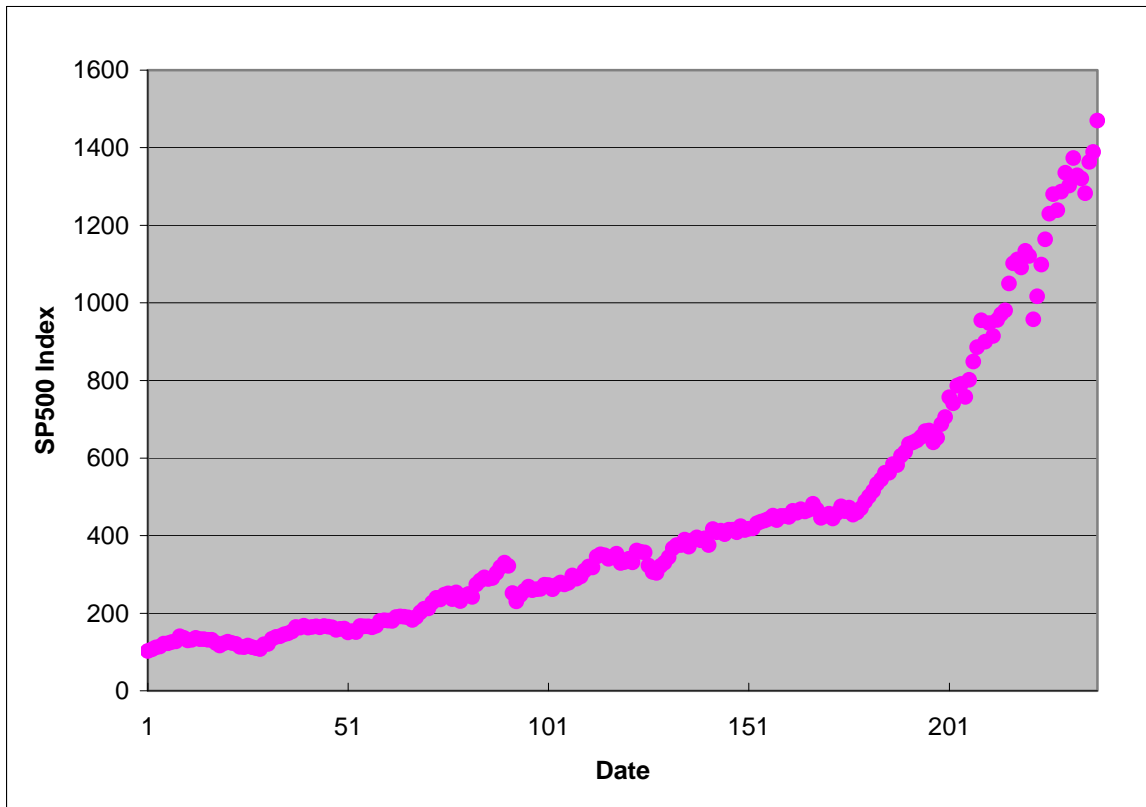


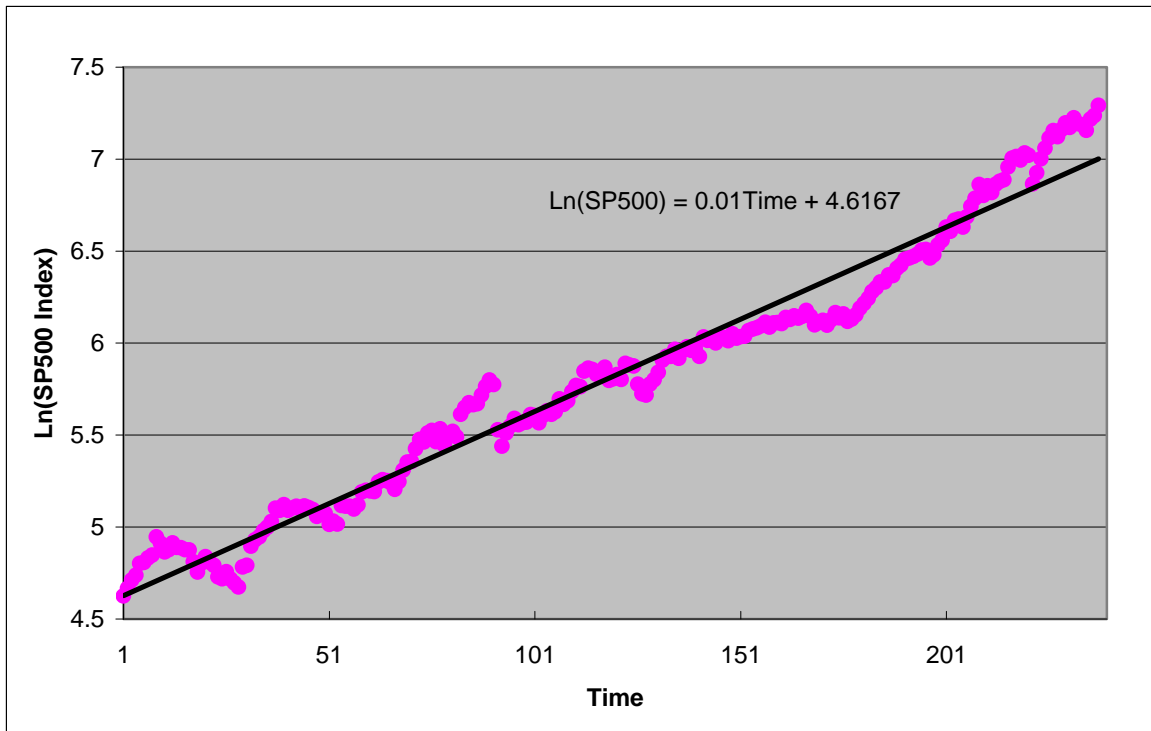
MBA Maths

Practice Quiz 5 Solution 2009

Question 1

The graph below plots the natural log of the monthly closing prices of the SP500 index from March 1980 to June 1999. The graph is plotted such that March 1980 corresponds to time, $T=0$, April 1980 $T=1$, ... , June 1999 $T=231$





- (i) What type of relationship describes the growth of the SP500 index over time?

Exponential

- (ii) What is the average **monthly** growth rate expressed continuously?

1%

- (iii) What is the average **annual** growth rate expressed continuously?

12%

(v) What was the value of the SP500 index at March 1980 (i.e. when $T=0$)?

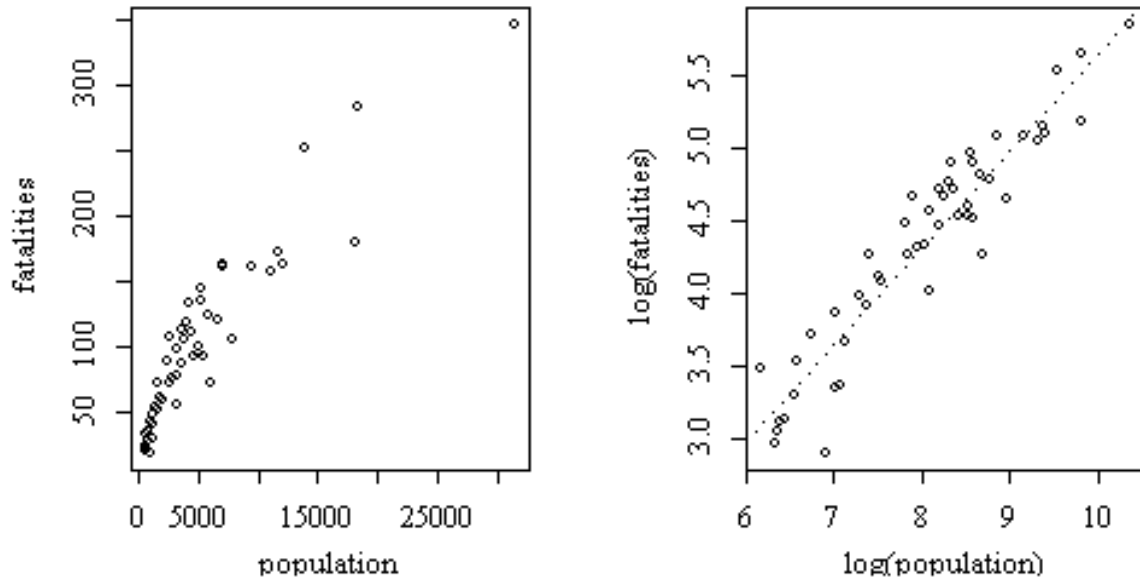
$$e^{4.6167} = 101$$

(vi) Let S be the value of the SP index. Express S as a function of time in months?

$$S = 101 \times e^{0.01t}$$

Question 2

The graphs below depict the number of car fatalities in a state as a function of that states' population (state population is given in 1000's)



The line of best fit for the data in the right panel is

$$y = -1.0 + 2x/3$$

- (i) What type of function best describes the relationship between the number of fatalities in a state and the population of that state?

The relationship between fatalities (F) and population (P) is a **power function** because a plot of log (F) vs ln(P) is linear

- (ii) Write down an expression for the number of fatalities as a function of population.

If the relationship between fatalities (F) and population (P) is a power function, then it can be written in the form

$$F = aP^m$$

If we take logs of both sides we see

$$\ln(F) = \ln(a) + m\ln(P)$$

Comparing this equation with the $y = -1.0 + 2x/3$ we see that

$$\begin{aligned}
 y &= \ln(F) \\
 x &= \ln(P) \\
 \ln(a) &= -1 \\
 a &= e^{-1} \\
 &= 0.368 \\
 m &= \frac{2}{3}
 \end{aligned}$$

Therefore

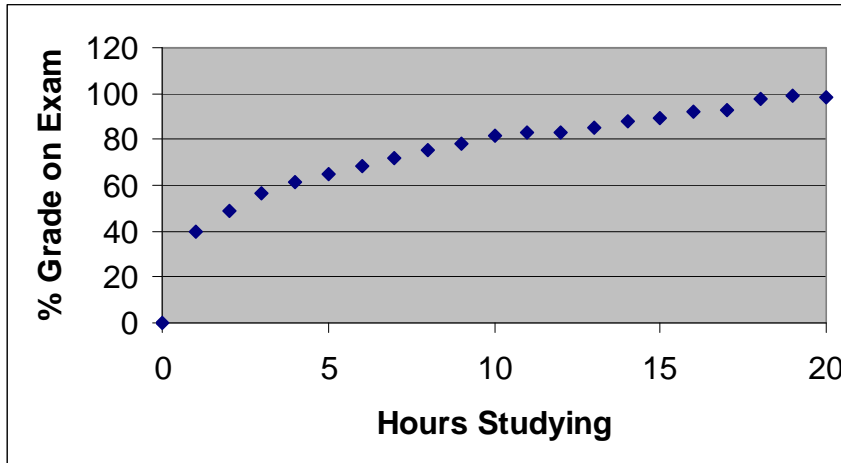
$$F = 0.368P^{\frac{2}{3}}$$

(iii) If you were told that a state's population was 15,000 (expressed in thousands) what would be your best guess of that state's car fatalities?

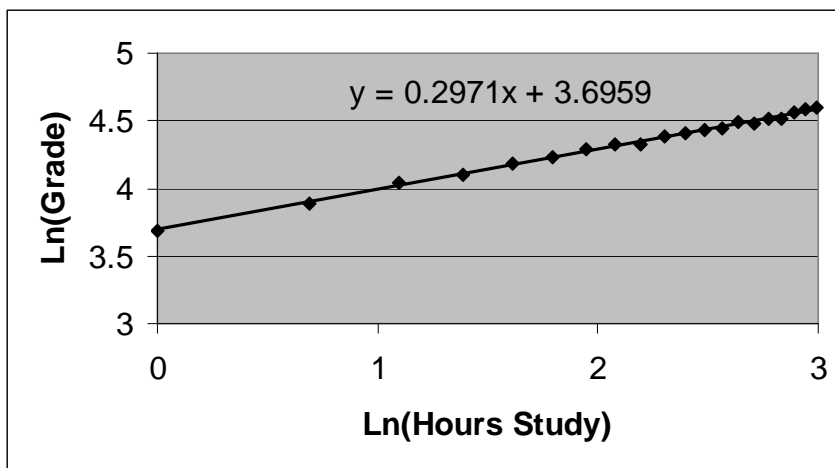
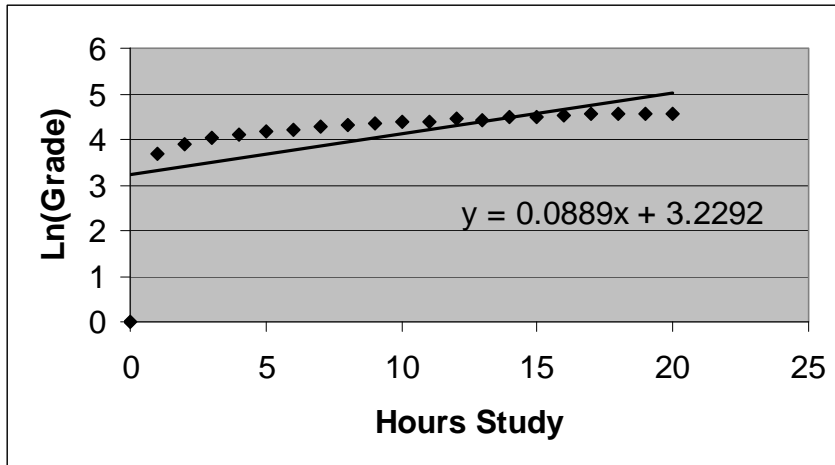
$$\begin{aligned}
 F &= 0.368P^{2/3} \\
 &= 0.368 \times 15000^{2/3} \\
 &= 223.7
 \end{aligned}$$

Question 3

A researcher decided to examine the effect of the length of time MBA students spent studying on the performance of these students on a maths exam. Students were given material that they had never seen before and examined on this material one week later. The data is summarized in the graph below



The relationship looks non-linear and to determine the nature of the relationship between "% Grade on Exam" and "Hours Spent Studying" the researcher decides to construct the following two plots. The first graph plots the " $\ln(\% \text{ Grade on final exam})$ " vs "Hours Spent Studying". The second graph plots " $\ln(\% \text{ Grade on Exam})$ " vs " $\ln(\text{Hours spent studying})$ "



- (i) On the basis on these graphs what type of function best describes the relationship between “% Grade on Exam” and “Hours spent studying”? Explain

We can describe this relationship as a power function because a plot of $\ln(\text{Grade})$ vs $\ln(\text{Hours})$ is linear

- (ii) Given your answer to part (a) write down the function that relates “% Grade on Exam” to “Hours spent studying”

If the relationship between Grade (G) and Hours (H) is a power function, then it can be written in the form

$$G = aH^m$$

If we take logs of both sides we see

$$\ln(G) = \ln(a) + m \ln(H)$$

Comparing this equation with the $y = 3.6959 + 0.2971x$ we see that

$$y = \ln(G)$$

$$x = \ln(H)$$

$$\ln(a) = 3.6959$$

$$a = e^{3.6959}$$

$$= 40.28$$

$$m = 0.2971$$

Therefore

$$G = 40.28H^{0.2971}$$

- (iii) If a student spends 3 hours studying what grade would that student expect.

$$G = 40.28H^{0.2971}$$

$$= 40.28 \times 3^{0.2971}$$

$$= 55.82$$

- (iv) If a student spends 20 hours studying what grade would that student expect.

$$G = 40.28H^{0.2971}$$

$$= 40.28 \times 20^{0.2971}$$

$$= 97.5\%$$

CORRECTION : $G = 98.1\%$

Question 4

Microsoft shares were trading for \$1.21 at the end of December in 1989. The annual growth rate of Microsoft shares for the 1990's was 57.9%, expressed as ordinary compound interest.

- (i) What was the annual growth rate expressed as a continuous compound rate?

Ordinary compound rate $r_o = 57.8\%$

$$\begin{aligned} r_c &= \ln(1 + r_o) \\ &= \ln(1.579) \\ &= 0.4567 \\ &= 45.67\% \text{ p.a.} \end{aligned}$$

- (ii) What was the monthly growth rate expressed continuously?

If the annual rate is 45.67% then the monthly rate is $45.67/12 = 3.8\% \text{ p.m.}$

- (iii) If the natural log of monthly prices of Microsoft for the 1990's were plotted against time (measured in months) we would observe a straight line. What would be the intercept of this line?

Let S be the share price of Microsoft. We are told that

$$\begin{aligned} S &= A(1 + r_o)^T \\ &= A \times 1.576^T \end{aligned}$$

Where A is the value of Microsoft share at time $T=0$, i.e. $A = \$1.21$. Equivalently we can write

$$\begin{aligned} S &= A e^{r_c T} \\ &= 1.21 \times e^{0.4567T} \end{aligned}$$

Taking logs of both sides we have

$$\ln(S) = \ln(A) + r_c T$$

Which is in the form of the straight line

$$y = mx + b$$

where

$$y = \ln(S)$$

$$b = \ln(A)$$

$$m = r_c$$

$$x = T$$

So that the intercept

$$b = \ln(A)$$

$$= \ln(1.21)$$

$$= 0.19$$

(iv) What would be the slope?

We can see from above that the slope $m=r_c=0.038$. Note that the monthly continuous interest rate was used because we are plotting monthly prices.

(v) Let Y be the natural log of the monthly closing price of Microsoft shares. Express Y as a function of time.

From above we have

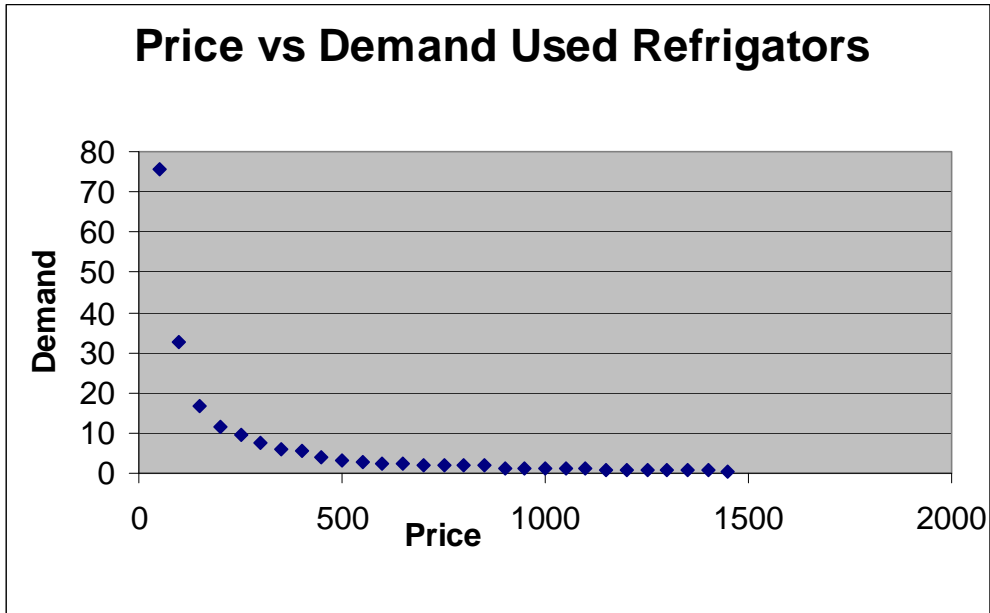
$$y = 0.19 + 0.038T$$

(vi) Let S be the monthly closing price of Microsoft shares. Express S as a function of time.

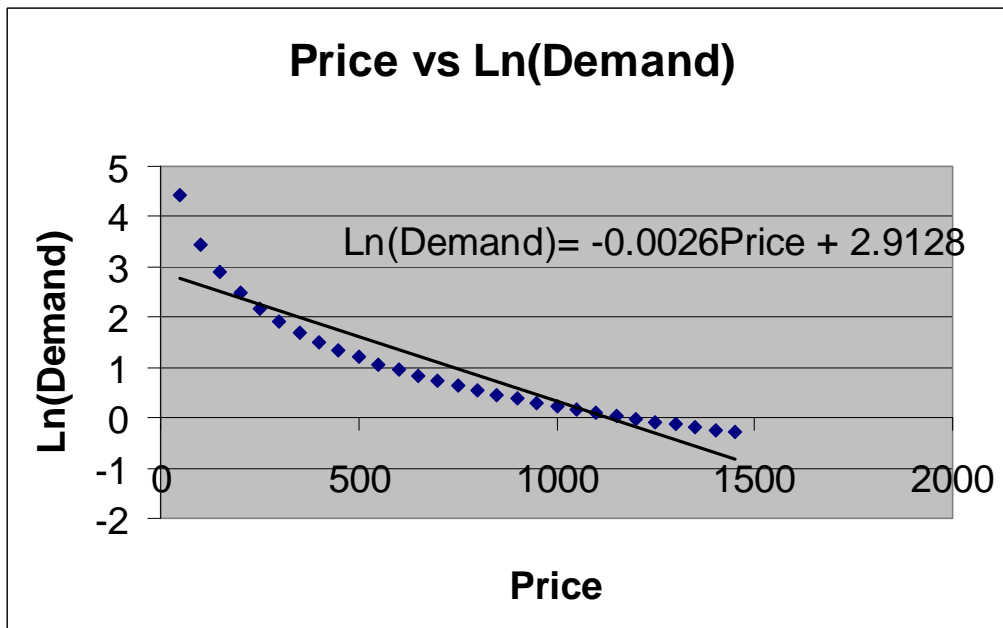
$$\begin{aligned} S &= Ae^{r_c T} \\ &= 1.21 \times e^{0.038T} \end{aligned}$$

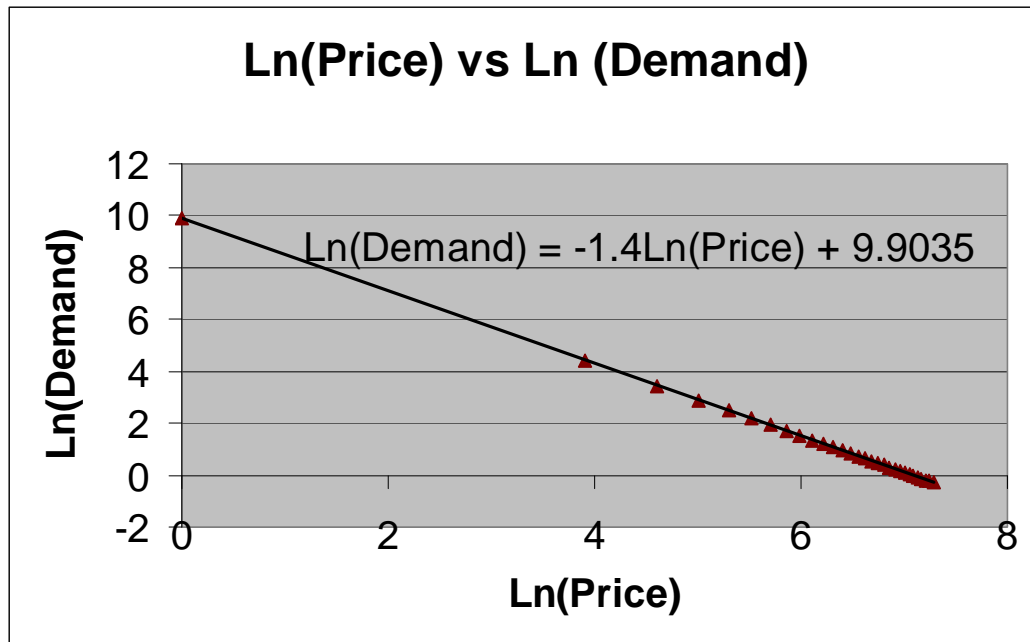
Question 5

Data were collected on the demand of used refrigerators versus the price charged and are plotted below



You decide to construct the following graphs in order to determine the relationship between Demand and Price.





- (i) What type of relationship best describes the relationship between Demand and Price?

A power function because a plot of $\ln(\text{Demand})$ vs $\ln(\text{Price})$ is linear

- (ii) Write down the relationship between Demand (Q), and Price (P)?

If the relationship between Demand (Q) and Price (P) is a power function, then it can be written in the form

$$Q = aP^m$$

If we take logs of both sides we see

$$\ln(Q) = \ln(a) + m\ln(P)$$

Comparing this equation with the

$\ln(\text{Demand}) = 9.9 - 1.4 \times \ln(\text{Price})$ we see that

$$\ln(a) = 9.9$$

$$a = e^{9.9}$$

$$= 19930$$

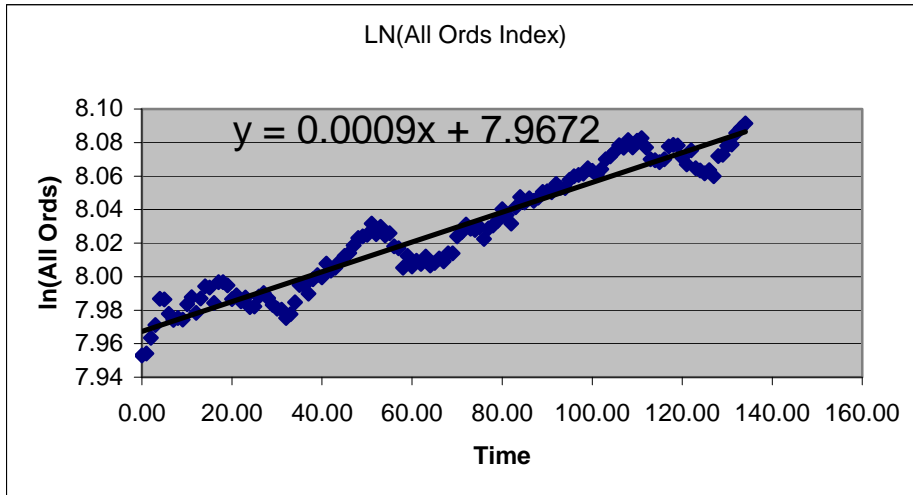
$$m = -1.4$$

Therefore

$$Q = 19930 \times P^{-1.4}$$

Question 6

The graph below plots the natural log of daily closing prices of the All Ords Index from April 2003 to Oct 2003. The graph is plotted such that April 1st 2003 corresponds to time, $T=0$, April 2nd 2003 $T=1$ etc.



(i) What is the average **daily** growth rate expressed **continuously**?

The daily growth rate is given by the slope, so that
 $r_c = 0.0009$ or 0.09%

(iii) What is the average **annual** growth rate expressed as **continuously** compound interest?

$365 \times 0.09\% = 32.85\%$

(iv)) What is the average **annual** growth rate expressed as an **ordinary** compound rate?

$$\begin{aligned}
 r_o &= e^{r_c} - 1 \\
 &= e^{0.3285} - 1 \\
 &= 0.3888 \\
 &= 38.88\%
 \end{aligned}$$

(v) What was the value of the All Ords Index in April 1st 2003 (i.e. when $T=0$)?

We know that $\ln(A)=b$, the intercept. Therefore $A=e^b$
 $=e^{7.9672}=2884$

(vi) Write down an expression for the value of the All Ords index as a function of time?

$$S = 2884 \times e^{0.3285T}$$

Where T is measured in years