

## MBA Maths Practice Quiz 4 2009 Solution

### Question 1

(a) The quantity demanded  $Q$  is a linear function of price. When price  $P=0$  the demand is 80. When price  $P=40$  the demand is zero. Give an equation for the quantity  $Q$  in terms of price  $P$  you should have  $Q$  on the left and an expression of the form  $b+mP$  on the right

$$m = -80/40 = -2 ; b = 80 ; Q = 80 - 2P$$

(b) The quantity supplied  $S$  is zero if the price is less or equal to than the unit cost of \$1.10. For every extra 30 cents above this, the quantity supplied rises by 1.8 units. Give an equation for the quantity supplied  $S$  in terms of price  $P$ . You should have  $S$  on the left and an expression of the form  $b+mP$  on the right.

$$m = 1.8/0.30 = 6 ; 0 = b + 6(1.10) ; b = -6.6 ; S = 6P - 6.6$$

### Question 2

Sketch the following functions

- a.  $-2x^2 + 2x + 4$
- b.  $2x^2 + 6x + 5$
- c.  $4x - 1 - 4x^2$

On each graph indicate the following

- (i) The point/s at which the function crosses the x-axis

This is asking us to solve the function by equating it to zero.

- (a) we can factorise to give  $-2(x-2)(x+1) = 0$ .

When  $x=2$  or  $x=-1$  the function is zero, hence the function crosses the x-axis at these points

- (b) This doesn't factorise and doesn't have any real roots at all where the function is zero. That is because in the formula for solving a quadratic equation,  $(b^2 - 4ac) = (6^2 - 4 \cdot 2 \cdot 5)$  is negative, and we cannot find square root of a negative number. We do not want to go into a discussion of imaginary roots!! Let us leave it to the mathematicians.

(c)  $4x - 1 - 4x^2 = 0$  ;  $4x^2 - 4x + 1 = 0$  .  $(2x - 1)^2 = 0$ . Thus  $x = 1/2$

(ii) The point at which the function crosses the y-axis.

Here we just need to set x to zero.

(a) 4 (b) 5 (c) - 1

(iii) The value of x which minimizes or maximizes the function.

For a quadratic function, a maximum is obtained when  $x = -b/2a$

(a)  $a = -2$ ,  $b = 2$  ; hence  $x = -2/2 \cdot -2 = 1/2$  and since a is negative,  $x = 1/2$  gives us the maximum

(b)  $a = 2$ ,  $b = 6$  ; hence  $x = -6/2 \cdot 2 = -3/2$  and since a is positive,  $x = -3/2$  gives us the minimum

(c)  $a = -4$ ,  $b = 4$  ; hence  $x = -4/2 \cdot -4 = 1/2$  and since a is negative,  $x = 1/2$  gives us the maximum

(iv) The minimum/maximum value of the function

Here we set x to be the value we calculated above back into the original equation:

(a) Maximum =  $-2(1/2)^2 + 2(1/2) + 4 = -1/2 + 1 + 4 = 4 \frac{1}{2}$

(b) Minimum =  $2(-3/2)^2 + 6(-3/2) + 5 = 9/2 - 9 + 5 = \frac{1}{2}$

(c) Maximum =  $4(1/2) - 1 - 4(1/2)^2 = 2 - 1 - 1 = 0$

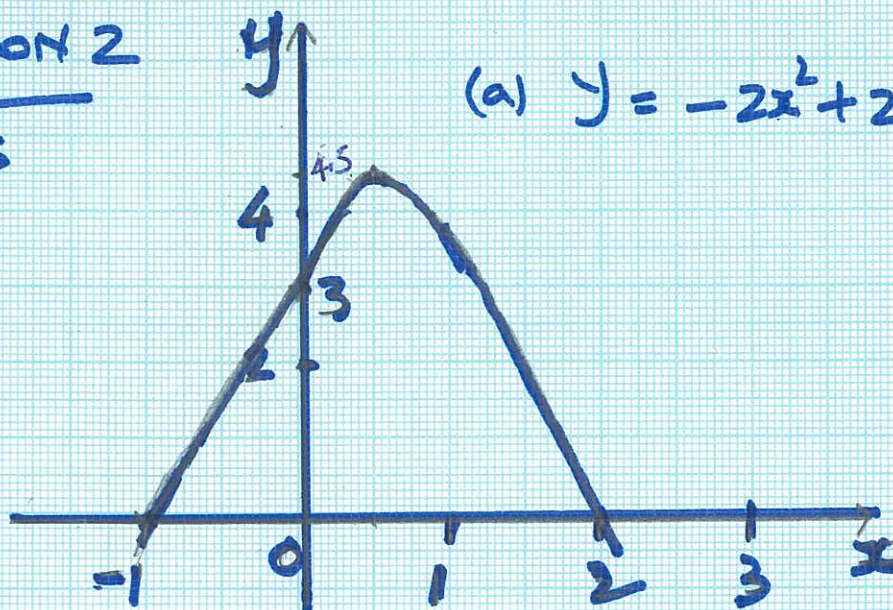


# PRACTICE QUIZ 4

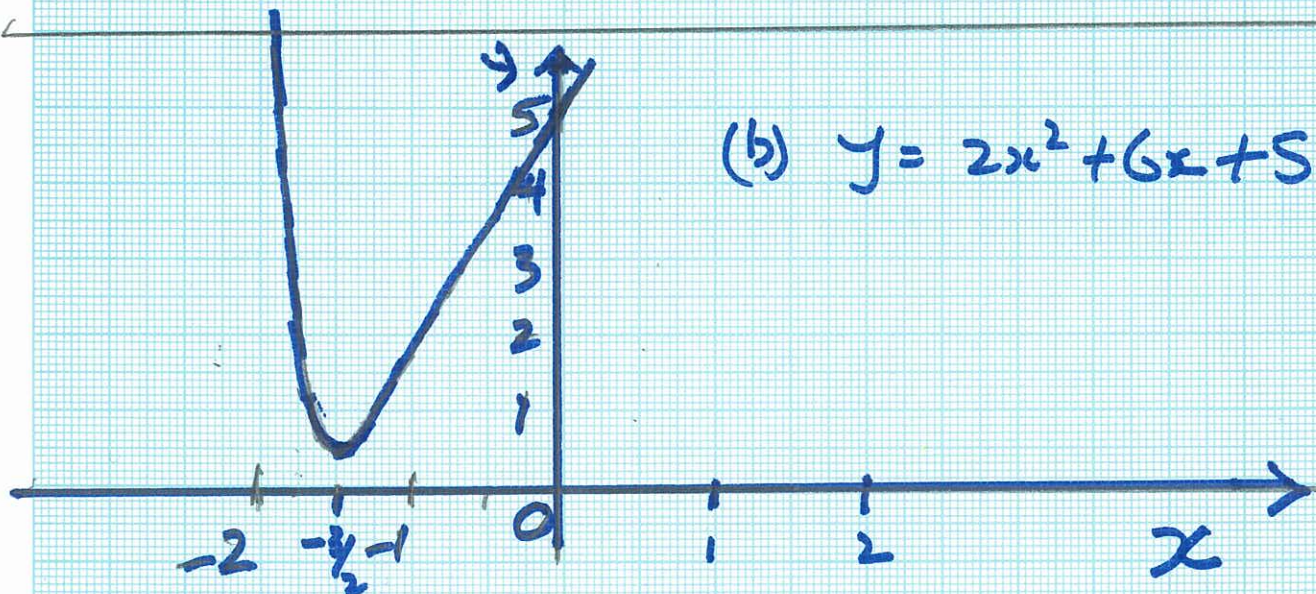
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## QUESTION 2

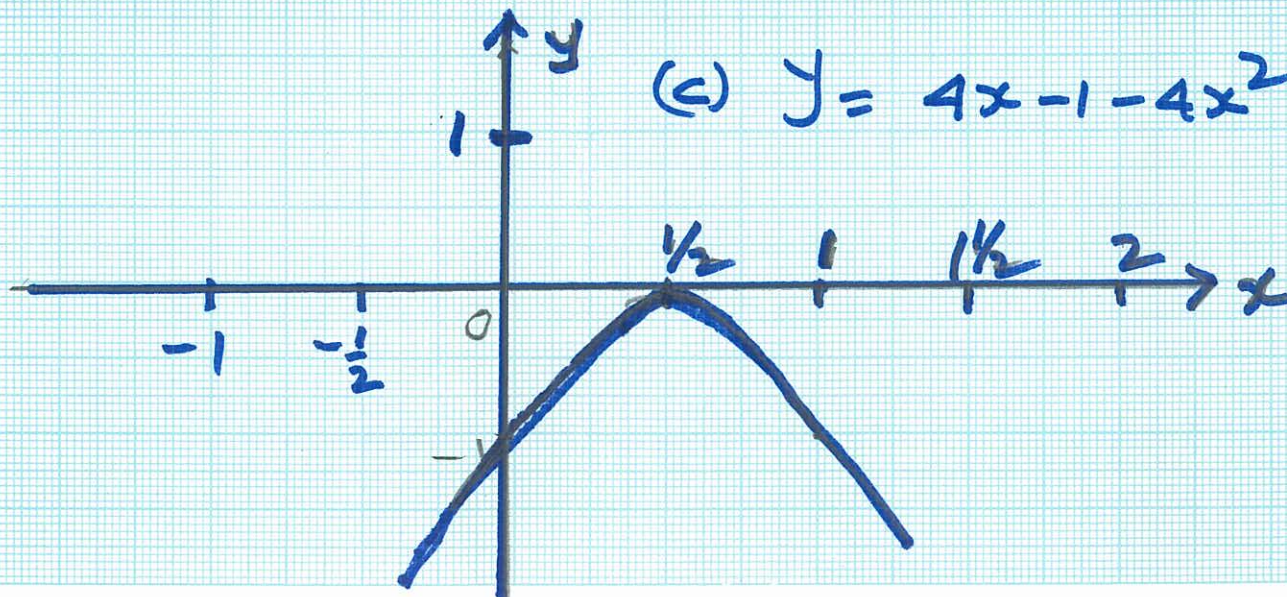
### GRAPHS



(b)  $y = 2x^2 + 6x + 5$



(c)  $y = 4x - 1 - 4x^2$





### Question 3

Suppose you are given the following information about the monthly demand for widgets.

- If the price is \$0, the demand for widgets will be 115,000 units per month. For every increase in price of \$3, the monthly demand will decrease by 15,000 units.
- Fixed costs are \$100,000 per month and variable costs are \$2 per unit sold.

(i) Write down the cost **C** of widgets as a function of demand **Q**.

$$C = 100000 + 2Q$$

(ii) Write down the demand **Q** for widgets as a function of price **P**.

$$\text{Slope } m = -15000/3 = -5000 ; b = 115000; Q = 115000 - 5000P$$

(iii) If revenue, **R** is given by **R=PQ**, write down revenue as a function of price.

$$R = PQ = P[115000 - 5000P] = 115000P - 5000P^2$$

(iv) Using your answers to (i) and (ii), write down cost, **C**, as a function of price, **P**.

$$C = 100000 + 2[115000 - 5000P] = 330000 - 10000P$$

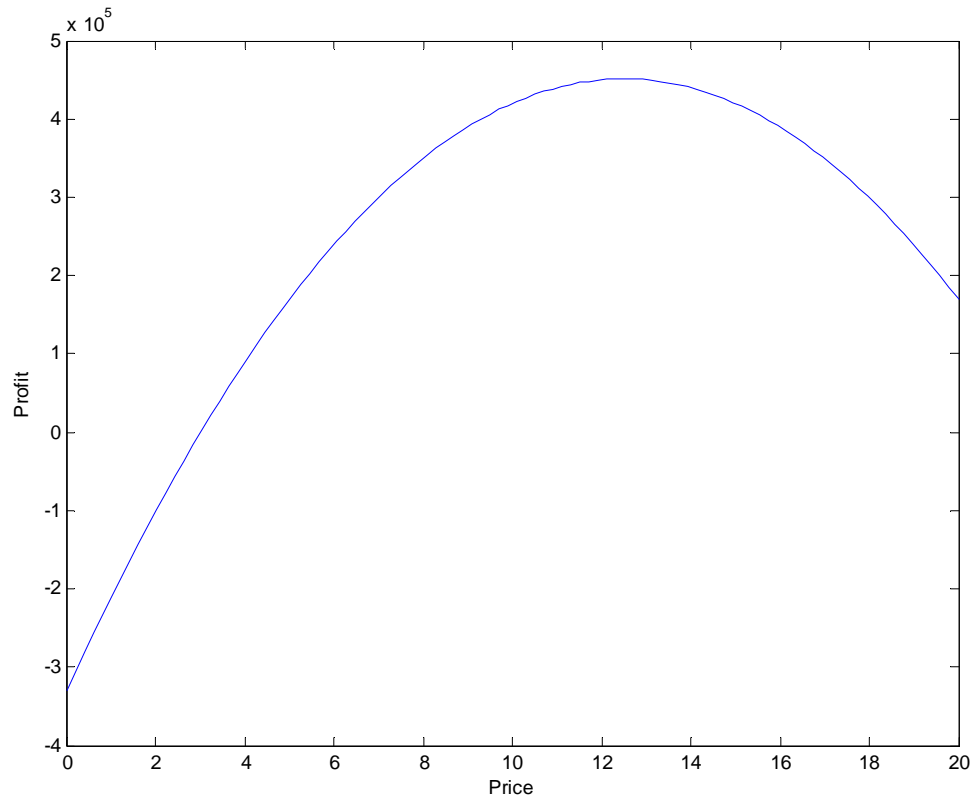
(v) If profit, **Pr**, is given by **Pr=R-C**, and write down an expression for profit as a function of price.

$$Pr = 115000P - 5000P^2 - [330000 - 10000P] = 125000P - 5000P^2 - 330000$$

(vi) Graph **Pr** vs **P**. On your sketch you should indicate

a. The prices at which revenue=0

We know that  $R=PQ$ , so  $P=0$  gives  $R=0$  and  $Q=115000-5000P$  from (ii) so  $P=23$  resulting in  $Q=0$  also gives  $R=0$



b. The price which maximizes revenue

Maximum revenue is obtained when  $P = -b/2a = -115000/2(-5000) = 11.5$

c. The value of revenue at this price.

Sticking this back into our revenue equation gives us 661250

$$\text{Profit Pr} = -5000P^2 + 125000P - 330000$$

For maximum profit,  $P = -125000/2(-5000) = 12.5$

$$\text{Maximum profit} = -5000(12.5)^2 + 125000(12.5) - 330000 = 451250$$

The graph is shown above for Profit versus Price.