

## 7. Multi-Attribute Decision Making

Many decisions are based on other attributes than price. Choosing a car, for instance, although you might be looking in a particular price band. Comfort, performance, reliability, size, safety, status, image — these are some attributes of cars.

*Example:* helping a family to buy a car

*Steps:* (1) Clarify problem; (2) Identify objectives; (3) Measurement of effectiveness.

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(1) <i>Clarify problem</i>	keep an older car? use public transport? constraints? — \$ manual transmission / auto? size? power steering? ? 1. driving kids to school ? 2. reliable & safe commuting vehicle? ? 3. status symbol ? 4. help on family holidays
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*Example (cont.):*

_____	(1) comfort 5A, or 1A + 5K	$S_1$
(2)	(2) safe & reliable	$S_2$
<i>Identify</i>	(3) status	$S_3$
<i>objectives</i>	given \$ constraint	
_____	(1) + (3) subjective—judgement	
(3)		intuition
<i>Measurement</i>		experience
<i>of effectiveness</i>	(2) more objective	
_____		

## Additive Valuation

1. Use scales for  $S_1, S_2, S_3$   
(1) (2) (3)

For each of the three attributes (1), (2), and (3), rank the cars from 0 to 1.

2. Subject to the \$ constraint,

now weight the three attributes: that is,

- how important is the first attribute (comfort) in the total decision?  $\rightarrow w_1$
- How important the second (safety and reliability)?  $\rightarrow w_2$
- The third (status)?  $\rightarrow w_3$

The three weightings  $w_1, w_2, w_3$  should be normalised:  $\sum w_i = 1$ .

3. From the rankings, each car  $j$  has a score for attribute  $i$ :

$x_{ij}$  : score of car  $j$  in attribute  $i$

Each car's total score can thus be calculated:  $\sum_i x_{ij} w_i \rightarrow$  score for car  $j$

4. Choose the car with the highest score, *or* iterate, until you feel happy with the scores, the weightings, and the final outcome.

## Multiattribute Problem

CBA a subset  
e.g. which bank ?

quality of service		interest rates		location
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<u>Comparing specific</u>		<b>outcomes projects</b>
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There are six ways:

(Perry & Dillon in Package)

1. Pairwise comparisons
2. "Satisficing"
3. Lexicographic ordering
4. Reducing search
5. Even swaps, or Pricing out
6. Additive value models

## 7.1 Pairwise comparisons

“eye-balling”:

- OK for small number of attributes
- ? OK number of alternatives?
- large number of alternatives or attributes
- no complete preference ordering
- - but – time consuming, costly
  - continuous variables
  - no information for *delegation*

## 7.2 “Satisficing”

- set minimum levels (“satisfy”) of all attributes but one (the “target” attribute)
- choose the project/outcome/action with the highest level of the target

→ iterative solution

if min levels too | *high*  
*low*

So: *useful, often used, attributes explicit*

## 7.3 Lexicographic Ordering

How to:

- rank attributes;
- choose project with the highest Attribute 1;
- only consider Attribute 2 if there is a tie in terms of Attribute 1.
- Using the letters of the alphabet in order, this is how dictionaries (or lexicons) order words — hence, lexicographic.
- Examine the table on the next page, where countries' performances at the Atlanta Olympics are tabulated lexicographically.

This means there is no trade-off between numbers of Silver medals and numbers of Golds, so that Denmark (4 G, 1 S, 1 B) is ranked nineteenth, while Great Britain (1 G, 8 S, 5 B) is ranked thirty-sixth.

- Or we could rank by total number of medals, which means equal trade-offs between Gold and Silver and Bronze.
- Or we could weight the medals, say, Gold = 3, Silver = 2, Bronze = 1, which still allows a trade-off, but not an equal trade-off.

**Lexicographically Ranked by Gold, Silver, Bronze Medals (Atlanta)**

	<b>Gold</b>	<b>Silver</b>	<b>Bronze</b>	<b>Total</b>
United States	44	32	25	101
Russia	26	21	16	63
Germany	20	18	27	65
China	16	22	12	50
France	15	7	15	37
Italy	13	10	12	35
Australia	9	9	23	41
Cuba	9	8	8	25
Ukraine	9	2	12	23
South Korea	7	15	5	27
Poland	7	5	5	17
Hungary	7	4	10	21
Spain	5	6	6	17
Romania	4	7	9	20
Netherlands	4	5	10	19
Greece	4	4	0	8
Czech Republic	4	3	4	11
Switzerland	4	3	0	7
Denmark	4	1	1	6
Turkey	4	1	1	6
Canada	3	11	8	22
Bulgaria	3	7	5	15
Japan	3	6	5	14
Kazakhstan	3	4	4	11
Brazil	3	3	9	15
New Zealand	3	2	1	6
South Africa	3	1	1	5
Ireland	3	0	1	4
Sweden	2	4	2	8
Norway	2	2	3	7
Belgium	2	2	2	6
Nigeria	2	1	3	6
North Korea	2	1	2	5
Algeria	2	0	1	3
Ethiopia	2	0	1	3
Great Britain	1	8	5	15
Belarus	1	6	8	15
Kenya	1	4	3	8
Jamaica	1	3	2	6
Finland	1	2	1	4
Indonesia	1	1	2	4
Yugoslavia	1	1	2	4
Iran	1	1	1	3

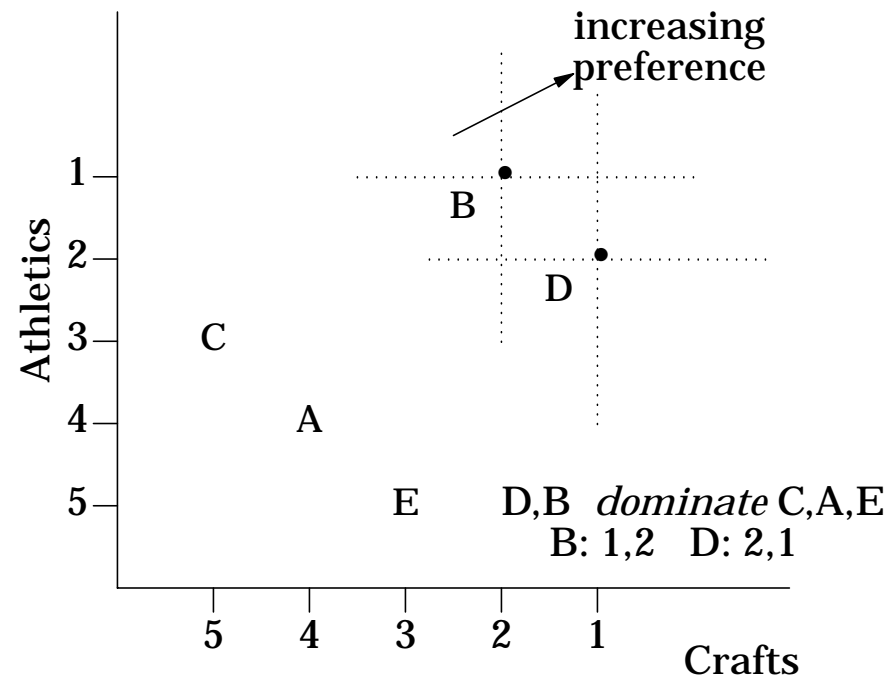


Slovakia	1	1	1	3
Armenia	1	1	0	2
Croatia	1	1	0	2
Portugal	1	0	1	2
Thailand	1	0	1	2
Burundi	1	0	0	1
Costa Rica	1	0	0	1
Ecuador	1	0	0	1
Hong Kong	1	0	0	1
Syria	1	0	0	1
Argentina	0	2	1	3
Namibia	0	2	0	2
Slovenia	0	2	0	2

## 7.4 Reducing Search

e.g. which building to choose, given the two main uses for the building of Athletics and Crafts?

Building	Rank (ordinal)	
	Athletics	Crafts
A	4	4
B	1	2
C	3	5
D	2	1
E	5	3



## 7.5 Even Swaps, or Pricing Out

[see the Hammond *HBR* reading.]

e.g. which of five jobs to choose, given the five attributes of each job?

Job	Attributes / Characteristics				
	Salary	Leisure Time	Working conditions	Co-workers	Where
<i>A</i>	2	3	3	2	2
<i>B</i>	3	4	4	1	2
<i>C</i>	3	3	2	3	3
<i>D</i>	3	1	2	1	1
<i>E</i>	1	2	1	2	2

Freda has ranked the jobs in terms of each attribute.

<i>EPA</i>		∴ Freda's comparison is reduced to <i>D, E</i>
<i>EPC</i>		
<i>DPB</i>		

## Even Swaps (cont.)

Spell out the measures of each attribute:

	Salary	Leisure	Working Conditions	Colleagues	Location
$D$	\$90k	8 days	$W_D$	$C_D$	$L_D$
$E$	\$100k	5 days	$W_E$	$C_E$	$L_E$

Q: How much of \$100K would Freda be prepared to give up to get 3 additional leisure days/year?

A: \$25K  $\rightarrow E'$

$D$	90k	8	$W_D$	$C_D$	$L_D$
$E'$	75k	8	$W_E$	$C_E$	$L_E$

from above  $W_E$  (1st)  $>$   $W_D$  (2nd)

Q: How much of \$90k would Freda be prepared to give up to get  $W_E$ ?

A: \$10k  $\rightarrow D'$

***“pricing out”***

**Even Swaps (cont.)**

$D'$	\$80k	8	$W_E$	$C_D$	$L_D$
$E'$	\$75k	8	$W_E$	$C_E$	$L_E$
$D'$	\$80k	8	$W_E$	$C_D$	$L_D$
$E''$	\$70k	8	$W_E$	$C_D$	$L_E$
$D''$	\$72.5k	8	$W_E$	$C_D$	$L_E$
$E''$	\$70k	8	$W_E$	$C_D$	$L_E$

i.e. all attributes “priced out” by Freda, whose choice is job  $D$

$D' ID'' - ?$

$E' IB'' - ?$

$D ID' - ?$

$E IB' - ?$

$E'' ID''$

$\therefore EID$

$D ID'' PE'' IE \Rightarrow DPE$

**Pitfalls** → Majone (ed.)

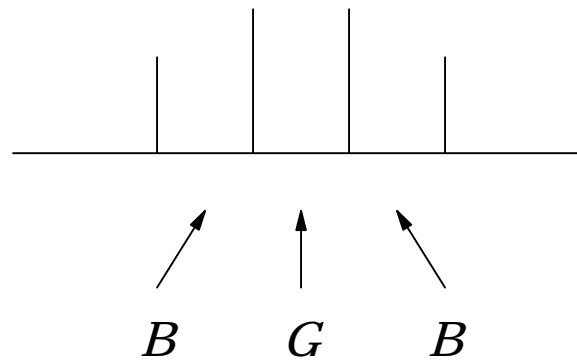
1. decision-maker's errors  
misunderstanding the concepts, questions expressing preferences etc.
2. analyst's errors  
testing for preference independence (see below)
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  - 
  -
3. attitudes might change (subjective)
4. situation might change (objective)

Olympics	Gold	Silver	Bronze	Total
USA	43	20	18	81
USSR (URS)	40	31	23	94
EG (GDR)	35	49	9	93
FRA	15	6	14	35
AUS	1	10	8	19
NZ	1	9	12	22



	<i>Goals</i>		<i>Behinds</i>		<i>Points</i>
Hawthorn	11	:	11	:	77
Melbourne	6	:	5	:	41

$$P = 6G + B$$



## 7.6 Additive Value Models

e.g. three projects: A, B, & CR

three attributes:

Net Present Value	$PV$	$\oplus$	the more the better
Time to Completion	$T$	$\ominus$	the less the better
Size	$S$	$\oplus$	

### ○ *Independence* ○

If the trade-off between  $\{PV \text{ \& } T\}$  is independent of the level of  $S$   
 & if the trade off between  $\{T, S\}$  is independent of the level of  $PV$   
 then  $\{PV \text{ \& } S\}$  are independent of  $T$ .

i.e. ***Preference Independence*** of  $PV, T, S$

## Value Function

$$V(\text{project } j) = \sum_i^{\text{attributes}} w_i [v_{ij}(x_{ij})]$$

- where  $x_{ij}$  is the level of attribute  $i$  in project  $j$
- where  $v_{ij}(\cdot)$  is a “relative value preference of attribute  $i$  for project  $j$ ”  
 $v_{ij} \in [0, 1]$
- where  $w_i$  are attribute weights,  $\sum w_i = 1$

Project  $j \rightarrow$  score  $V_j$  & can compare projects :  $V_j$  to obtain ranking

e.g.	$w_i$	$A \quad v_{i1}$ $j=1$		$B \quad v_{i2}$ $j=2$		$C \quad v_{i3}$ $j=3$	
$NPV$	0.9	\$20m	0.5	\$15m	0	\$25m	1
$T$	0.06	8y	0.6	5y	1	12y	0 (–ve)
$S$	0.04	200k	0.8	300k	1	100k	0

e.g.  $x_{23}$  = level of attribute  $T$  in Project 3 = 12.

$\sum w_i = 1, w_i \geq 0$  attribute weights

$$\begin{aligned}
 \text{project A: } V_A &= 0.9 \times 0.5 + 0.06 \times 0.6 + 0.04 \times 0.8 = 0.518 \\
 V_B &= 0.9 \times 0 + 0.06 + 0.04 = 0.1 \\
 \rightarrow V_C &= 0.9 \times 1 + 0 + 0 = 0.9
 \end{aligned}$$

	A l t e r n a t i v e s				
	Job A	Job B	Job C	Job D	Job E
<i>Objectives</i>					
Monthly salary	\$2000	\$2400	\$1800	\$1900	\$2200
Flexibility	mod	low	high	mod	none
Business skills	computer	people man.	operations	org.	time man.
Development		computer	computer		multitasking
Annual leave	14	12	10	15	12
Benefits	health, dental retirement	health, dental	health retirement	health	health, dental
Employment	great	good	good	great	boring

## 8. Landsburg

1. Tax revenues are not a net benefits (when looking from society's viewpoint) and a reduction in tax revenues is not a net cost.
2. A cost is a cost, no matter who bears it.
3. A good is a good, no matter who owns it.
4. Voluntary consumption is a good thing.
5. Don't double count.

Only individuals matter

+

All individuals matter equally  
(or: a \$ is a \$, no matter whose)

## 9. Real Options

(See Dixit & Pindyck and Bruun & Bason — handouts)

Disadvantages of NPV/DCF (especially for private firms):

1. positive-NPV opportunities might be bid away as firms enter (strategic rivalry)
2. allocation of overhead costs in a multi-project setting is non-trivial
3. assumption of reinvestment at the entire project's rate is questionable
4. the risk adjustment ( $\beta$ ) of the discount rate depends on: project life, growth trend in the expected DCF, etc.
5. interdependencies among projects: spillovers, asymmetric (skewed) outcomes, etc.
6. investments are sunk (sometimes assumed not)
7. the Winner's Curse when choosing one of several:  
the estimates of future costs and benefits are not unbiased in the most attractive project (highest benefits – costs): possibility of negative NPV.

**What if there are options present:**

- timing: wait
- operational: flexibility & discretion once underway
- growth: future options contingent on this project

Then NPV/DCF:

1. *with timing options:*  
if projects are exclusive or investment budgets limited, then projects effectively compete with themselves over time.
2. *with operational options:*  
including
  - temporary shutdowns
  - expanding or scaling down operations
  - switching between inputs, outputs, or processes

Can create value, but skew the return distribution: must use options techniques.

3. *with growth options:*  
or follow-on investments, with distant and uncertain payoffs. Often, learning more about future options is most valuable.

## **Why not use Decision Analysis?**

Plus: an Influence Diagram or Decision Tree does model asymmetries and paths, but

Minus: as the value of the underlying asset (the project) changes over time, so does its risk and so the correct risk premium.

Answer: the principles of risk-neutral valuation with the Black-Scholes option pricing techniques.