MBA Maths

Practise Quiz 2 solutions 2009

Question 1

Evaluate

(a) $\log_7 343$

3 because

$7^3=343$

(b) $\log_5 0.04$

-2 because

$5^{-2}=1/5^2$

$=1/25$

$=0.04$

(c) $\log_{10} 0.0001$

-4 because

$10^{-4}=1/10^4$

$=0.0001$

(d) $\log_8 64$

2 because $8^2=64$

(e) $\log_{16} 0.50$

-1/4 because

$16^{-1/4}=1/16^{1/4}$

$=1/2$

$=0.5$

(f) $\log_6 1/9 + \log_6 1/4$

-2 because

$\log_6 1/9 + \log_6 1/4 = \log_6 1/36$

and

$6^{-2} = 1/36$

(g) $\log_{10} 5 + \log_{10} 20$

2 because

$\log_{10} 5 + \log_{10} 20 = \log_{10} 100$

and

$10^2 = 100$

(h) $\log_9 405 - \log_9 5$

2 because

$\log_9 405 - \log_9 5 = \log_9 81$

and

$9^2 = 81$

(i) $\log_3 \sqrt[3]{108} - \log_3 \sqrt[3]{4}$

1 because

$\log_3 \sqrt[3]{108} - \log_3 \sqrt[3]{4} = \log_3 \sqrt[3]{27}$

$= \log_3 3$

$= 1$

(j) $\ln e^{2/3} + \ln e^{4/3}$

2 because

$\ln e^{2/3} + \ln e^{4/3} = \frac{2}{3}\ln e + \frac{4}{3}\ln e$

$= \frac{2}{3} + \frac{4}{3}$

$= \frac{6}{3}$

$= 2$

Question 2

(a) If the growth factor for 1 year is 1.075 what is the interest rate expressed as

i. An annual compound rate?

$F = (1+r)^T$

$= 1.075$

$T = 1$ year

$r = 1.075 - 1$

$= 0.075$ or 7.5%

ii. A quarterly compound rate?

$F = (1+r)^T$

$= 1.075$

$T = 4$ quarters

$1.075 = (1+r)^4$

$r = 1.075^{1/4} - 1$

$= 0.018$ or 1.8%

(b) If an initial investment of 1550 is worth 2200 at the end of 18 months, what was the interest rate expressed as

i. A monthly compound rate?

$$F = \frac{2200}{1550}$$
$$= 1.42$$
$$F = (1+r)^T$$
$$T = 18 \text{ months}$$
$$1.42 = (1+r)^{18}$$
$$r = 1.42^{1/18} - 1$$
$$= 0.0196 \text{ or } 1.96\% \text{ per month}$$

ii. An annual compound rate?

$$F = \frac{2200}{1550}$$
$$= 1.42$$
$$F = (1+r)^T$$
$$T = 18 \text{ months}$$
$$= 1.5 \text{ years}$$
$$1.42 = (1+r)^{1.5}$$
$$r = 1.42^{1/1.5} - 1$$
$$= 0.263 \text{ or } 26.3\% \text{ per year}$$

(c) Referring to part (b)

i. How much will the investment be worth at the end of 2 years?

$$S = P(1+r)^T$$
$$P = 1550$$
$$T = 2 \text{ years}$$
$$r = 0.263 \text{ p.a.}$$
$$S = 1550 \times (1.263)^2$$
$$= 2472$$

ii. How much was the investment worth after 3 months?

$S = P(1+r)^T$

$P = 1550$

$T = 3$ months

$r = 0.0196$ p.a.

$S = 1550 \times (1.0196)^3$

$= 1643$

(d) If you invest $10,000 at a semi-annual interest rate of 3.5%, how much is the investment worth after

i. One year?

$S = P(1+r)^T$

$P = 10000$

$r = 0.035$ per 6 months

$T = 1$ year

$= 2 \times 6$ months

$S = 10000 \times (1.035)^2$

$= 10712$

ii. After 15 months?

$S = P(1+r)^T$

$P = 10000$

$r = 0.035$ per 6 months

$T = 15$ months

$= 2.5 \times 6$ months

$S = 10000 \times (1.035)^{2.5}$

$= 10898$

(e) If you invest $5000 at a month compound interest rate of 1%

i. how long will it take for your investment to triple?

$$S = P(1+r)^T$$

$$F = (1+r)^T$$

Tripling your investment means

$$F = 3$$

$$3 = (1+r)^T$$

$$\ln(3) = \ln(1+r)^T$$

$$= T\ln(1+r)$$

$$T = \frac{\ln(3)}{\ln(1+r)}$$

$r = 0.01$ per month

$$T = \frac{\ln(3)}{\ln(1.01)}$$

$= 110.4$ months

ii. If you invested \$10,000 (instead of \$5000) how would your answer to part (e)-(i) change?

It wouldn't change at all