstra nets \$140,000 while Optus nets only \$25,000, and vice versa.**

**If both Advertise Moderately, then Telstra nets \$120,000 and Optus nets \$90,000.**



## **Payoffs in the Telstra/Optus Game:**

**Let's assume if both Advertise Heavily then Telstra nets \$70,000, while Optus nets \$50,000.**

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**What to do?**

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**What to do?**


**Consider the payoff matrix:**

## The Advertising Game

		<i>Optus</i>	
		Heavy	Moderate
<i>Telstra</i>	Heavy	70, 50	140, 25
	Moderate	25, 140	120, 90

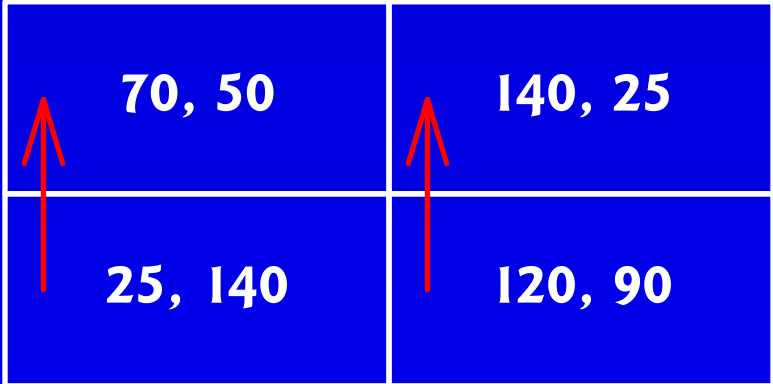
## The Advertising Game

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## The Advertising Game

		<i>Optus</i>	
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The table displays the payoffs for the Advertising Game. The rows represent Telstra's strategies (Heavy and Moderate) and the columns represent Optus's strategies (Heavy and Moderate). The payoffs are given as (Telstra, Optus). Red arrows indicate the best response for each player: Telstra's best response is Heavy (70, 50) and Optus's best response is Heavy (140, 25).

## The Advertising Game

		<i>Optus</i>	
		Heavy	Moderate
<i>Telstra</i>	Heavy	70, 50	140, 25
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The table displays the payoffs for the Advertising Game between Telstra and Optus. The strategies are Heavy and Moderate for both players. The payoffs are given as (Telstra, Optus).

- If both choose Heavy: (70, 50)
- If Telstra chooses Heavy and Optus chooses Moderate: (140, 25)
- If Telstra chooses Moderate and Optus chooses Heavy: (25, 140)
- If both choose Moderate: (120, 90)

Red arrows indicate best responses:

- Telstra's best response is Heavy (70, 50) > (25, 140) and (140, 25) > (120, 90).
- Optus's best response is Moderate (140, 25) > (70, 50) and (120, 90) > (25, 140).

The Nash equilibrium is (Heavy, Moderate) with payoffs (140, 90).

## The Advertising Game

		<i>Optus</i>	
		Heavy	Moderate
<i>Telstra</i>	Heavy	70, 50	140, 25
	Moderate	25, 140	120, 90



## The Advertising Game

		<i>Optus</i>	
		Heavy	Moderate
<i>Telstra</i>	Heavy	70, 50	140, 25
	Moderate	25, 140	120, 90

The matrix shows the following payoffs (Telstra, Optus):

- (Heavy, Heavy): 70, 50
- (Heavy, Moderate): 140, 25
- (Moderate, Heavy): 25, 140
- (Moderate, Moderate): 120, 90

**Both choose Heavy advertising, although each would be better off with Moderate advertising.**

## The Advertising Game

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		Heavy	Moderate
<i>Telstra</i>	Heavy	70, 50	140, 25
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***A Prisoner's Dilemma.***

## The Advertising Game

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		Heavy	Moderate
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**Both choose Heavy advertising, although each would be better off with Moderate advertising.**

***A Prisoner's Dilemma.***

**The arrows show each player has a dominant strategy of H.**

## With Pure Strategies, Rankings are Sufficient:


Or, could rank outcomes for each player:  
4 is best, 1 is worst.

		<i>Optus</i>	
		Heavy	Moderate
<i>Telstra</i>	Heavy	2, 2	4, 1
	Moderate	1, 4	3, 3

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**Important:** When strategies are “pure” (deterministic), then we needn’t have exact knowledge of the payoffs, just their rankings.

## **Ex: The Capacity Game**

***Players: two firms Alpha and Beta***

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**Strategies:**

**Allow three choices for each of the two players:**



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**∴**

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**Greater capacity → more sales → lower prices. Profits?**



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**∴ A  $3 \times 3$  payoff matrix (POM)**

**Greater capacity → more sales → lower prices. Profits?**

**The payoff matrix (in net returns '000) for simultaneous moves is:**

## The Capacity Game

		<i>Beta</i>		
		<b>DNE</b>	<b>Small</b>	<b>Large</b>
<i>Alpha</i>	<b>DNE</b>	<b>\$18, \$18</b>	<b>\$15, \$20</b>	<b>\$9, \$18</b>
	<b>Small</b>	<b>\$20, \$15</b>	<b>\$16, \$16</b>	<b>\$8, \$12</b>
	<b>Large</b>	<b>\$18, \$9</b>	<b>\$12, \$8</b>	<b>\$0, \$0</b>

## The Capacity Game

		<i>Beta</i>		
		DNE	Small	Large
<i>Alpha</i>	DNE	\$18, \$18	\$15, \$20	\$9, \$18
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Red arrows indicate best responses for each player:

- Alpha's best responses: DNE (from DNE), Small (from Small), Large (from Large).
- Beta's best responses: Small (from DNE), Small (from Small), Large (from Large).



## The Capacity Game

		<i>Beta</i>		
		DNE	Small	Large
<i>Alpha</i>	DNE	\$18, \$18	\$15, \$20	\$9, \$18
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The payoff matrix (Alpha, Beta).

## The Capacity Game

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The payoff matrix (Alpha, Beta).

**N.E. at {Small, Small}, although both would prefer {DNE, DNE}.**

## The Capacity Game

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		DNE	Small	Large
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The payoff matrix (Alpha, Beta).

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**Large is a dominated strategy for both players.**

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The payoff matrix (Alpha, Beta).

N.E. at {Small, Small}, although both would prefer {DNE, DNE}.

Large is a dominated strategy for both players.

What if the payoffs were the *differences* in returns? (an envious game)

Then the game is changed to an “envious” game..




## 2. One Player Has a Dominant Strategy

		<i>RBA</i>	
		Low	High
<i>Gov't</i>	Balanced	3, 4	1, 3
	Deficit	4, 1	2, 2

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Red arrows indicate dominant strategies: a vertical arrow pointing down from 'Balanced' to 'Deficit', a horizontal arrow pointing left from 'High' to 'Low', and a horizontal arrow pointing right from 'Low' to 'High'.

## 2. One Player Has a Dominant Strategy

		<i>RBA</i>	
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<i>Gov't</i>	Balanced	3, 4	1, 3
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Red arrows indicate dominant strategies: Gov't chooses Deficit (4 > 3), RBA chooses Low (4 > 3) if Gov't chooses Balanced, and RBA chooses High (2 > 1) if Gov't chooses Deficit.

## 2. One Player Has a Dominant Strategy

		<i>RBA</i>	
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	Deficit	4, 1	2, 2

**Players:**

**Gov't:** fiscal policy (taxes, govt. expenditure)

**RBA:** monetary policy (interest rates)

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Red arrows indicate dominant strategies: Gov't chooses Deficit (4 > 3), RBA chooses High (3 > 2). The outcome (2, 2) is circled in green.

### *Players:*

**Gov't:** fiscal policy (taxes, govt. expenditure)

**RBA:** monetary policy (interest rates)

### *Actions:*

**Gov't:** either balanced budget or deficit

**RBA:** high or low interest rates

## 2. One Player Has a Dominant Strategy

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		Low	High
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Diagram illustrating a game matrix between the Government (*Gov't*) and the Reserve Bank of Australia (*RBA*). The *Gov't* chooses between Balanced and Deficit, and the *RBA* chooses between Low and High interest rates. The payoffs are (Gov't, RBA). The outcome (2, 2) is circled in green, indicating it is the dominant strategy outcome for the *RBA*.

### *Players:*

**Gov't:** fiscal policy (taxes, govt. expenditure)

**RBA:** monetary policy (interest rates)

### *Actions:*

**Gov't:** either balanced budget or deficit

**RBA:** high or low interest rates

### *Preferences?*



## **Ex: The Macroeconomics Game**

**The RBA's best strategy depends on the Gov't's strategy. Dislikes inflation, High rates.**

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**The Gov't prefers spending (and a budget deficit).**

**The RBA realises that {Deficit} is a dominant strategy for Gov't.**

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**∴ RBA should choose {High}.**

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∴ Payoffs of (2,2), although {Balanced, Low} → (3,4) is jointly better.

## Ex: The Macroeconomics Game

The RBA's best strategy depends on the Gov't's strategy. Dislikes inflation, High rates.  
The Gov't prefers spending (and a budget deficit).

The RBA realises that {Deficit} is a dominant strategy for Gov't.

∴ RBA should choose {High}.

∴ Payoffs of (2,2), although {Balanced, Low} → (3,4) is jointly better.

Many countries have a loose fiscal policy and a tight monetary policy at {Deficit, High interest rates}.

### 3. Successive Elimination of Dominated Strategies

		<i>C o l u m n</i>		
		<b>Le</b>	<b>Ce</b>	<b>Ri</b>
<i>R o w</i>	<b>T</b>	<b>3, 1</b>	<b>2, 3</b>	<b>10, 2</b>
	<b>H</b>	<b>4, 5</b>	<b>3, 0</b>	<b>6, 4</b>
	<b>L</b>	<b>2, 2</b>	<b>5, 4</b>	<b>12, 3</b>
	<b>B</b>	<b>5, 6</b>	<b>4, 5</b>	<b>9, 7</b>

### 3. Successive Elimination of Dominated Strategies

		<i>C o l u m n</i>		
		<b>Le</b>	<b>Ce</b>	<b>Ri</b>
<i>Row</i>	<b>T</b>	<b>3, 1</b>	<b>2, 3</b>	<b>10, 2</b>
	<b>H</b>	<del><b>4, 5</b></del>	<del><b>3, 0</b></del>	<del><b>6, 4</b></del>
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### 3. Successive Elimination of Dominated Strategies

*Column*

		<b>Le</b>	<b>Ce</b>	<b>Ri</b>
<i>Row</i>	<b>T</b>	<del>3, 1</del>	2, 3	10, 2
	<b>H</b>	<del>4, 5</del>	<del>3, 0</del>	6, 4
	<b>L</b>	<del>2, 2</del>	5, 4	12, 3
	<b>B</b>	<del>5, 6</del>	4, 5	9, 7

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	<b>L</b>	<del>2, 2</del>	<del>5, 4</del>	<del>12, 3</del>
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### 3. Successive Elimination of Dominated Strategies

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For Row, H is dominated (by B): eliminate H;

### 3. Successive Elimination of Dominated Strategies

		<i>Column</i>		
		<b>Le</b>	<b>Ce</b>	<b>Ri</b>
<i>Row</i>	<b>T</b>	<del>3, 1</del>	<del>2, 3</del>	<del>10, 2</del>
	<b>H</b>	<del>4, 5</del>	<del>3, 0</del>	<del>6, 4</del>
	<b>L</b>	<del>2, 2</del>	<b>5, 4</b>	<del>12, 3</del>
	<b>B</b>	<del>5, 6</del>	<del>4, 5</del>	<del>9, 7</del>

**For Row, H is dominated (by B): eliminate H;**  
**For Column, Le is dominated (by Ri);**

### 3. Successive Elimination of Dominated Strategies

		<i>Column</i>		
		<b>Le</b>	<b>Ce</b>	<b>Ri</b>
<i>Row</i>	<b>T</b>	<del>3, 1</del>	<del>2, 3</del>	<del>10, 2</del>
	<b>H</b>	<del>4, 5</del>	<del>3, 0</del>	<del>6, 4</del>
	<b>L</b>	<del>2, 2</del>	<b>5, 4</b>	<del>12, 3</del>
	<b>B</b>	<del>5, 6</del>	<del>4, 5</del>	<del>9, 7</del>

For Row, H is dominated (by B): eliminate H;  
 For Column, Le is dominated (by Ri);  
 For Row, T and B are now dominated (by L).

### 3. Successive Elimination of Dominated Strategies

		<i>Column</i>		
		<b>Le</b>	<b>Ce</b>	<b>Ri</b>
<i>Row</i>	<b>T</b>	<del>3, 1</del>	<del>2, 3</del>	<del>10, 2</del>
	<b>H</b>	<del>4, 5</del>	<del>3, 0</del>	<del>6, 4</del>
	<b>L</b>	<del>2, 2</del>	<b>5, 4</b>	<del>12, 3</del>
	<b>B</b>	<del>5, 6</del>	<del>4, 5</del>	<del>9, 7</del>

**For Row, H is dominated (by B): eliminate H;**

**For Column, Le is dominated (by Ri);**

**For Row, T and B are now dominated (by L).**

**Which now leaves Row with L, and Column chooses Ce.**



### 3. Successive Elimination of Dominated Strategies

		<i>Column</i>		
		Le	Ce	Ri
<i>Row</i>	T	<del>3, 1</del>	<del>2, 3</del>	<del>10, 2</del>
	H	<del>4, 5</del>	<del>3, 0</del>	<del>6, 4</del>
	L	<del>2, 2</del>	5, 4	<del>12, 3</del>
	B	<del>5, 6</del>	<del>4, 5</del>	<del>9, 7</del>

For Row, H is dominated (by B): eliminate H;

For Column, Le is dominated (by Ri);

For Row, T and B are now dominated (by L).

Which now leaves Row with L, and Column chooses Ce.

Not every game is *dominance solvable*, but the POM perhaps becomes smaller.

## What if there are ties?

It's possible to eliminate using *weak dominance* ( $\leq$ ) instead of *strict dominance* ( $<$ ), but this successive elimination of weakly dominated strategies might throw out some N.E.

(See Dixit & Skeath, p. 97.)

## 4. Best-Response Analysis (BRĀ) to Find N.E.

**Dixit & Skeath use circles to show the best response.  
Row looks at for highest payoff in each column, and  
Column looks for the best payoff in each row.**

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When BRĀ fails to find a N.E., there is no N.E. in pure strategies (but see Lecture 11).

In Lecture 5, we derive best-response curves with continuous strategies.

## Pure Coordination Games

Common interests, but independent choices → issues.


		Starbucks	<i>Sally</i>	Local
<i>Harry</i>	Starbucks	1, 1		0, 0
	Local	0, 0		1, 1



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Two N.E., with equal payoffs: need to coordinate.

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How?

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		Starbucks	Local
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Two N.E., with equal payoffs: need to coordinate.

How? Without communication, to a *focal point*.




# Assurance Games

		<i>Sally</i>	
		Starbucks	Local
<i>Harry</i>	Starbucks	1, 1	0, 0
	Local	0, 0	2, 2

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# Assurance Games

		<i>Sally</i>	
		Starbucks	Local
<i>Harry</i>	Starbucks	1, 1	0, 0
	Local	0, 0	2, 2

The table illustrates a game between Harry and Sally. The strategies for Harry are Starbucks and Local, and the strategies for Sally are Starbucks and Local. The payoffs are (Harry, Sally). Red arrows point to the (1, 1) and (2, 2) cells, indicating that these are the only outcomes where both players receive a positive payoff.

# Assurance Games

		<i>Sally</i>	
		Starbucks	Local
<i>Harry</i>	Starbucks	1, 1	0, 0
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# Assurance Games

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# Assurance Games

		<i>Sally</i>	
		Starbucks	Local
<i>Harry</i>	Starbucks	1, 1	0, 0
	Local	0, 0	2, 2

The table illustrates a game between Harry and Sally. The payoffs are (Harry, Sally). The outcomes are: (1, 1) if both choose Starbucks, (0, 0) if one chooses Starbucks and the other Local, and (2, 2) if both choose Local. Red arrows indicate that (1, 1) and (2, 2) are the only outcomes where both players receive a positive payoff, representing the assurance game structure.

## Assurance Games

		<i>Sally</i>	
		Starbucks	Local
<i>Harry</i>	Starbucks	1, 1	0, 0
	Local	0, 0	2, 2

Now a shared preference for the Local, over Starbucks.

## Assurance Games

		<i>Sally</i>	
		Starbucks	Local
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Now a shared preference for the Local, over Starbucks.  
 This needs to be *common knowledge*.



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But also need a *convergence of expectations* of actions.

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Now a shared preference for the Local, over Starbucks.

This needs to be *common knowledge*.

But also need a *convergence of expectations* of actions.

Need enough certainty or *assurance* to get to (Local, Local).

# The Battle of the Sexes

***A coordination game:***

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*A coordination game:*

**e.g. video VHS v. Sony's Betamax;**

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**and the high-definition DVD:  
Blu-ray DVD v. HD-DVD.**

***The Players & Actions:***

- **a man (Hal) who wants to go to the Theatre and**
- **a woman (Shirl) who wants to go to a Concert.**

**While selfish, they are deeply in love, and would, if necessary, sacrifice their preferences to be with each other.**



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**While selfish, they are deeply in love, and would, if necessary, sacrifice their preferences to be with each other.**

**The payoff matrix (measuring the scale of happiness) is below.**

**What are all equilibria?**


**(i.e. Which pairs of actions are mutually best response?)**

## The Battle of the Sexes

		<b>Theatre</b>	<i>Shirl</i>	<b>Concert</b>
<i>Hal</i>	<b>Theatre</b>	2, 1		-1, -1
	<b>Concert</b>	-1, -1		1, 2

## The Battle of the Sexes

		<b>Theatre</b>	<i>Shirl</i>	<b>Concert</b>
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The payoff matrix (Hal, Shirl).

## The Battle of the Sexes

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The payoff matrix (Hal, Shirl).

A non-cooperative, positive-sum game,  
with two Nash equilibria.



## The Battle of the Sexes

**There is no iterated dominant strategy equilibrium.**

**There are two *Nash equilibria*:**

- **(*Theatre, Theatre*): given that Hal chooses *Theatre*, so does Shirl.**
- **(*Concert, Concert*), by the same reasoning.**

## The Battle of the Sexes

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**There are two *Nash equilibria*:**

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- **(*Concert, Concert*),** by the same reasoning.

**How do the players know which to choose?**

**(A coordination game.)**

## **Players' choices.**

**If they do not talk beforehand, Hal might go to the Concert and Shirl to the Theatre, each mistaken about the other's beliefs.**

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**Focal points?**

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**Focal points?**

**Repetition?**

**Each of the Nash equilibria is collectively rational (efficient): no other strategy combination increases the payoff of one player without reducing that of the other.**

## Market analogues ?



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Emerging standards mean choice and decisions for  
early adopters.**
- **others?**

## No Equilibrium in Pure Strategies?

		<i>Serena</i>	
		DL	CC
<i>Venus</i>	DL	50	80
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**$\therefore$  No N.E. in pure strategies.**

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**Zero-sum game: serving Venus's percentage of wins against Serena.**

**Play Down the Line, or Cross Court.**

**$\therefore$  No N.E. in pure strategies. Why?**

**(See Lecture 11 later.)**

# Chicken!

Here “Bomber” and “Alien” are matched.

		<i>Bomber</i>	
		Veer	Straight
<i>Alien</i>	Veer	Blah, Blah	Chicken!, Winner
	Straight	Winner, Chicken!	Death? Death?

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	<b>Straight</b>	Winner, Chicken!	Death? Death?

Diagram illustrating the game outcome for Bomber and Alien strategies:

- If Bomber chooses **Veer** and Alien chooses **Veer**, the outcome is **Blah, Blah**.
- If Bomber chooses **Straight** and Alien chooses **Veer**, the outcome is **Chicken!, Winner**.
- If Bomber chooses **Veer** and Alien chooses **Straight**, the outcome is **Winner, Chicken!**.
- If Bomber chooses **Straight** and Alien chooses **Straight**, the outcome is **Death? Death?**.

Red arrows indicate a path from the top-left cell to the top-right cell, and from the bottom-left cell to the top-right cell.

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	<b>Straight</b>	Winner, Chicken!	Death? Death?

Diagram illustrating a game matrix for Bomber and Alien. The matrix shows outcomes for different strategies (Veer and Straight) for both players. Red arrows indicate a path from the top-right cell (Chicken!, Winner) to the bottom-left cell (Winner, Chicken!). A green circle highlights the outcome "Chicken!, Winner".



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<i>Alien</i>	<i>Veer</i>	Blah, Blah	Chicken!, Winner
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**No dominant strategies: what's best for one depends on the other's action.**

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**No dominant strategies: what's best for one depends on the other's action.**

**Nash Equilibrium where?**

## **Six Steps to Help:**

- 1. What is the strategic Issue?**
- 2. Who are the Players?**
- 3. What are each player's strategic Objectives?**
- 4. What are each player's potential Actions?**
- 5. What is the likely Structure of the game?**
  - simultaneous or sequential (who's on first?)?**
  - one-shot or repeated?**
- 6. Simultaneous: Rank each player's Outcomes across all combinations of the actions of both.**

# What Have We Learnt?

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**Rule 3:** Eliminate any dominated strategies from consideration, and go on doing so successively.

**Rule 4:** Look for an equilibrium, a pair of strategies in which each player's action is the best response to the other's.