A Theory of Choice

Here we build a theory of rational choice in order to understand how the consumer chooses between alternatives in a world with scarcity. Six parts:

1. *Utility theory*: provides a framework for modelling rational choice, under the assumption that the consumer acts to maximise his or her utility, or satisfaction. **H&H:** Chap. 3.1–3.2.
2. *Indifference Curves*: allow us to avoid asking "by how much more does John prefer *A* to *B* than does Mary?" Allow us to solve the choice problem of the consumer faced by prices and limited income. **H&H:** Chap. 3.3–3.5.
3. *Choice Set*: how can we characterise the optimum choice of the consumer? What if several consumers face the same prices, but with different incomes? **H&H:** Chap. 4.1.
4. *Gains from Trade*: how does voluntary exchange make both parties better off? How does exchange add value? **H&H:** Chap. 13.1.
5. *Demand functions*: the effects of changes in income, and changes in price (own price and related goods' prices). The Slutsky equation and income effects. Gross and true substitutes. **H&H:** Chap. 4.2–4.4, 5.1–5.3.
6. *Market Demand*: horizontal summing individual consumers' demands **H&H:** Chap. 4.5.

1. Utility

Question:

What is the *best* combination of consumer goods (and services) for any individual?

or, equivalently,

What choice(s) or bundles of goods and services maximize the consumer's *utility*?

The Law of Preference

The formalities — Two axioms:

1. *Axiom of Preference*

   each comparison of any two *bundles* *A* and *B* of goods and services results in *one* of:

   (1) bundle *A* preferred to bundle *B* (*A P B*)
   (2) bundle *B* preferred to bundle *A* (*B P A*)
   (3) indifference between bundles *A* & *B* (*A I B*).

   (*P* : "is preferred to")
   (*I* : "is indifferent to")

   Example of a bundle?

   What does this axiom rule out?

2. *Axiom of Transitivity*

if $A\ P\ B$ and $B\ P\ C$ then $A\ P\ C$.

- Then axioms 1 & 2 result in

the *Proposition of Rank Ordering of Preferences:*

all possible bundles of goods can be consistently ranked in order of preference by the consumer.

A ***good*** : is a commodity more of which is preferred to less.

(A "***bad***" is the opposite.) ["bad" is slang]

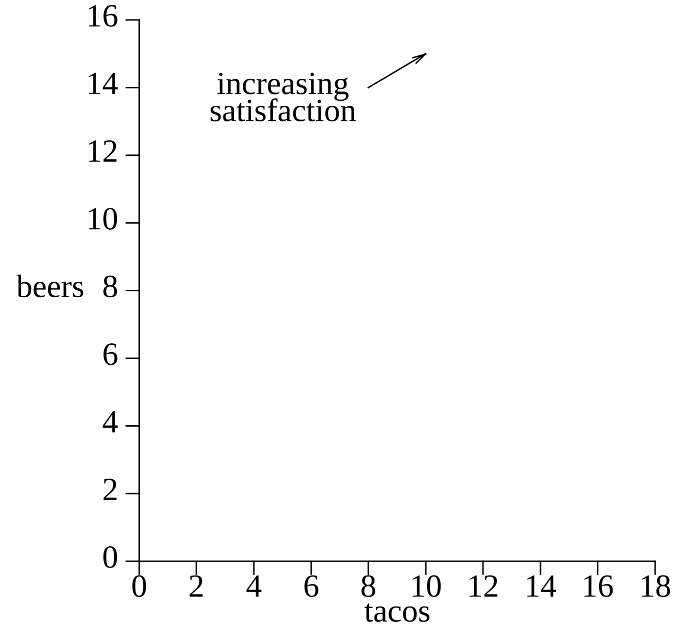
We assume *non-satiation* in general, that is, consumers are always happy for more.

We say that the individual consumer chooses *the most preferred bundle*, and this is as though he was maximizing his "*utility*" over the choice of bundles, subject to any constraints on his choice.

such as his budget
availability
etc.

Revision

A volunteer?



A:	*E*: 20 tacos, 0 beer
B:	*F*: tacos, 5 beers
C:	*G*: tacos, 1 beer
D:	*H*:

A *good:*
something of which more is preferred, or better.

A *"bad":*
something of which less is preferred, or worse.

1.1 Utility Functions

The *utility* of a consumer is a function of the bundle of goods and services chosen (all the goods & services he (she) chooses).
The utility of bundle $\mathbf{x}$ is written $U(\mathbf{x})$, where $\mathbf{x} = (x_1, x_2, x_3, x_4, \ldots)$.

e.g. $U = U$ (beer, tacos)

The utility of bundle $\mathbf{x}$ is greater than the utility of bundle $\mathbf{y}$ if and only if (= iff) bundle $\mathbf{x}$ is preferred to bundle $\mathbf{y}$.

$$U(\mathbf{x}) > U(\mathbf{y}) \Leftrightarrow \mathbf{x}\, P\, \mathbf{y}$$

$$U(\mathbf{x}) \geq U(\mathbf{y}) \Leftrightarrow \mathbf{x}\, R\, \mathbf{y}$$

(The individual prefers $\mathbf{x}$ to $\mathbf{y}$ or is indifferent: $\mathbf{x}\, R\, \mathbf{y}$.)

So now we can work with the numbers associated with the utilities of different bundles. But this implies *cardinal* choice, *(by how much*?) whereas $\mathbf{x}\, P\, \mathbf{y}$ ($\mathbf{x}$ is preferred to $\mathbf{y}$) is *ordinal* choice:

(bundles can be ranked without asking *by how much?*)

Ordinal choice: a ranked ordering of preferences.

Cardinal choice: a measure of *how much* one bundle is preferred to another.

Example:

{ Christmas 1985 was hotter than Christmas 1984 } ordinal

{ It was 10 °C hotter 18 °F } cardinal

Utility functions ⇒ cardinal choice, which may be more than we need to model consumers' behaviour.

Utility of One Good: e.g. U (beer)

Non-satiation implies that:

we always want more, or that more is preferred to less

⇔ increasing utility

⇔ positive marginal utility of good

$$\frac{\Delta U}{\Delta \text{beer}} > 0 \text{ or}$$

$$\frac{dU}{d\,\text{beer}} > 0$$

The *marginal utility* of good j is written MU_j, where

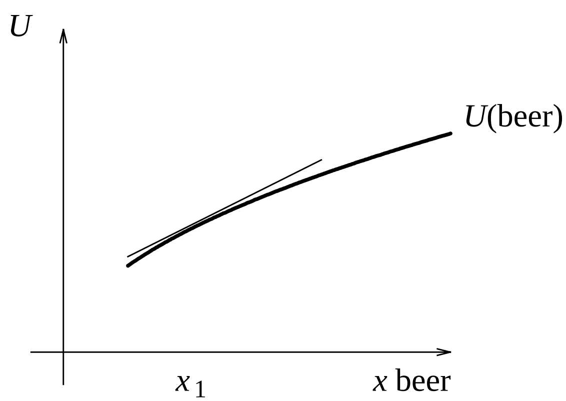
$$MU_j \equiv \frac{\partial U(\mathbf{x})}{\partial x_j} = \frac{\partial U(x_1 \ \cdots \ x_j \ \cdots \ x_n)}{\partial x_j}$$

$$= \lim_{\Delta x_j \to 0} \left. \frac{\Delta U}{\Delta x_j} \right| \text{ all else equal, (ceteris paribus)}$$

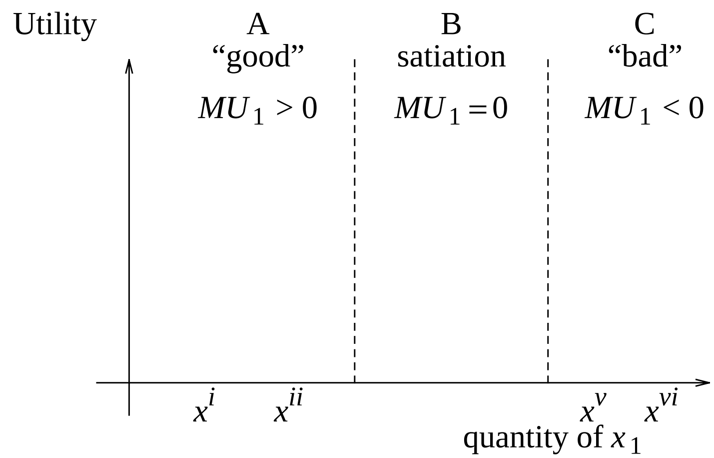
= the utility associated with an additional (marginal)

unit of x_j

$$\frac{\partial U}{\partial x_{beer}} > 0$$



Ceteris paribus: holding constant:
- the number of tacos, and
- everything else.



A. $U(x^{ii}) > U(x^i) \Leftrightarrow x^{ii} \ P \ x^i$

$MU_1 > 0$, positive slope

but diminishing MU_1: $\dfrac{\partial^2 U}{\partial x_1^2}$

$$\equiv \frac{\partial MU_1}{\partial x_1} < 0$$

B. $MU_1 = 0$: satiation, unchanging utility
i.e. horizontal slope in B

C. $MU_1 < 0$: “bad”, negative slope

$$U(x^v) > U(x^{vi}) \Leftrightarrow x^v \ P \ x^{vi}$$

i.e. less is preferred to more
i.e., x^v is preferred to x^{vi}

Marginal utility (MU_1) is the slope of the utility function U as the amount of good 1 increases, cet. par.

Marginal utility is the slope of the total utility U curve.

Consider Joe in the desert, craving for an Internet connection, oops, no, craving for a nice cold refreshing drink.

He crests a sandhill, and what should he see before him but a couple of kids selling home-made lemonade, which Joe just happens to looove.

He takes a glass, and drains it. Hmmm, that was goood.

He takes a second glass, and drinks it; and a third, and a fourth. He's really thirsty and can still drink a fifth, a sixth, and a seventh.

But by his eighth glass, although he's still enjoying the sweet, cold liquid in the hot, dusty desert, he can't honestly say that the next drink is as delightfully thirst-quenching and satisfying as the first or the second.

If you understand Joe's reaction to the ninth glass of home-made lemonade, then you understand the phenomenon we call *diminishing marginal utility*.

Empirically, we observe *diminishing marginal utility*:

$$\frac{\partial MU_j}{\partial x_j} \equiv \frac{\partial^2 U}{\partial x_j^2} < 0,$$

which accords with intuition.

That is, additional units provide ever less utility.
(If we drew the MU curve, it would have negative slope.)

But in general:

- non-satiation: $MU_i \equiv \dfrac{\partial U}{\partial x_i} \neq 0$

  for a good i: $MU_i > 0$
  for a "bad" j: $MU_j < 0$

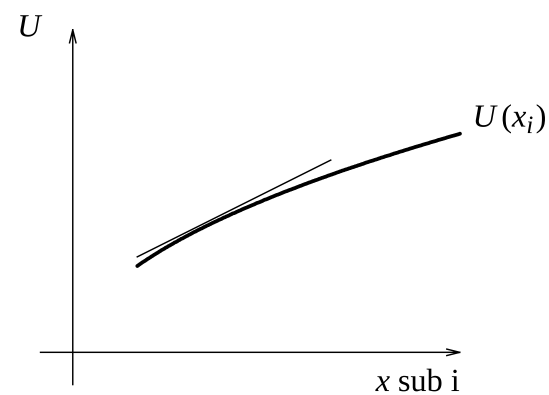
- diminishing marginal utility of a good:

  $MU_i > 0$

  but falling:

  $\dfrac{\partial^2 U}{\partial x_i^2} < 0$ (diminishing marginal utility)

- for a good, $U > 0$



$\Delta U_1 > \Delta U_2$:
diminishing marginal utility
Slope of the tangent = MU_i at x_i^2.

>> Include H&H Fig 3.3, 3.4 <<

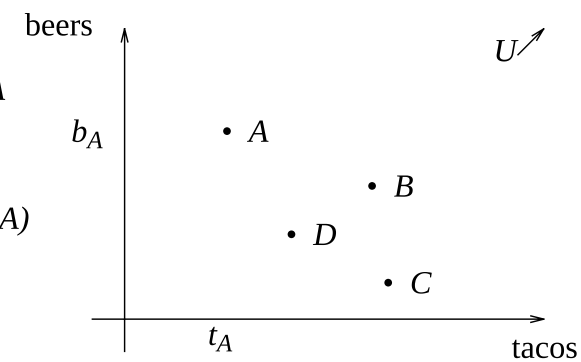
2. Indifference Curves

Indifference curves: locuses of equal-utility (iso-utility) contours join bundles to which the individual is *indifferent*.

(Projection of equal utility contours onto the plane of the bundles.)

B is preferred to *A*

$B \; P \; A$
$\Leftrightarrow$
$U(B) > U(A)$



B is on a "higher" indifference curve than is *A* $(U_2 > U_1)$

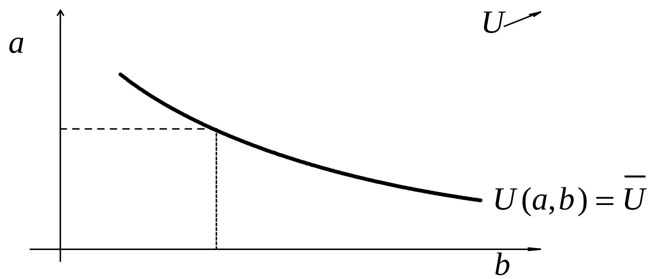
$$
\begin{array}{lcl}
A \; I \; C & \Leftrightarrow & U(A) = U(C) \\
A \; I \; D & \Leftrightarrow & U(A) = U(D) \\
\therefore \; C \; I \; D & \Leftrightarrow & U(C) = U(D)
\end{array}
$$

A and *C* are on the same indifference curve
A and *D* are on the same indifference curve

2.1 Five Characteristics of Indifference Curves

1. Need an ordinal ranking only: indifference curves don't require cardinal ("*by how much*?").
   Cardinal $\Rightarrow$ ordinal, but
   ordinal $\not\Rightarrow$ cardinal.

2. For two *goods* (increasing utility, remember), indifference curves are negatively sloped: An indifference curve is a set of points along which utility is constant (a and b are both *goods*).



Differentiate $U(a,b)$ = constant *totally*:

$$dU = \frac{\partial U}{\partial a}\,da + \frac{\partial U}{\partial b}\,db$$

$$= MU_a da + MU_b db$$

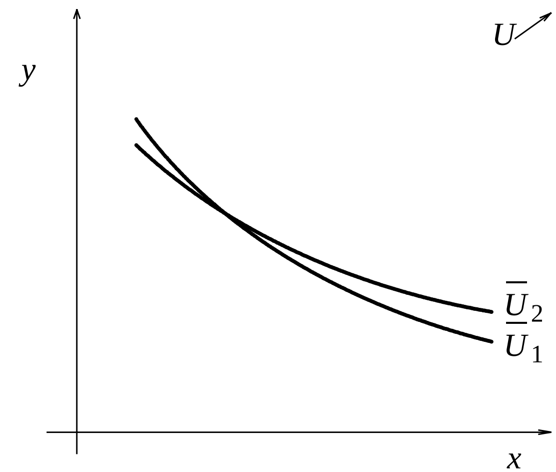
but $dU = 0$ along indiff. curve $\bar{U}\,(a,\ b)$

$$\therefore\ \left.\frac{da}{db}\right|_{\bar{U}} = -\frac{\partial U/\partial b}{\partial U/\partial a} = -\frac{MU_b}{MU_a} < 0$$

i.e. the slope of the indifference curve,
when holding utility constant at $\bar{U}$
along the indifference curve; both are goods:
the slope is negative if MU_b , $MU_a > 0$.

So long as both goods are *goods* ($MU > 0$) not "*bads*" ($MU < 0$), getting more of one but staying indifferent (equal utility) will require getting less of the other.

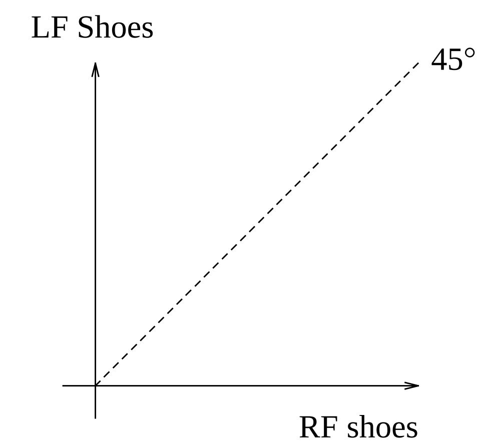
3. Indifference curves cannot intersect.



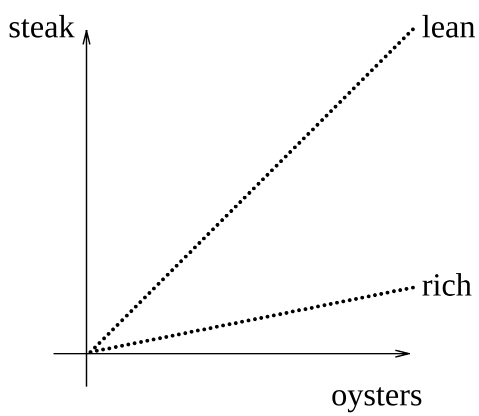
$A\ I\ Q$	$\because$	both on $\bar{U}_1$
$A\ I\ R$	$\because$	both on $\bar{U}_2$
transitivity	$\Rightarrow$	$R\ I\ Q$
but $R\ P\ Q$	$\because$	R above and to the right of Q and x & y are goods ($MU > 0$) for both x & y
$\therefore$ contradiction	$\Rightarrow$	Indifference curves can't cross.

$Q \rightarrow T$: $MU_x > 0$, $\therefore\ T\ P\ Q$
$T \rightarrow R$: $MU_y > 0$, $\therefore\ R\ P\ T$
$\therefore\ R\ P\ Q$ — a contradiction

Indifference Curves of Complementary Goods

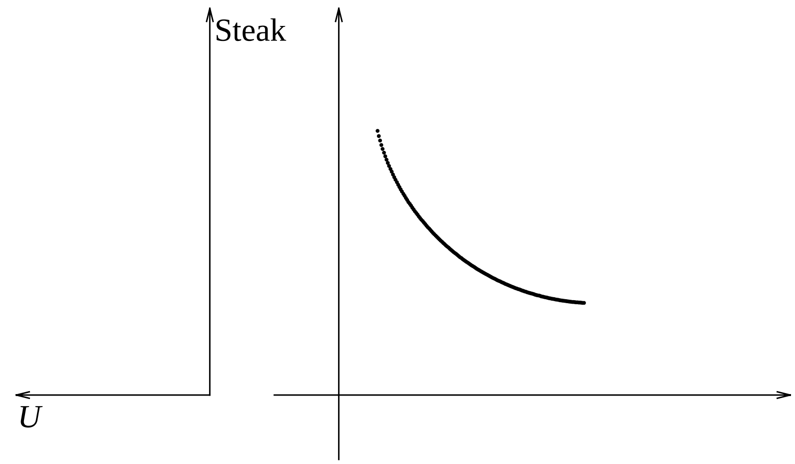


perfect complementary goods



(A “carpet-bagger” steak is a steak stuffed with oysters.)

Satiation of Both Goods



Bliss point: satiation in both steak and caviar.

Marginal Rate of Substitution (MRS) in consumption

Define MRS $\equiv$ amount of y to give up per unit of x gained, at constant utility

$$= \text{ratio of } \left.\frac{\Delta y \text{ given up}}{\Delta x \text{ gained}}\right|_{\bar{U}} > 0$$

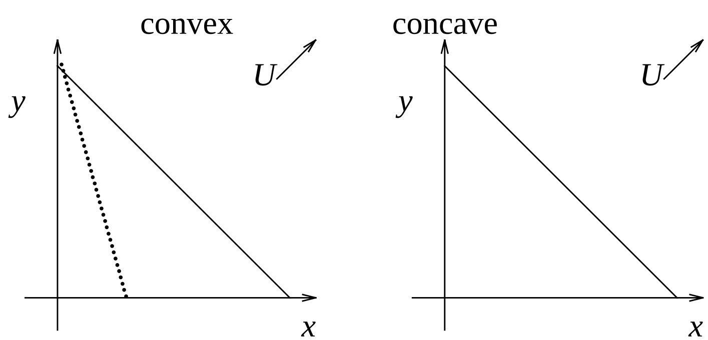
$=$ –slope of the indifference curve

> 0 for two *goods*.

4. Indifference curves "cover the space" (+ve orthant).

   From the Axiom of Comparison all bundles can be compared
   $\therefore$ each bundle lies on an Indifference curve.

5. Indifference curves are convex to the origin.

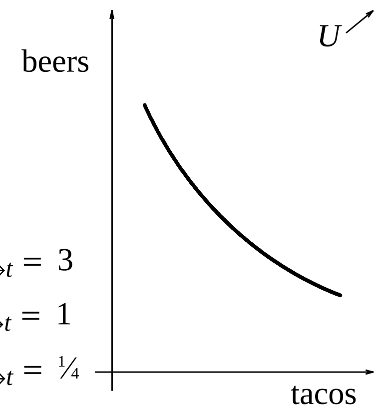


We assert this: it doesn't follow from axioms.

Convexity implies: *diversity in consumption*, which we observe, while concavity always means a corner solution: no diversity.

e.g. Bundle *A* (many beers, few tacos)
$\rightarrow$ give up several beers for one more taco

Bundle *B* (few beers, many tacos)
$\rightarrow$ give up only few beers for one more taco



A: $MRS_{b \to t} = 3$

B: $MRS_{b \to t} = 1$

C: $MRS_{b \to t} = \frac{1}{4}$

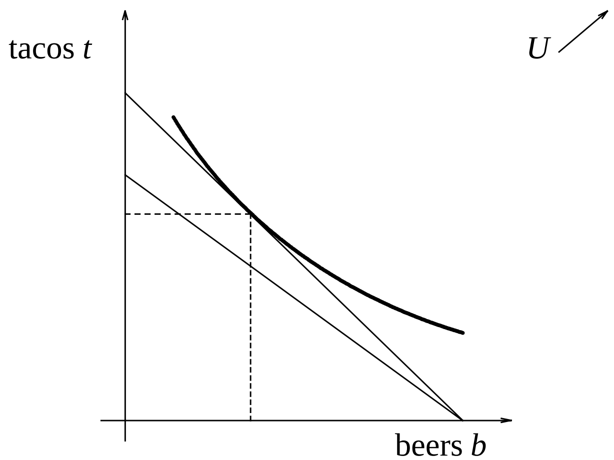
But with two goods, and indifference curves *concave* to the origin, only one good will be chosen or bought, and this isn't observed (see later). (Not convex.)

2.2 Feasible Set

Feasible Set (FS): the set of all bundles *obtainable* (affordable) by the chooser, subject to constraints.

The most common constraint is the *budget constraint*, where the budget line is a function of price and income;

(although others exist: availability...).



price of beers P_b	=	\$1.50/beer
price of tacos P_t	=	\$1.00/taco
income (budget) I	=	\$12.00
(new) price of beer	=	\$1.50/beer unchanged
new price of tacos	=	\$1.20/taco increased

The equation of the Feasible Set is:

expenditure ≤ budget

i.e. $P_b b + P_t t \leq I$

$\therefore$ the budget line is $t = -\dfrac{P_b}{P_t} b + \dfrac{I}{P_t}$.

slope of budget line

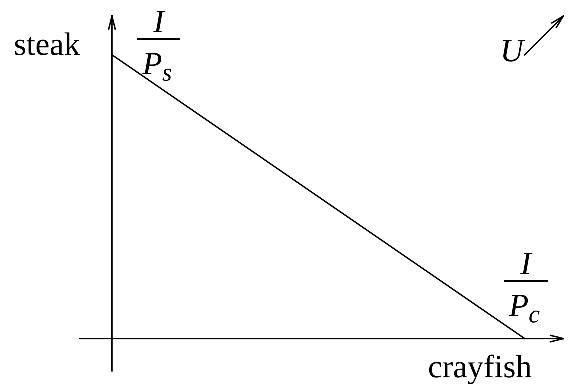
Q: Does feasible set increase or decrease as the price of one good falls?

A: It increases: a lower price for beers ⇒ a greater number of feasible (affordable) bundles, cet. par.

3. Choice Sets

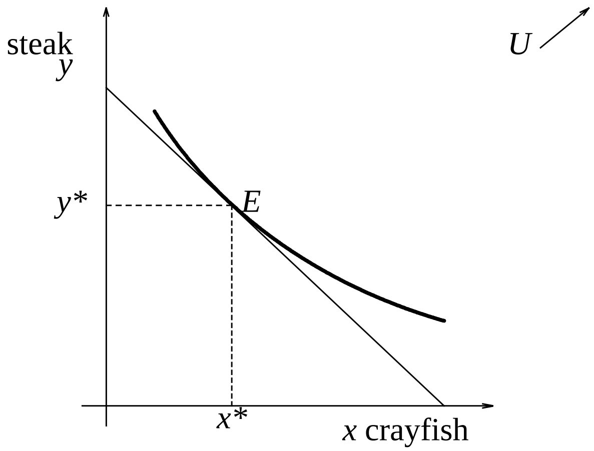
Choice Set: is the *set of most preferred bundle(s)* of the Feasible Set, determined by preferences, (as encoded in the Indifference Curves).

Problem: Given one's preferences (indifference curves) and
given one's income budget I and
given the prices of steak P_s and
given the prices of crayfish P_c, then
→ what is the Choice Set? Feasible Set? Budget Line?



given *B P A*, *B I D*, *C P D*, *E P D*
→ *F* chosen (set) point, ∵ no other point on the budget line is preferred to *F*

Important: we model the consumer's choice with *quantities* on the *x* and *y* axes, not *expenditures* — this separates preferences from prices and income, which come in through the feasible set.



Utility is maximized (⇒ choice set) at the point or points of tangency between the budget line and a convex indifference curve (if this point exists).

At the (interior) *Choice Set* (*E*):
the slope of the Indifference Curve equals the slope of the budget line:

$$\text{I.C.: } U(x,y) = \bar{U} \Rightarrow \left.\frac{dy}{dx}\right|_{\bar{U}} = -\frac{MU_x}{MU_y} = -\frac{\partial U/\partial x}{\partial U/\partial y}$$

is the slope of indifference curve < 0

budget line: $I = P_x x + P_y y$

$\therefore\ dI = 0 \Rightarrow \left.\dfrac{dy}{dx}\right|_I = -\dfrac{P_x}{P_y}$ the slope of the budget line i.e. the slope of budget line < 0

$\therefore$ at choice set:

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}, \ \textbf{or}\ \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

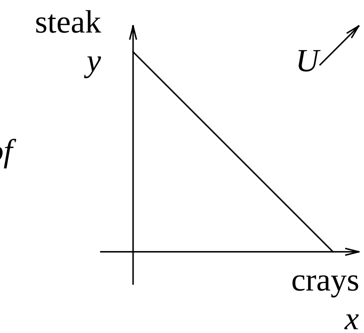
The optimality condition $\dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y}$ says that:

the marginal utility per \$ spent on crayfish *x* equals the marginal utility per \$ spent on steak *y*,
that is, the last dollar spent must yield an equal increase in satisfaction (or marginal utility) from crayfish or steak.

Moreover, (two definitions)
- minus the slope of the highest (feasible) indifference curve

$= \dfrac{MU_x}{MU_y}$

is the *Marginal Rate of Substitution in Consumption, MRSC*:



the ratio at which the individual is just *willing* to substitute a small amount of *y* (steak) for a unit of (crayfish) *x*.

- minus the slope of the budget line = P_x/P_y is the *Marginal Rate of Substitution in Trading, MRST*: the ratio at which individual is *able* to substitute units of *y* (steak) for a unit of *x* (crayfish), given the prices P_x and P_y.

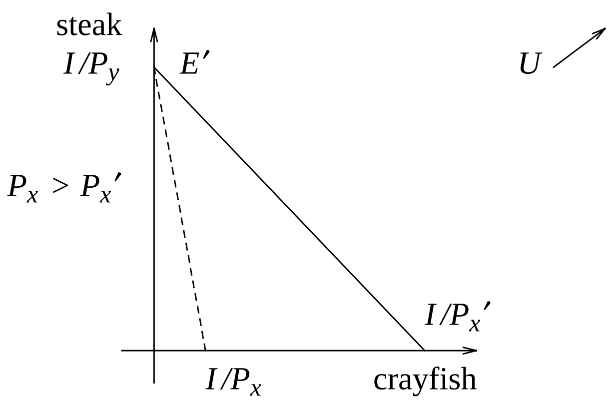
So: at the choice point

MRS in consumption	=	MRS in trading
or, willingness to substitute (taste, preferences)	=	ability to substitute (budget and prices)

[at least for “interior” solutions: E

- the choice point not at an axis
- for any good consumed at all (choice point at axis $\Rightarrow$ at least one good not consumed).]

Corner Solutions are possible: E'



With high P_x, he may choose to spend all on steak — the tangency conditions don’t hold! (At E'.)

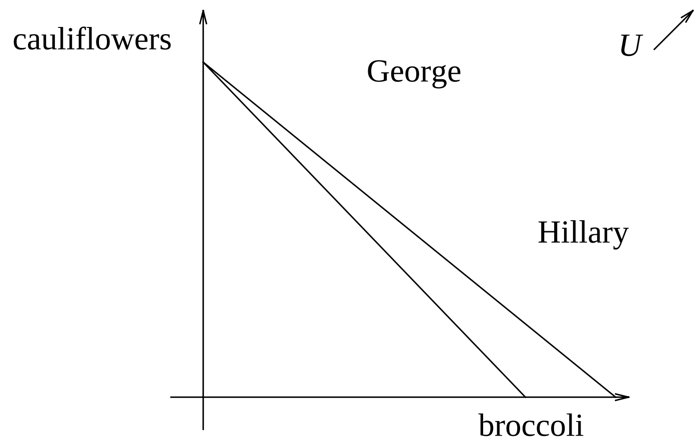
With lower P_x', again he chooses to buy some of each. (At E.)

Note: as price of crayfish rises, the feasible set shrinks.

Proposition: If all consumers face the same prices, then the Marginal Rate of Substitution in Consumption MU_B / MU_C is equal for *all* consumers.
Necessary condition for maximising one’s utility:

$$\text{MRS} = \frac{MU_B}{MU_C} = \frac{P_B}{P_C} \qquad \textbf{\textit{or}} \qquad \frac{MU_C}{P_C} = \frac{MU_B}{P_B} = \cdots$$

(tangency condition) (equal additional utility per \$ spent)



Price of one falls (say broccoli cheaper).
Can buy old bundle + \$ left over.
$\therefore$ real income rises.

Moral: (tangency condition) for everyone, *the marginal rates of substitution of broccoli and cauliflower are EQUAL*, and equal to the price ratio which everyone faces, whatever I_1 and I_2 and their preferences.

and MRSC = MRST $\because$ prices the same.

Revision

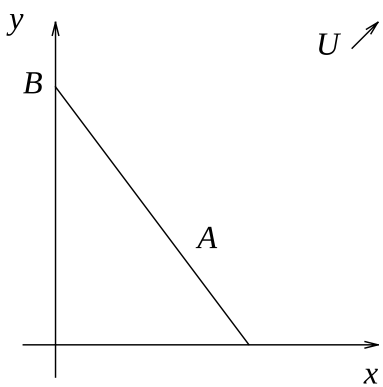
• The condition $\dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y}$ I

is a First-Order condition, *necessary* for *optimizing* utility (given that both x and y are consumed).

• The condition that the Indifference Curves are *convex* is necessary for *maximizing* utility (2nd order conditions). II

Counter example:

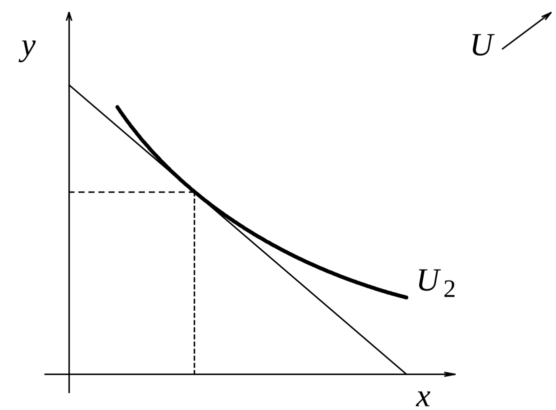
Concave ICs
$\Rightarrow$ *minimum* utility A
but Choice Point is B.



At the point of tangency, First-Order, necessary condition holds, but utility is a *minimum*, with concavity.

So, if all consumers face the same prices, then the ratios of the marginal utilities of all pairs of products are *equal* for all consumers.

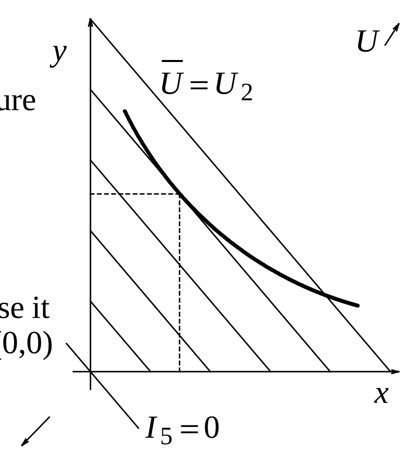
Mathematically: a constrained maximization:



maximize utility $U(x,y)$
subject to budget constraint:
$P_x x + P_y y \le I$

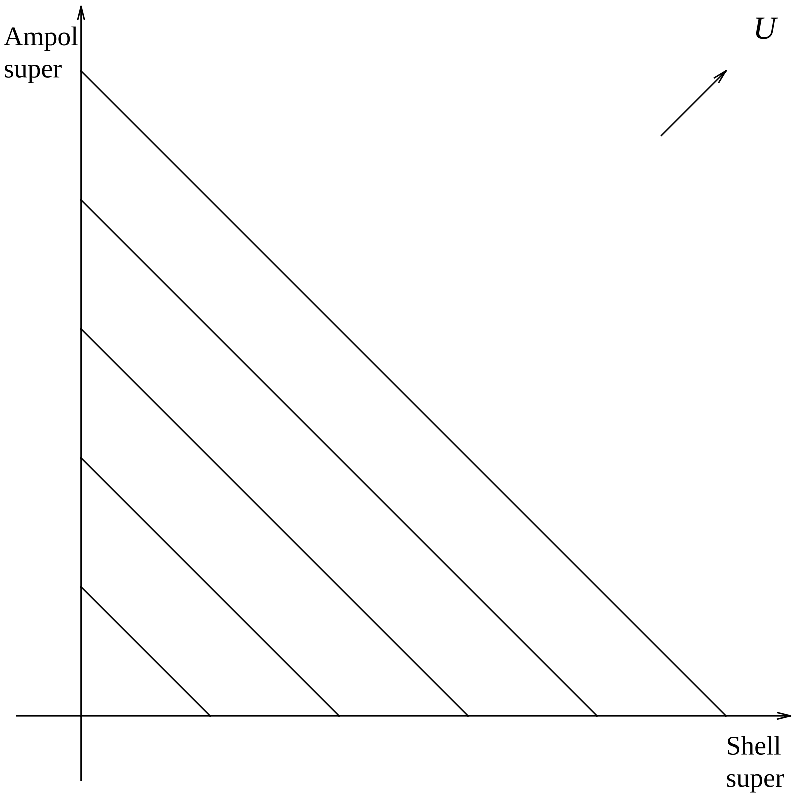
or equivalently ("the dual"):

minimize expenditure
$P_x x + P_y y$
subject to attaining
target utility, $\overline{U}$
$U(x,y) \ge \overline{U}$
Note: $I_5 = 0$ because it includes the point (0,0)



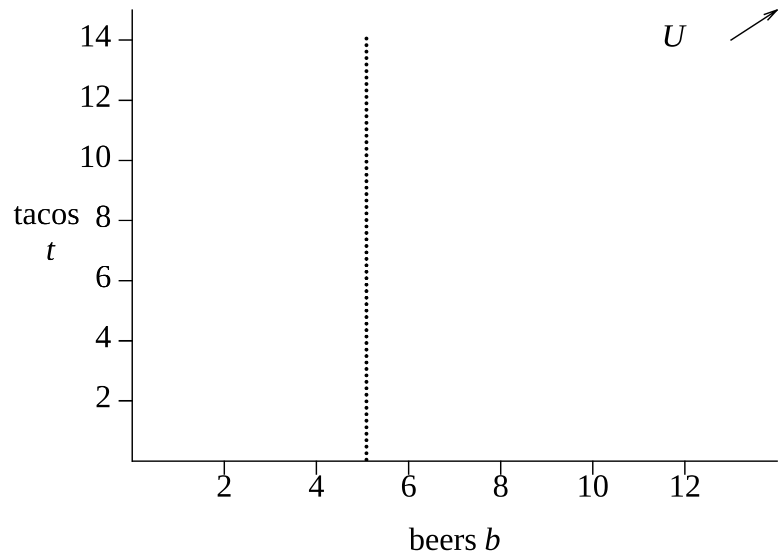
lower expenditure

Indifference Curves of Perfect Substitutes



The choice of which to buy depends solely on the price ratio $\frac{P_{Ampol}}{P_{Shell}}$, if they are perfect subsitutes.

What if Prices aren't Constant?



$$P_t = \$1.00/t$$

$$P_b = \$1.50/b \qquad b \leq 5 \quad (1,2,3,4,5)$$

$$= \$0.75/b \qquad b > 5 \quad (6,7,\cdots)$$

$$I = \$12$$

So, it is possible, even with convex Indifferent Curves, to have more than one Choice Point.

$\because$ a kinked budget line.

4. Gains from Trade

Question 1:

_________ is more partial to beer than to tacos, and

_________ is more partial to tacos than to beer.

(But for both people, both beer and tacos provide positive marginal utility: for both people, both are "goods".)

Both people have initial endowments of

6 beers + 6 tacos

Q: Is there the potential for mutually beneficial voluntary trade or exchange between the two people?

Question 2:

Both ________ and ________ have the same preferences for beer and tacos, and neither of them regards either beer or tacos as a "bad".

_________'s endowment is 1 beer + 11 tacos

_________'s endowment is 11 beers + 1 taco

Q: Is there the potential for mutually beneficial voluntary trade or exchange between the two people?

In the first case trade opportunities occur because of: *different preferences, but the same endowments.*

In the second case trade opportunities occur because of: *different endowments, but the same preferences.*

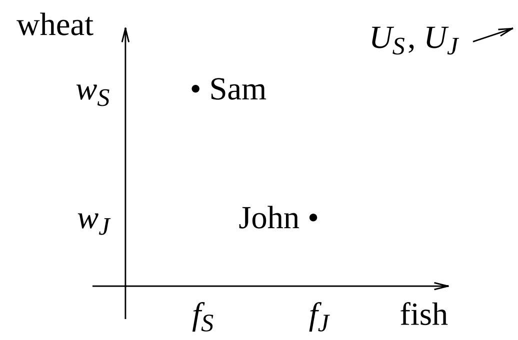
TRADE MAKES BOTH PARTIES BETTER OFF

Consider two individuals:

- Sam with much wheat but with little fish, and
- John with much fish but little wheat.

Given normal preference for diversity, both stand to gain from exchange or trading: there are *gains from trade*.

Diagrammatically:



Each person has:

- a *family of indifference curves*, shown passing through
- the *person's endowment.*

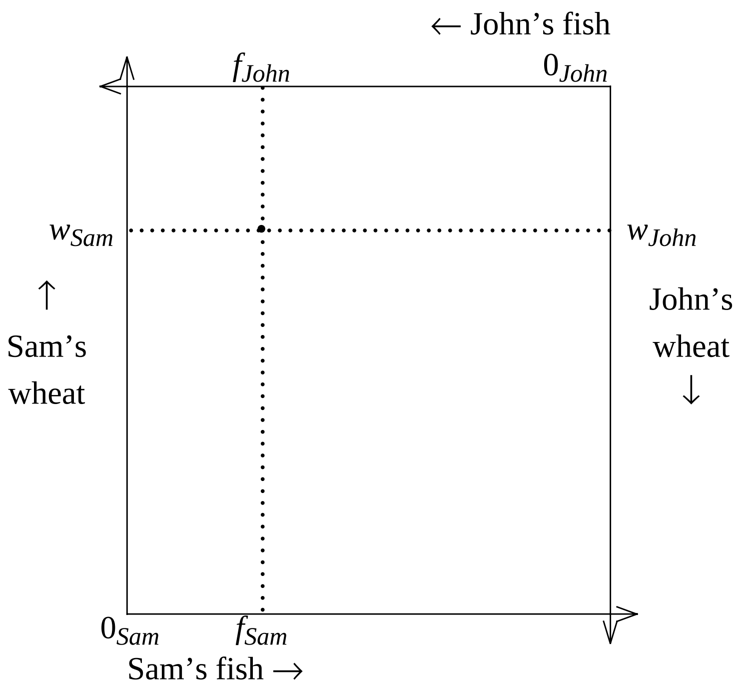
By trading some wheat for fish, Sam can move to a higher indifference curve,

and by trading some fish for wheat, John can also increase his utility.

4.1 An Edgeworth Box

(See H&H Ch. 13.1)

- Sam's origin is at the SW corner, and John's at the NE corner.
- The height of the Box corresponds to the total endowment of wheat: $w_S + w_J$
- The width is the total endowment of fish: $f_S + f_J$
- trade is movement of a point within the box:
- the point is the (pre-trade) position of the two traders in wheat and fish.



Look for Efficient (or Pareto Optimal) allocations of fish & wheat.

The initial endowment: the bullet, through which the two indifference curves pass.

The area between these curves is *the lens of trade*:

allocations of wheat and fish with which both Sam and John would be better off (on higher indifference curves) than with the initial endowment.

There exist gains to trade at any allocation at which the indifference curves are not tangents.

The locus of points of tangency of Sam's and John's indifference curves is the *contract curve*;

on the contract curve it is not possible to change the allocation to make one trader better off without making the other worse off.

Thus these allocations are *efficient* (or Pareto Optimal) along the contract curve.

In a market, when both traders are price takers, the choice point is the set of allocations at which the traders' indifference curves are tangent (on the contract curve) and have a slope equal to minus the price ratio.

This is consistent with our analysis of the individual's constrained maximisation of utility subject to the budget constraint, above.

An *inefficient (or non-Pareto Optimal) allocation* is one where we can change the allocation (the shares of wheat and fish) to make (at least) one person better off without making anyone worse off.

NB: This page is not examinable!

Maximize utility:

max $U(x,y)$ constrained

s.t. $P_x x + P_y y \le I$

Form the **Lagrangian** using λ*: the Lagrange multiplier*

$$L \equiv U(x,y) + \lambda(P_x x + P_y y - I),$$

then maximize the *unconstrained* Lagrangian.

1st Order Conditions:

$$\frac{\partial L}{\partial x} = MU_x + \lambda P_x = 0 \qquad (1)$$

$$\frac{\partial L}{\partial y} = MU_y + \lambda P_y = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = P_x x + P_y y - I = 0 \qquad (3)$$

(3) $\Rightarrow$ maximizing L is equivalent to maximizing U, and that the budget constraint is binding (i.e. on the boundary of the Feasible Set).

(1) & (2) $\Rightarrow$ $\dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y} = -\lambda$

$$\text{solution} \rightarrow \left\{ \begin{array}{l} x^* = x^*(P_x,\, P_y,\, I) \\ y^* = y^*(P_y,\, P_x,\, I) \end{array} \right\}$$

$\rightarrow$ individual *demand functions*, given preferences.

5. Demand Functions

These can be written as:

$$x^* = x^*(P_x,\ \cdots\ ,\, P_y,\, I)$$

which says that quantity x^* demanded is a function of:

- its own price P_x,
- the price P_y of related goods (substitutes and complements), and
- the budget or income I,

and that x^* is derived from maximising the utility of the chosen bundle:

$$\max_{x,y}\ U(x,y),$$

subject to the budget constraint:

that $P_x x + P_y y = I.$

(We assume that tastes, preferences are given, and unchanging.)

This constrained maximisation can be solved using Lagrange multipliers.

the optimality condition: $-\lambda = \dfrac{MU_x}{P_x} = \dfrac{MU_y}{P_y}$

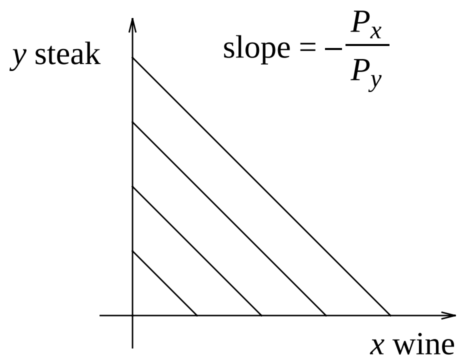
to obtain $\rightarrow$ *the demand functions:*

$$x^* = x^*(P_x,\, P_y,\ \cdots\ ,\, I)$$
$$y^* = y^*(P_x,\ \cdots\ ,\, P_y,\, I)$$

Question: How is demand for x affected by changes (1) in I or (2) in P_x or P_y ? (The comparative statics.)

5.1 The Income Effect:

income $I_2 > I_1$



As income rises, the demand for both rises along the *income-expansion curve* (or Engel curve) because both steak and wine are "*normal*" goods.

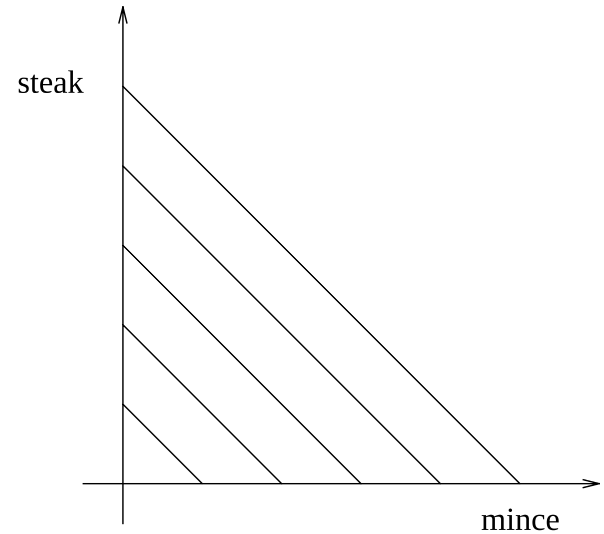
Remember, for a "normal" good: $\dfrac{\partial x^*}{\partial I} > 0$

or $\varepsilon \equiv \dfrac{\partial x^*}{\partial I}\,\dfrac{I}{x} > 0$ is the *income elasticity of demand* for a *normal* good.

e.g. mince is an "*inferior*" good

$$\therefore \quad \frac{\partial\, \text{mince}^{D*}}{\partial I} < 0$$

(against mince, steak is *ultra-superior*)



along the income-expansion curve

Mince may become inferior as the individual's income rises sufficiently.

5.2 The Effects of Price Changes on Demand:

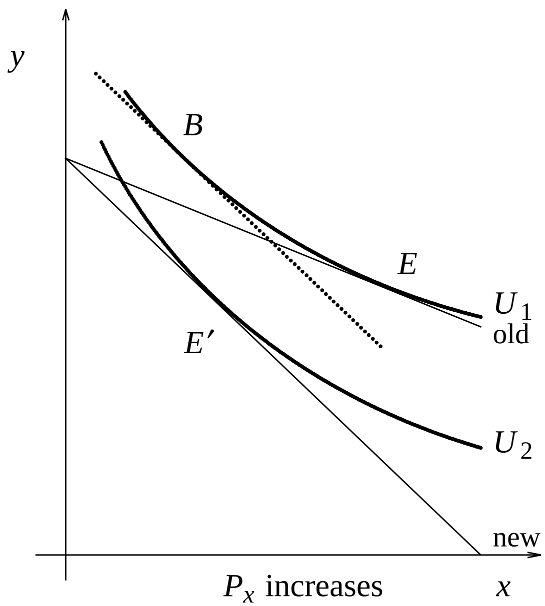
$x^* = x^*(P_x, P_y, \cdots, I)$ the *demand function* for x

$\dfrac{\partial x^*}{\partial P_x}$? There are two components:

(cet. par.)

(1) *substitution effect* $E \to B$

(2) *income effect* $B \to E'$



From E to B: solely because of the change in P_x:

(1) the *substitution* effect.

From B to E' along the income-expansion curve: solely because of the change in real income associated with higher P_x:

(2) the *income* effect.

5.3 Individual Demand

From the optimality condition, we can obtain *demand functions*

$$x_i^* = x_i^*(\mathbf{P}, I)$$

For the consumer, the amount of good i chosen, x_i^*, is a function of all prices $\mathbf{P}$ and his money income I, given his preferences.

Notes: (1) if all prices and his income change proportionately, then no change in x_i^*.

(2) the demand function x_i^* is single-valued (from the convexity of the indifference curves, of the preference set).

We want to know how the amount (x_i^*) of good i chosen varies with changes in P_i, $P_{j\neq i}$ (other prices), and money income I.

That is, we want to derive the Slutsky equation.

comparative statics : $\dfrac{\partial x_i^*}{\partial P_i}, \dfrac{\partial x_i^*}{\partial P_j}, \dfrac{\partial x_i^*}{\partial I}$?

(Partial differentials $\Rightarrow$ ceteris paribus.)

max. U s.t. budget constraint $\rightarrow$ demand functions
or
min. expenditure s.t. $\overline{U}$ $\rightarrow$ *income-compensated demand functions*
NFX

The substitution effect:

the increase in price of x induces the consumer to substitute relatively lower-priced good y for the now relatively higher priced good x (at constant utility).

The income effect:

but as the price of x rises, the consumer's real income (purchasing power) falls (and the Feasible Set FS is smaller); as a result the consumer is worse off and tends to buy less of all (normal) goods.

For a "*normal*" good x: as P_x rises,

the substitution and income effects work together $\rightarrow$ less of good x demanded.

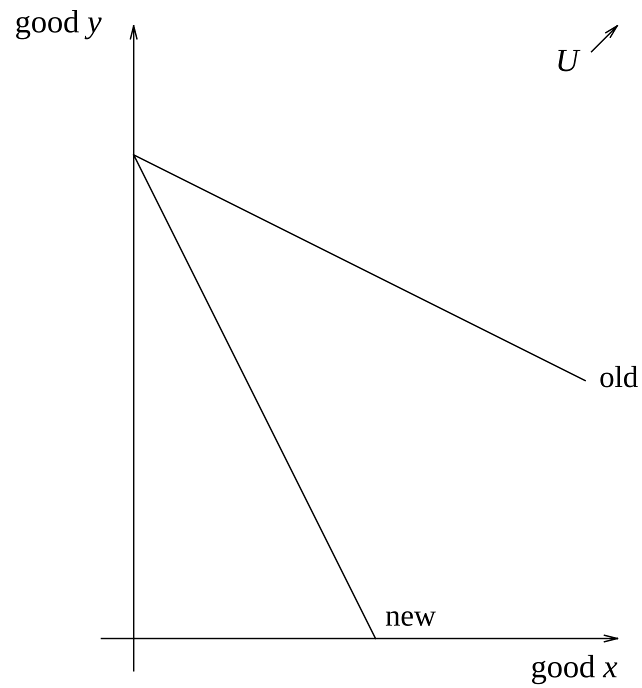
For an "*inferior*" good x: as P_x rises,

substitution effect	$\rightarrow$	less of good x demanded, &
income effect	$\rightarrow$	more of good x demanded.
$\therefore$ total effect		? (but *generally* less of good x demanded).

Generally the substitution effect dominates the income effect, and so the Law of Demand holds. (Exception: mythical Giffen goods.)

The Mythical Giffen Good

(Own price rises, and so does the amount of good demanded!)



(From A to C) If $x_C > x_A$ then the income effect is greater than the substitution effect and the good is "inferior": the increase in P_x results in a *rise* in amount of x chosen!

[but this Giffen good—a mythical Irish creation—is very rare, and only for inferior goods, when the price and the amount demanded rise, cet. par.]

5.4 The Slutsky Equation

The Slutsky equation separates the effect of a price change on demand (cet. par.) into a substitution effect and an income effect.

Price change $\rightarrow$ substitution effect + income effect.

in elasticity terms:

$$\eta_P^x \quad = \quad \eta_P^x \big|_{\bar{U}} \quad - \quad f_x \quad \times \quad \varepsilon^x$$

η_P^x	$\eta_P^x \big\|_{\bar{U}}$	f_x	ε^x
price elasticity of demand	income-compensated price elasticity "substitution elasticity" < 0	fraction of income spent of x $\equiv \dfrac{P_x \times x}{I}$	income elasticity of demand

For derivation of the Slutsky equation (NOT FOR EXAM), see Alasdair Smith (1982, pp. 97–103).

A *rise* in P_x with constant I,

$\rightarrow$ (1) makes good x relatively dearer
substitution effect:
a fall in x if U unchanged

$\rightarrow$ (2) reduces real income by reducing the Feasible Set
income effect:

We can see that *the effect of a price rise on demand depends on the income effect*: if the good is an inferior good, the income effect (which in that case is positive) may dominate the price effect of substitution.

$$\eta_P^x = \eta_P^x \Big|_{\bar{U}} \; - \; f_x \times \varepsilon^x$$

(–)	–	(+)	(+)	if "normal" < 0
(–)	–	(+)	(–)	if "inferior" ? 0
sub-stitution		income		

$\therefore$ for a normal good x: $\quad \eta_P^x < 0$

$\therefore$ for an inferior good x: $\quad \eta_P^x \gtrless 0$?

because the income and price effects conflict,
but almost always the net effect of a price rise is negative.

If f_x, the share of the expenditure on x in the total budget, is sufficiently large, then x may be a mythical Giffen good, with the income effect dominating the substitution effect so that

$$\eta_P^x > 0,$$

but this is really only a theoretical possibility (!)

As well as the *own-price effects* above, we can also derive a Slutsky equation for *cross-price effects*, and we can see that measuring whether two related goods are substitutes or complements can be confounded by income effects too.

5.5 The Slutsky Equation for Cross-Price Effects:

How does an increase in the price P_y of a *related* good affect the demand for x?

In algebraic terms, what is the sign of

$$\partial x\,(\mathbf{P},\, I) / \partial P_y \ \ ?$$

The Slutsky cross-price equation is (in partial differential terms):

$$\frac{\partial x(\mathbf{P},\, I)}{\partial P_y} = \frac{\partial \tilde{x}(\mathbf{P},\, \overline{U})}{\partial P_y} - y\ \frac{\partial x(\mathbf{P},\, I)}{\partial I}$$

the first term: > 0 "gross" substitutes
< 0 "gross" complements

the second term: > 0 true substitutes
< 0 true complements

(The income-compensated price effect is symmetrical.) (The third term is the income effect.) So, in the Introduction we now see we were using *gross* complementarity and *gross* substitutability.

In elasticity terms: (multiply both sides by P_y / x)

$$\eta^x_{P_y} = \eta^x_{P_y}\Big|_{\overline{U}} - f_y \times \varepsilon^x.$$

In general $\dfrac{\partial x(\mathbf{P},\, I)}{\partial P_y} \neq \dfrac{\partial y(\mathbf{P},\, I)}{\partial P_x}$ $\because$ income effects

Remember that the cross elasticities are only equal for income-compensated demand functions.

5.6 Example: Demand for Grapes

The Economic Research Service of the Department of Agriculture has reported the results of a study of the effects of the price of various types of grapes on the rate at which they were bought. In particular, three types of grapes were studied:

- Sultana,
- Waltham Cross, and
- Black Muscat.

In nine test supermarkets in Geelong, the researchers varied the price of each of these types of grapes for a month. The observed effect of a 1% rise in the price of each type of grape on the rate of purchase of this and each of the other types of grapes is shown below.

A 1% rise in the price of:	Results in the following percentage change in the rate of purchase of: *Sultana*	*Waltham Cross*	*Black Muscat*
Sultana	–3.1	+1.6	+0.01
Waltham Cross	+1.2	–3.0	+0.1
Black Muscat	+0.2	+0.1	–2.8

For example, a 1% rise in the price of Sultana grapes (ceteris paribus) resulted in a 3.1% fall in the rate of purchase of Sultana grapes, a 1.6% rise in the rate of purchase of Waltham Cross grapes, and a 0.01% rise in the rate of purchase of Black Muscat grapes.

The diagonal elements are the *own-price* elasticities; the off-diagonal elements are the *cross-price* elasticities.

Questions:

a. What is the definition of the *income elasticity of demand* of a good?

b. What is the difference between *gross substitutability and true substitutability*? (See the Slutsky equation.)

c. What does the *own-price elasticity of demand* for each type of grape seem to be?

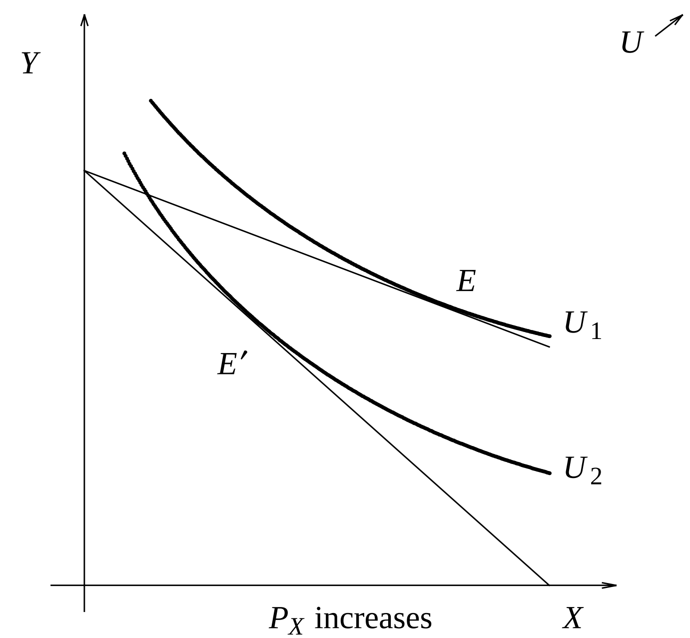
d. What does the *cross-price elasticity of demand* for each pair of types of grape seem to be? Why might the measured pairs of cross-price elasticities not be expected to be symmetrical?

e. Which pair of types of grape seem to be the *closest substitutes*?

f. Of what *use* might these results be to grape producers?

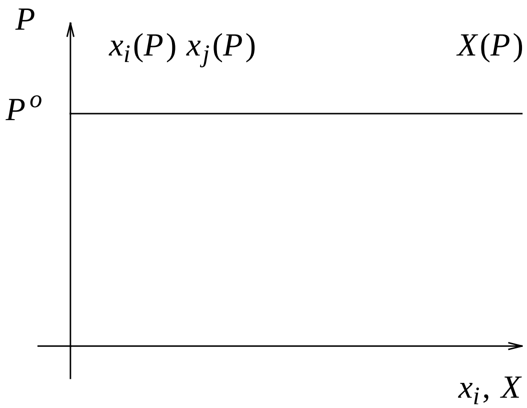
Note: The definition of substitutes, using the cross-price elasticity of demand, is in terms of % change in Q in response to a 1% change in P, *not* in terms of absolute changes in Q and P.

5.7 Demand Curve Derivation.



6. Market Demand

Market demand for a good is obtained by *horizontally* summing individual demand functions, $x_i(P)$.



At any price P^o, market demand $X(P^o)$ is the sum of the demands of each person i for the good:

$$X(P^o) = \sum_{i=1}^{n} x_i(P^o) \quad \text{for all } P \qquad (1)$$

and

$$\eta_P^X = \sum_{i=1}^{n} \frac{x_i}{X} \eta_P^{x_i} \qquad (2)$$

The *market price elasticity of demand* is a weighted sum of individual price elasticities of demand, weighted by $\frac{x_i}{X}$.

How can we derive (2) from (1)?

6.1 Example: Subsidy v. Voucher

- in effect it lowers the price of education
- consequences?

- can only be used for education
- consequences?

(eg) SCC $75 electricity refund

>> =Include H&H Fig 4.21, 4.22 <<

7. Summary

This section has built a theory of market demand from a small number of axioms of rational choice. On the way we have considered:

- A *theory of rational choice*, as stated in the Proposition of Rank Ordering of Preferences, and *Modern utility theory*: utility functions, marginal utility, goods, “bads”, satiation.
- Utility bundles and *indifference curves*: properties of indifference curves. The Marginal Rate of Substitution in Consumption.
- *Constrained maximisation of utility*: the feasible set, the budget line, the choice set.
- The *gains from trade*: the Edgeworth Box, the lens of trade, the contract curve.
- The *individual demand* function: the effect of income changes, the definitions of substitutes, complements, “normal” and inferior goods.
- Its *comparative statics*, including the Slutsky equation, the own-price elasticity of demand, the cross-price elasticity of demand, the income elasticity of demand.
- The *market demand* curve and relevant elasticities.

“People don’t turn down money — that’s what separates us from the animals” *Jerry Seinfeld*.