MBA Maths
Practice Quiz 3 Solutions
2009

Question 1

In each case an investment grows from an initial value P to a final value S over T years. Give both the annual ordinary compound growth rate and the annual continuously compounded growth rate

(a) P=100, S=17000 T=34
(b) P=3.84 S=7.38 T=3
(c) P=13.35 S=425.6 T=45

P	S	F=S/P	T	ro=F^(1/T)-1	rc=ln(1+ro)
100	17000	170	34	0.16	0.15
3.84	7.38	1.921875	3	0.24	0.22
13.35	425.6	31.88015	45	0.08	0.08

Question 2

Consider the following two equations

$$S = P(1+r)^T \quad (1)$$

and

$$S = Pe^{r_c T} \quad (2)$$

(a)

a. If a contract specifies a continuously compounding rate of 5%p.a. how long until the investment increases by 80%?

Increases by 80% means that the growth factor S/P=1.8

$$\begin{aligned}
\frac{S}{P} &= e^{r_c T} \\
&= 1.8 \\
r_c T &= \ln(1.8) \\
T &= \frac{\ln(1.8)}{r_c} \\
&= \frac{\ln(1.8)}{0.05} \\
&= 11.75 years
\end{aligned}$$

b. If the contract specifies an ordinary compounding rate of 5%p.a. how long until the investment increases by 80%?

$$
\begin{aligned}
\frac{S}{P} &= (1+r_o)^T \\
&= 1.8 \\
(1+r_o)^T &= 1.8 \\
T\ln(1+r_o) &= \ln(1.8) \\
&= \frac{\ln(1.8)}{\ln(1.05)} \\
&= 12.05\,years
\end{aligned}
$$

(b)

a. If contract specifies a continuously compounding rate and you wish your investment to double within 10 years, what annual rate do you require?

$$
\begin{aligned}
\frac{S}{P} &= e^{r_c T} \\
&= 2 \\
r_c T &= \ln(2) \\
r_c &= \frac{\ln(2)}{T} \\
&= \frac{\ln(2)}{10} \\
&= 0.0693 \\
&= 6.93\%
\end{aligned}
$$

b. Do the same for a contract which specifies ordinary compounding.

$$
\begin{aligned}
\frac{S}{P} &= (1+r_o)^T \\
&= 2 \\
1+r_o &= 2^{1/T} \\
r_o &= 2^{1/T}-1 \\
&= 2^{1/10}-1 \\
&= 0.0718 \\
&= 7.18\%
\end{aligned}
$$

(c) You have the choice of the following three investments
   a. 1.5% per quarter, continuously compounding
   b. 0.55% per month ordinary compounding
   c. 3.2% semi annually ordinary compounding

   Which investment would you choose to maximize your return?

Need to choose a common interest rate, say we choose r_c per quarter then
(a) r_c=1.5% per quarter
(b) $r_c=\ln(1+r_o)=\ln(1.0055)=0.00548$ per month. Therefore quarterly $r_c=3\times0.00548=0.0165=1.65\%$
(c) $r_c=\ln(1+r_o)=\ln(1.032)=0.0315$ per 6 months. Therefore quarterly $r_c=0.0315/2=0.0157=1.57\%$

So investment (b) maximizes return.

Question 3

Give an equation of the form y=mx+b.

(a) When x=0, y=2. For each increase in x, y decreases by 2.

We are told the slope m=-2. The intercept is the point at which the line crosses the y-axis. This is the value of y when x=0, which are also told is 2, so b=2. Therefore equation of the straight line is

$$y = 2 - 2x$$

(b) When x=3, y=0 while when x=0, y=4

Intercept b=4

$$Slope = m = \frac{\Delta y}{\Delta x}$$

$$\Delta y = 0 - 4$$
$$= -4$$
$$\Delta x = 3 - 0$$
$$= 3$$
$$m = \frac{-4}{3}$$

So equation of line is

$$y = 4 - \frac{4}{3}x$$

(c) When x=2 y=5 and when x=9 y=5

$$\text{Slope} = m = \frac{\Delta y}{\Delta x}$$

$$\Delta y = 5 - 5$$
$$= 0$$
$$\Delta x = 2 - 9$$
$$= -7$$
$$m = \frac{0}{-7}$$
$$= 0$$
$$b = y - mx$$
$$= 5 - 0 \times 2$$
$$= 5$$

So equation of line is
y=5

(d)When y=0 x=4 and for each 2 unit increase in x y decreases by 1 unit

$$\text{Slope} = m = \frac{\Delta y}{\Delta x}$$

$$\Delta y = -1$$
$$\Delta x = 2$$
$$m = -\frac{1}{2}$$
$$b = y - mx$$
$$= 0 - \left(-\frac{1}{2} \times 4\right)$$
$$= 2$$

So equation of line is
y=2-*x*/2

(e) When price x equals the unit cost 8 the quantity y supplied is 0. For each 50 cents above this price, the quantity supplied increases by 3 units.

$$\text{Slope} = m = \frac{\Delta y}{\Delta x}$$

$$\Delta y = 3$$
$$\Delta x = 0.5$$

$$m = \frac{3}{0.5}$$
$$= 6$$

$$\begin{aligned} b &= y - mx \\ &= 0 - 6 \times 8 \\ &= -48 \end{aligned}$$

So equation of line is
y=6x-48
where y= quantity supplied and x=price.

Question 4

(a) Suppose the demand function is given by Q=200-10P and the supply function is S=5P-25. Find the equilibrium price and quantity.

Equilibrium occurs when S=Q

$$\begin{aligned} 5P - 25 &= 200 - 10P \\ 15P &= 225 \\ P &= 15 \end{aligned}$$

When P=15, Q=200-15×10=50

(b) Suppose that demand decreases by 5 units for every increase in price of $2, and that when price equals $60 the demand is zero. Suppose further that the quantity supplied is zero for the price is less then or equal to $20 and that for each $1 increase in price the quantity supplied increases by 4 units. Find the equilibrium quantity and price.

<u>The demand Function</u>

$$\text{Slope} = m = \frac{\Delta Q}{\Delta P}$$

$$\Delta P = 2 \;\; ; \;\; \Delta Q = -5$$

$$\begin{aligned} m &= \frac{-5}{2} \\ &= -2.5 \end{aligned}$$

Q = mP + b ; 0 = -2.5x60 + b ;

b = 150

So equation of line is

Q = 150-2.5P

The Supply Function

Slope $= m = \frac{\Delta S}{\Delta P}$

$\Delta P = 1$
$\Delta S = 4$

$$m = \frac{4}{1}$$
$$= 4$$

$$b = S - mP$$
$$= 0 - 4 \times 20$$
$$= -80$$

So equation of line is

S=4P-80

If S = Q then 150-2.5P=4P-80 ; 230=6.5P

P = 230/6.5 = $35.38

Q = 150 – 2.5x230/6.5 OR 4X230/6.5 – 80 = 61.5 units