1.2 Strategic Interactions

Game theory → a game plan, a specification of actions covering all possible eventualities

Strategic situations: "influence, to outguess, or to adapt to the decisions or lines of behaviour that others have just adopted or are expected to adopt" — Schelling.

Look forward and reason backwards.

Game theory is the study of rational behaviour in situations involving interdependence.

- May involve common interests: coordination
- May involve competing interests: rivalry
- *Rational behaviour*: players do the best they can, in their eyes;
- Because of the players' interdependence, a rational decision in a game must be based on a prediction of others' responses.
  By putting yourself in the other's shoes and predicting what action the other person will choose, you can decide your own best action.

Game theory provides links: many diverse situations have the same essential structure.

— a *procurement manager* trying to induce a subcontractor to search for cost-reducing innovations

— an *entrepreneur* negotiating a royalty arrangement with a manufacturing firm to license the use of a new technology

— a *sales manager* devising a commission–payments scheme to motivate salespeople

— a *production manager* deciding between piece-rate and wage payments to workers a firm's compensation committee designing a managerial incentive system

— a manager's decision on *how low to bid* for a government contract

— an antique collector's decision on *how high to bid* in an auction

— a *takeover raider's* decision on what price to offer for a firm

— a *negotiation* between a corporation and a foreign government over the setting up of a manufacturing plant

— the *haggling* between a buyer and seller of a used car

— *collective bargaining* between a trade union/employees and an employer

1.2.1 Auctioning a Dollar

(See O'Neal in the Package.)

Rules:

- First bid: 5¢
- Lowest step in bidding: 5¢
- Auction lasts until the clock starts ringing.
- Highest bidder pays bid and gets $1 in return.
- Second-highest bidder also pays, but gets nothing.

Write down the situation as seen by

1. the high bidder, and
2. the second highest bidder.

What happened?

Escalation and entrapment

Examples?

1.2.2 Schelling's Game

Rules:

- single play, $4 to play
- vote "C" (Coöperate) or "D" (Defect).
- sign your ballot. (and commit to pay the entry fee.)
- If x% vote "C" and (100 – x)% vote "D":
  - then "C"s' payoff = $\left(\frac{x}{100} \times \$6\right) - \$4$
  - then "D"s' payoff = "C" payoff + $2
- Or: You needn't play at all.

WHAT HAPPENED?

- numbers & payoffs.
- previous years?

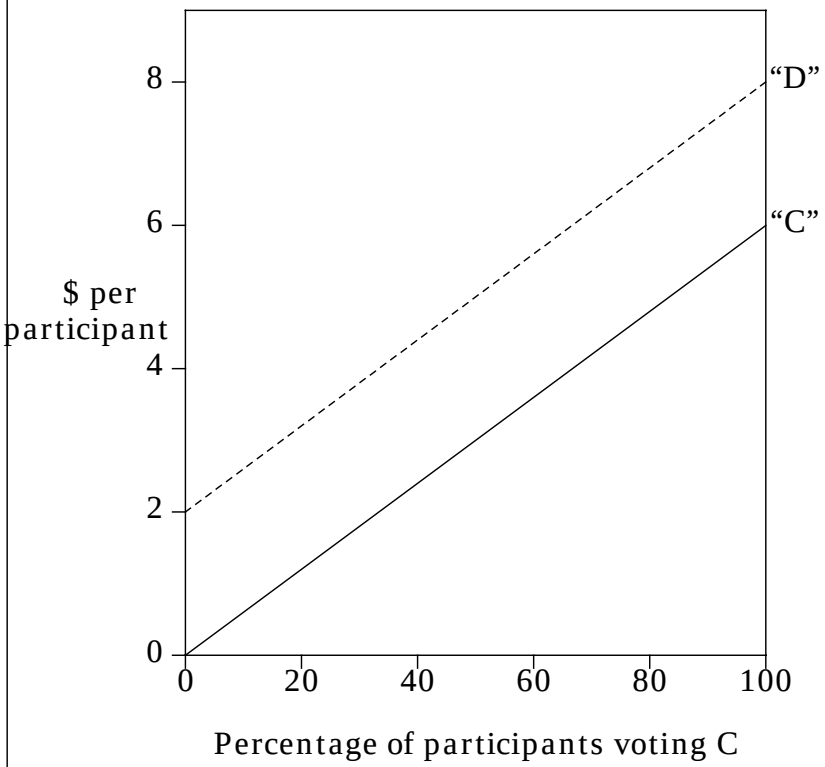
Dilemma: $\left\{ \begin{array}{l} \text{coöperate for the common good } \textit{or} \\ \text{defect for oneself} \end{array} \right.$

Public/private information

Examples?

Schelling's Game

(See Schelling in Package.)

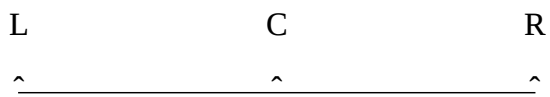


Payoffs per participant

Note: the game costs $4 to join.

1.2.3 The Ice-Cream Sellers

(See Marks in the Package)



- Demonstration
- Payoff matrix
- Incentives for movement?
- Examples?

We can model this interaction with a simplification: each firm can either:

- move to the centre of the beach (M), or
- not move (stay put) (NM).

The share of ice-creams each sells (to the total population of 80 sunbathers) depends on its move and that of its rival.

Since each has two choices for its location, there are $2 \times 2 = 4$ possibilities.

We use *arrows* and a **payoff matrix**[1], which clearly outlines the possible actions of each and the resulting outcomes.

What are the sales if neither moves (or both NM)? Each sells to half the beach.

What are the sales if You move to the centre (M) and your rival stays put at the three-quarter point?

What if you both move?

Given the analysis, what *should* you do?

Does the model imply necessary reversion to the mean?

No. (See the Marks in the Package.)

1. See the **Glossary** in the Package for new meanings.

The Ice-Cream Sellers

		The other seller	
		M	NM
You	M	**40, 40**	50, 30
	NM	30, 50	40, 40

TABLE 1. The payoff matrix (You, Other)

A non-cooperative, zero-sum game,
with a **dominant strategy**,
or dominant move.

1.2.4 The Prisoner's Dilemma

(See Marks in the Package)

The Payoff Matrix:

- The Cheater's Reward = 5
- The Sucker's Payoff = 0
- Mutual defection = 2
- Mutual coöperation = 4

These are chosen so that:

$$5 + 0 < 4 + 4$$

so that C,C is **efficient** in a repeated game.

Need for
communication
coördination
trust
or?

Efficient Outcome: there is no other combination of actions or strategies that would make at least one player better off without making any other player worse off.

Try your hand at playing the Prisoner's Dilemma over the 'Net:
`http://serendip.brynmawr.edu/~ann/pd.html`

The Prisoner's Dilemma

		The other player	
		C	D
You	C	4, 4	0, 5
	D	5, 0	**2, 2**

TABLE 2. The payoff bimatrix (You, Other)

A non-cooperative, positive-sum game, with a dominant strategy.

Efficient at _____

Nash Equilibrium at _____

(See above for a definition of Nash Equilibrium.)

1.2.5 The Capacity Game

Two firms each produce identical products and each must decide whether to Expand (E) its capacity in the next year or not (DNE).

A larger capacity will increase its share of the market, but at a lower price.

The simultaneous capacity game between Alpha and Beta can be written as a payoff matrix.

The Capacity Game

		Beta	
		DNE	Expand
Alpha	DNE	$18, $18	$15, $20
	Expand	$20, $15	**$16, $16**

TABLE 3. The payoff matrix (Alpha, Beta)

A non-cooperative, positive-sum game, with a dominant strategy.

Efficient at _____

Nash Equilibrium at _____

At a **Nash equilibrium**, each player is doing the best it can, given the strategies of the other players.

We can use arrows in the payoff matrix to see what each player should do, given the other player's action.

The Nash equilibrium is a self-reinforcing focal point, and expectations of the other's behaviour are fulfilled.

The Nash equilibrium is not necessarily efficient.

The game above is an example of the Prisoner's Dilemma: in its one-shot version there is a conflict between collective interest and self-interest.

Example

Given the social costs associated with litigation, why is it increasing?

David Ogilvy has said, "Half the money spent on advertising is wasted; the problem is identifying which half."

Is the explanation for the amount of advertising a Prisoner's Dilemma?

1.3 Modelling Players' Preferences

In *two-person games*, each of the two players has only n possible *actions*:

$\therefore$ represent the game with a $n \times n$ payoff matrix.

Two actions per player: $n = 2$.

$\therefore$ Each player faces four possible combinations.

For a one-shot game in *pure strategies*
(i.e., no dice rolling or mixing of pure strategies):

need only *rank* the four combinations:

best, good, bad, worst:

$\rightarrow$ payoffs of 4, 3, 2, and 1, respectively

Larger numbers of possible actions:

harder to rank the larger number of outcomes
(with three actions there are $3 \times 3 = 9$),
but ranking sufficient.

(i.e. *ordinal preferences*, instead of asking "by how much is one outcome preferred to another?")

Later, with **mixed strategies** (probabilistic or dice-throwing) and unpredictability:

- use *probabilities* over actions and
- the *expected values* of the possible outcome
- use *cardinal measures* over the amounts, usually dollar amounts, which are unambiguous, *and the numbers matter*!

Simple Games:

- The Dollar Auction
- The n-person Prisoner's Dilemma
- The Ice Cream Sellers
- The Prisoner's Dilemma
- The Battle of the Bismark Sea
- Chicken!
- The Boxed Pigs
- The Battle of the Sexes
- The Gift of the Magi
- Bargaining
- Vickrey auction

Further games, from Gardiner (pp.):

- Battle of the Networks: 38–41
- Video System Coordination (2P & 3P): 50–51, 99–100
- Cigarette Advertising on TV: 51–53
- Matching Pennies: 68–69
- Market Niche (Symm. & Asymm, 2P & 3P): 69–70,79–81,97–99
- Everyday Low Pricing — Sears: 81–84
- 3P Competitive Advantage: 96–97
- Stonewalling Watergate: 102–104
- Tragedy of the Commons: 108–111
- Cournot Competition: 124–127
- Bertrand Market Game: 129–133
- Telex versus IBM: 149–151
- Conscription: Reluctant Volunteers: 157–159
- Mutually Assured Destruction — MAD: 160–164
- Hawk versus Dove: 213–215
- Caveat Emptor (Buyer Beware): 240–242
- Depositor versus Savings & Loan: 286–288

- Voting Game: 440–443

1.4 Concepts and Tools

The following **concepts & tools** are introduced:

The Ice-Cream Sellers:
- payoff matrix
- incentives to change — use arrows!
- dominant strategy

The Prisoner's Dilemma:
- possibility of repetition
- efficiency — Pareto Optimality
- bimatrix of payoffs — both players'
- non-zero-sum game
- inefficient equilibria

The Battle of the Bismark Sea:
- zero-sum game
- iterated dominant strategy
- Nash equilibrium

Boxed Pigs:
- rationality
- game in normal form (payoff matrix)
- weakness may be strength
- efficient equilibria
- game in extensive form (game tree)
- information set

The Battle of the Sexes:
- coordination, not rivalry
- first-mover advantage
- focal points

See the Definitions in Lecture 6 (from Rasmusen), and the Glossary in the Package (from Baird Gertner & Picker).