3. Shadow Pricing, Direct Price Effects

3.1 Shadow Pricing

[DoF Ch. 3; FP Ch. 1.6, 6; S&W Ch. 8]

The NPV formula can be written as

$$NPV = \Sigma \frac{b(t) - c(t)}{(1+r)^t}$$

where $b(t) = \Sigma\, b_i\ p_i$

and $c(t) = \Sigma\, c_j\ p_j$.

Now, we assume here that we have the the quantities b_i benefits and c_j costs.

What of the prices p_i and p_j?

We want the true costs.

True prices reflect opportunities forgone (by suppliers, by consumers) $\rightarrow$ *shadow prices.*

Obtained here by adjusting (distorted) market prices.

Consider the five possible distortions:

1. of a tax,
2. of a price change,
3. of a tax with a price change,
4. of unemployment with minimum wages,
5. of a tariff (a tax on imports).

To determine the true or shadow prices, use:

- Willingness To Pay (demand curve) for *consumption*
- Opportunity Costs (supply curve) for *inputs*

But beware whether increases or decreases.

What is a shadow price?

A *shadow price* better approximates the true *opportunity cost* or *marginal valuation* of a product or resource or service.

Five cases in which market prices are distorted, so that we must dig a little to obtain the shadow price, the true opportunity cost or valuation:

1. taxes,
2. price changes,
3. prices changes with taxes,
4. a labour market with minimum wage laws, and
5. a tariff on imports.

3.1.1 *Example 1*: **A tax** [S&W Ch. 8.3]

Q: A remote electricity-generation project pays $1/litre for its fuel oil, the costliest input to the project. The FA (financial appraisal) gives an NPV close to zero, but there is a tax on the fuel oil of 45¢/litre. What is the shadow price of fuel oil, to be used in the CBA NPV?

A: Since the tax is a transfer (paying 45¢/litre for nothing), ignore it in a CBA. The shadow price is 55¢/litre, and the CBA NPV will be positive, because of the lower opportunity cost of fuel oil at the shadow price.

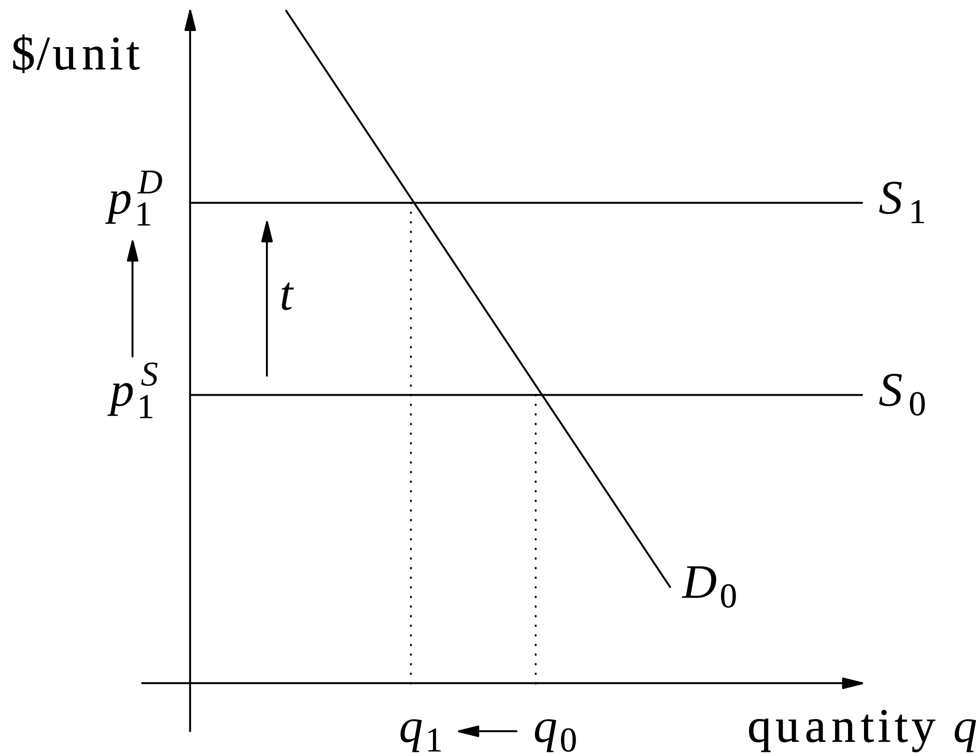
A (specific) tax on a good supplied in a competitive market:

- places a wedge between the marginal cost (supply) and price (demand)

∴ the single (equilibrium) price p_0^S can no longer represent both valuation and cost

- Suppose the good is an input into a project:

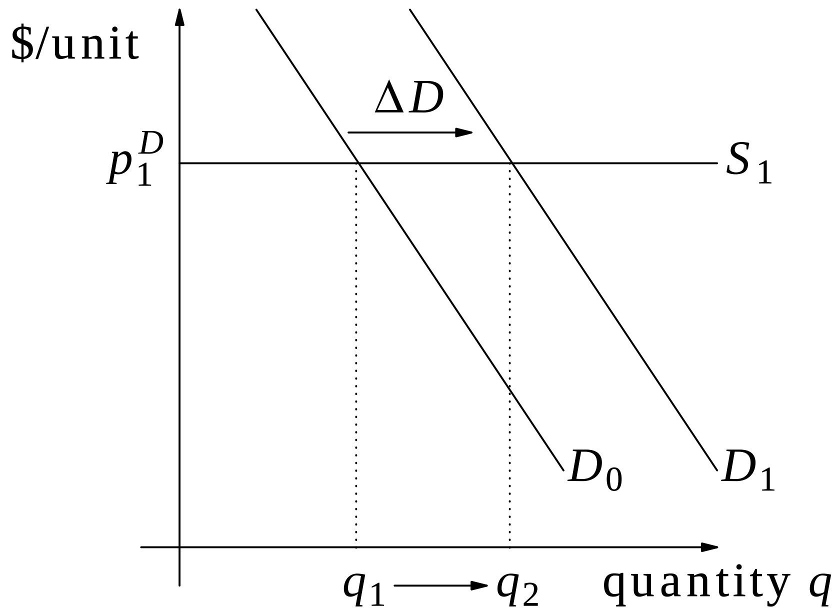
A Tax (cont.) — *Infinitely elastic supply*



Because of the tax, the (tax-inclusive) demand price p_1^D is greater than the (tax-exclusive) supply price p_0^S.

The diagram shows the Tax Revenue and the Dead-Weight Loss as the tax is imposed, pushing up the effective supply, and reducing the quantity demanded, from q_0 to q_1.

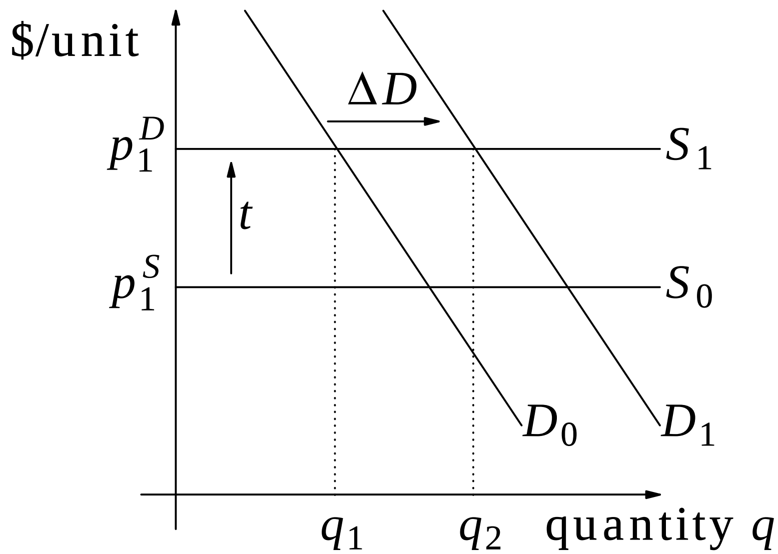
A Tax (cont.)



Let's say the project results in an expansion of demand, from D_0 to D_1.

- But there is no change in p_1^D with the increase in demand.
- Because of the tax, the project pays the higher, tax-inclusive price p_1^D.
- Is this *the shadow price*?
- Does p_1^D reflect the opportunity cost associated with the extra quantity?
- No, in general, but it depends on the purpose of the tax (revenue or "green" tax?).

A Tax (cont.) — e.g. oil at world price plus a local excise of t:



Consumers value the increase in demand (ΔD) at the tax-inclusive price p_1^D.

Suppliers' price is unchanged at p_1^S.

Shadow price = $p_1^D - t = p_1^S$ = unchanging tax-exclusive price, p_1^S.

The tax revenue ( ) is a transfer, and so changes in the tax revenue (changes in a transfer) are not changes in opportunity cost ( ).

This is only the case if there is *no effect on existing purchasers of output*, since there is no increase in price with perfectly elastic supply.

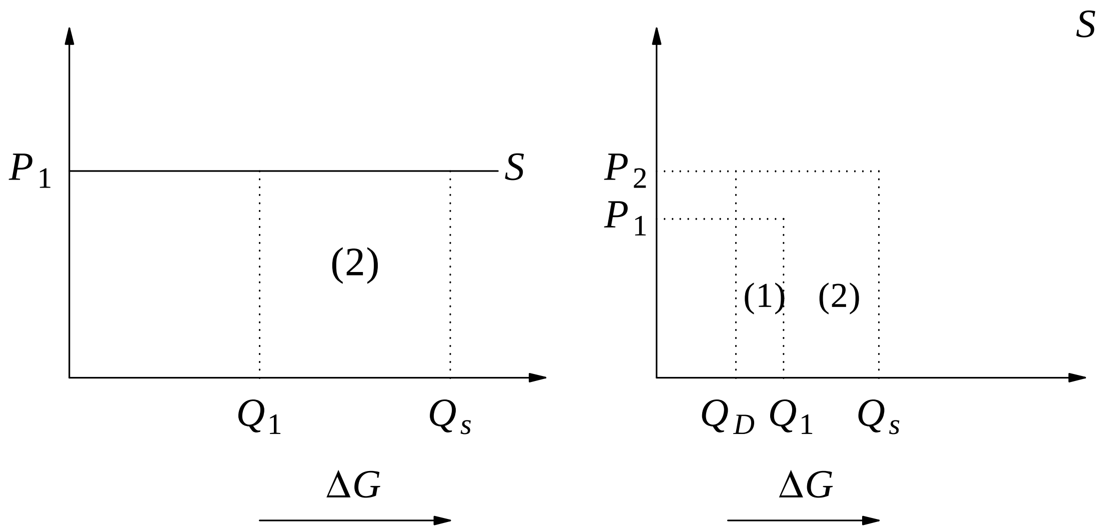
3.1.2 *Example 2:* Shadow prices and opportunity costs when prices change

[FP Ch. 2.2.2, 2.3.2; DoF 3.4]

Q: There is a local market for irrigation water. The going price is \$50/megalitre. A new cotton farm is planned, but its size and thirst for water are such that the going price of water will rise to \$60/megalitre, given its demand of 10,000 megalitres/year. At the lower price the NPV of the project is positive, but at the higher price negative. At the higher price the incumbent users cut their consumption by 1000 megalitres/year.

A: The shadow price is between \$50 and \$60/megalitre, say \$55 (assuming linear supply and demand curves). The existing users bear a cost of \$55 × 1000 = \$55,000/year for the water they can no longer afford (the *displaced* water). The shadow cost to the new farm is \$550,000/year, which includes \$55,000 to outbid the exiting users for 1000 gigalitres/year, and the opportunity cost of \$495,000 to induce the increased supply of 9000 megalitres of water (the *incremental* water).

Prices Change (cont.)



Case 2.1

(No price change)

Case 2.2

(Price change)

Case 2.1: (No price change)

resource opportunity cost = total social costs for increased factor supply (2)

$= P_1 \bullet \Delta G$

Case 2.2: (Price change)

resource opportunity cost = total social costs
$P_1 \bullet \Delta G$ for increased factor supply (2)

\+ value of reduced use of inputs in the rest of society as a response to higher prices (1)

$$P_1 \bullet \Delta G < \text{Area } [(1) + (2)] < P_2 \bullet \Delta G$$

$$\therefore \; P_1 < P_s < P_2$$

→ P_s is the "effective or shadow price":
$P_s \bullet \Delta G$ is the resource cost = area (1) + area (2)

Prices Change (cont.)

Note: be conservative

if NPV > 0 with P_2 ~ cost then GO

if NPV < 0 with P_1 ~ cost then STOP

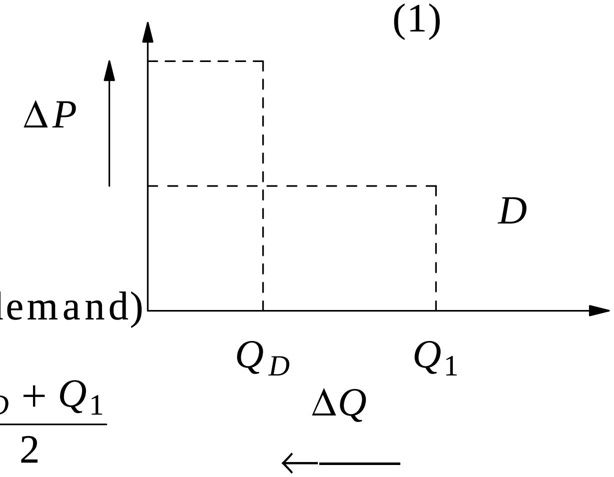
$$\text{if} \begin{cases} NPV(P_2) < 0 \\ NPV(P_1) > 0 \\ NPV(P_s)\ ? \end{cases} \qquad \text{then must find } P_s$$

The point is so avoid the cost and effort of deriving a better estimate of the shadow price P_s if it won't make any difference to the decision.

Some Equations and Harberger's Method

Area under Demand Curve

(1)



area

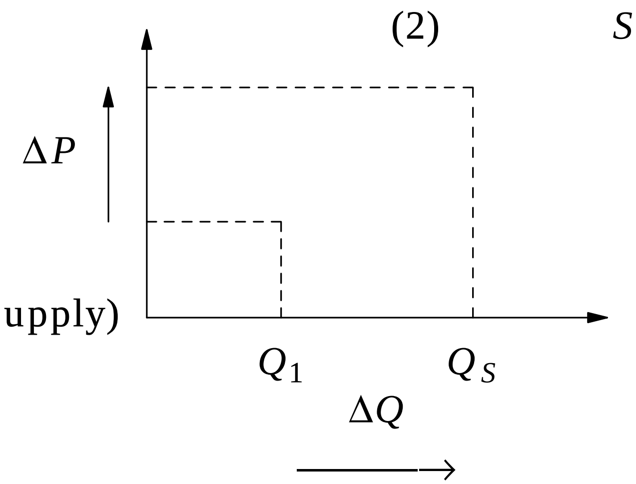
$\simeq \eta^P \bar{Q}_D \Delta P$

(η^P: price elasticity of demand)

& $\bar{Q}_D = \dfrac{Q_D + Q_1}{2}$

Area under Supply Curve

(2)



area

$\simeq \kappa^P \bar{Q}_S \Delta P$

(κ^P: price elasticity of supply)

& $\bar{Q}_S = \dfrac{Q_1 + Q_S}{2}$

Case 2.2: Prices change. (NFX: Not For Exam)

From above, social cost (1) + (2) $= P_s \bullet \Delta G$

$$= \Delta P\ (\eta \bar{Q}_D + \kappa \bar{Q}_s)$$

$$\therefore\ P_s = \frac{\Delta P\ (\eta \bar{Q}_D + \kappa \bar{Q}_S)}{\Delta G} \quad \text{shadow price}$$

$$= \frac{\Delta P \left(\eta (\frac{Q_1 + Q_D}{2}) + \kappa (\frac{Q_1 + Q_S}{2}) \right)}{\Delta G}$$

(if $\eta = \kappa$)

$$= \frac{\Delta P \eta \left(Q_1 + \frac{Q_D}{2} + \frac{Q_S}{2} \right)}{\Delta G}$$

— a means of obtaining the shadow price P_s from Q_1, ΔP, η, κ, Q_D, Q_S, and ΔG.

3.1.3 *Example 3:* **Prices change with a tax wedge.** [DoF 3.5]

Q: In the cotton-farm example, assume that the prices of \$50 (before) and \$60/megalitre (after) include a tax of \$10/megalitre, perhaps to pay for environmental protection. What now is the shadow cost of water to the new farm?

A: We have to adjust for both the induced price increase and the tax wedge between suppliers of water and users of water.

The value of the *displaced* 1000 megalitres of water for the existing farmers who cannot afford to pay \$60/megalitre is still \$55,000 year: we use the tax-inclusive price of \$55/megalitre because they evidently value this water at \$50/megalitre at least, but not at \$60/megalitre.

The shadow cost of the *incremental* water is between \$40 and \$50/megalitre (the tax-exclusive prices), since that's what the suppliers of water receive to induce them to increase supply; say $\$45 \times 9000$ megalitres/year = \$405,000/year.

$\therefore$ Total shadow cost = \$460,000/year.

Price Change & Tax (cont.)

Elastic supply

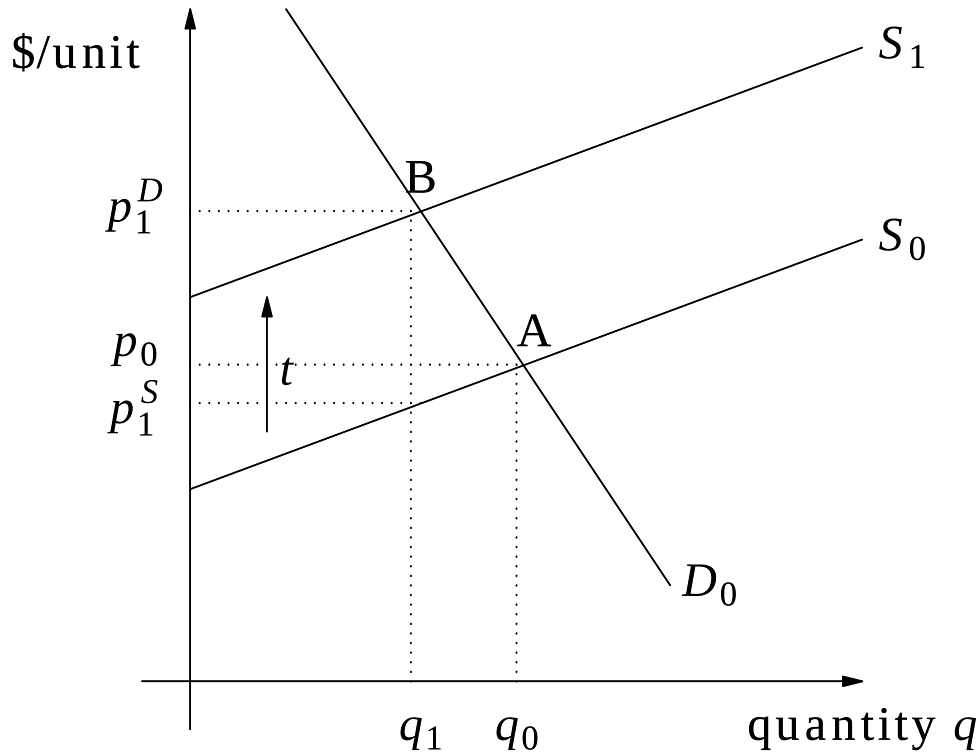


Figure 1

Price Change & Tax (cont.)

In Figure 1 above:

- p_0, q_0 is the initial price at A
- A specific tax of $p_1^D - p_1^S = t$ is imposed
- This is perceived by consumers as a shift in supply from S_0 to S_1

∴ buyers pay (tax-inclusive) p_1^D and producers receive (tax-exclusive) p_1^S

- The tax revenue is $(p_1^D - p_1^S)q_1 = tq_1$
- Consumption falls from q_0 to q_1 (by b)
- The tax revenue is a *transfer* from consumers of the product to consumers in general (via tax receipts and government expenditure)

Price Change & Tax (cont.)

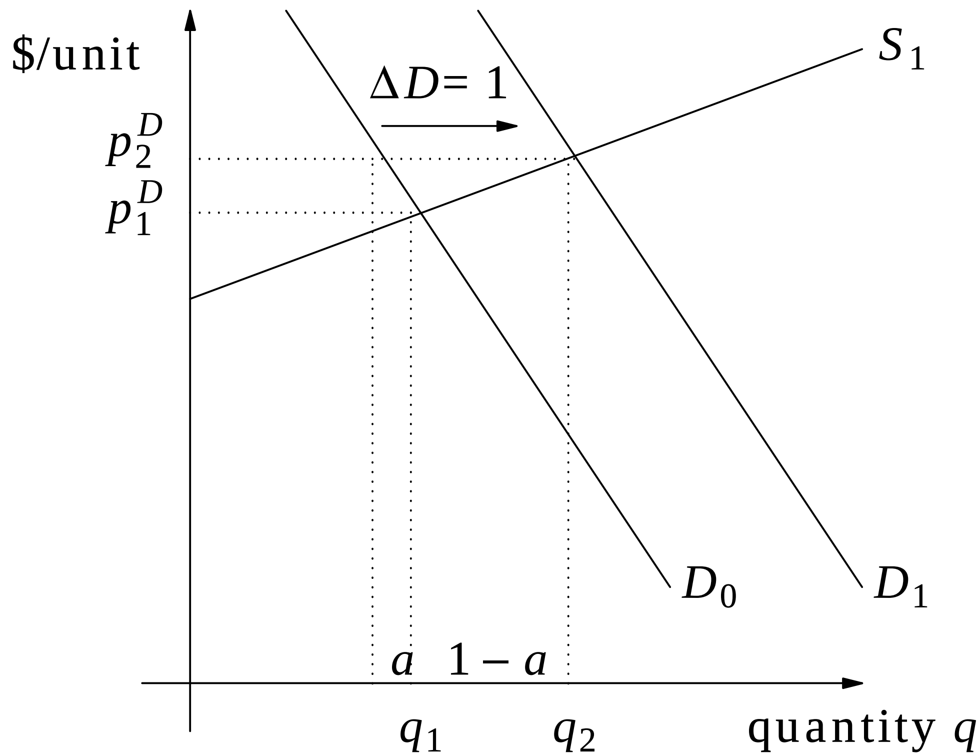


Figure 2

Price Change & Tax (cont.)

In Figure 2:

- p_1 is the initial tax-inclusive price = p_1^D
- because supply S_1 includes the tax t
- The project to be evaluated shifts the demand for the input to the right from D_1 to D_2 (assume $\Delta D = 1$)

∴ the tax-inclusive price is forced up from p_1 to $p_2 = p_2^D$

- and production goes up in total by $1 - a$
- The price increase induces other uses of the good to release an amount a which is absorbed by the project

∴ total usage of the input is $a + 1 - a = 1$ in the project.

- The expansion in output takes place at the tax-exclusive cost $S_1 - t$ or $p_2 - t$.
- The gain to the taxpayer is simply a transfer ∴ ignore it.
- Existing consumers value the reduction in a at the tax-inclusive price p_2^D that they pay

∴ unit social cost = a(gross-of-tax price) + $(1 - a)$(net-of-tax price)

$$= a\, p_2^D + (1 - a)(\, p_2^D - t\,) = \text{shadow price}$$

Hence shadow price = a weighted average of the tax-inclusive and tax-exclusive prices, p_2 and $p_2 - t$, respectively.

Price Change & Tax (cont.)

How do we calculate the weights a and $1 - a$?:

$\eta_D =$ initial price elasticity of demand at p_1

$$\eta_D = -\frac{a/q_1}{(p_2 - p_1)/p_1} \text{ (using initial-point convention)}$$

= % change in quantity ÷ % change in price

Similarly:

$$\kappa_S = \text{ supply elasticity } = \frac{(1-a)/q_1}{(p_2 - p_1)/p_1}$$

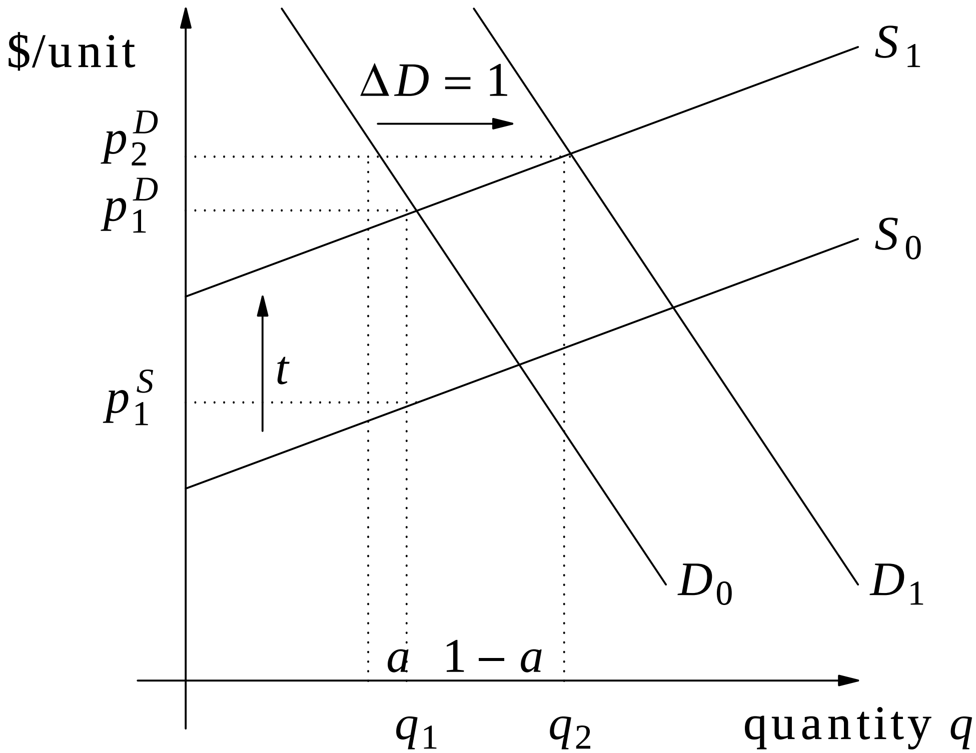
Hence $\dfrac{\eta_D}{\kappa_S} = -\dfrac{a}{1-a}$

and $\therefore\ a = -\dfrac{\eta_D}{\kappa_S - \eta_D}$

and Shadow Price = $ap_2 + (1-a)(p_2 - t)$:

- for an increased demand for the input
- also for an increase in supply of the good if the project results in more of the good

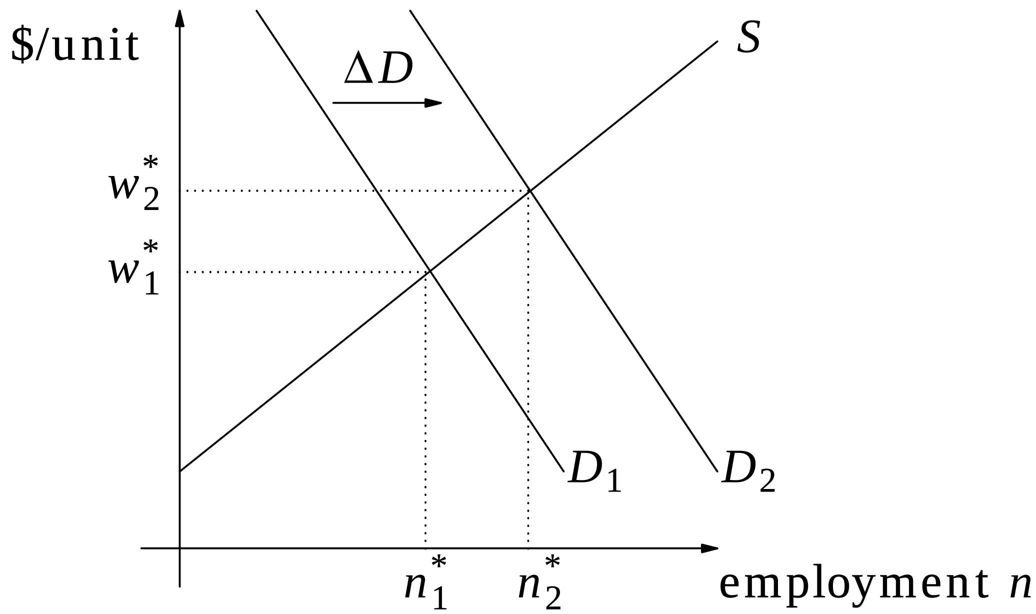
Price Change & Tax (cont.) — **Figure 3**



- Valuation of the increased supply $1 - a$ depends on S_0 (tax-exclusive)
- Valuation of the demand shifted to the project a depends on the shaded area under D_1
- Figure 3 adds the tax-exclusive supply curve S_0 to Figure 2
- So $\Delta D \times p_S$ = the sum of the two areas

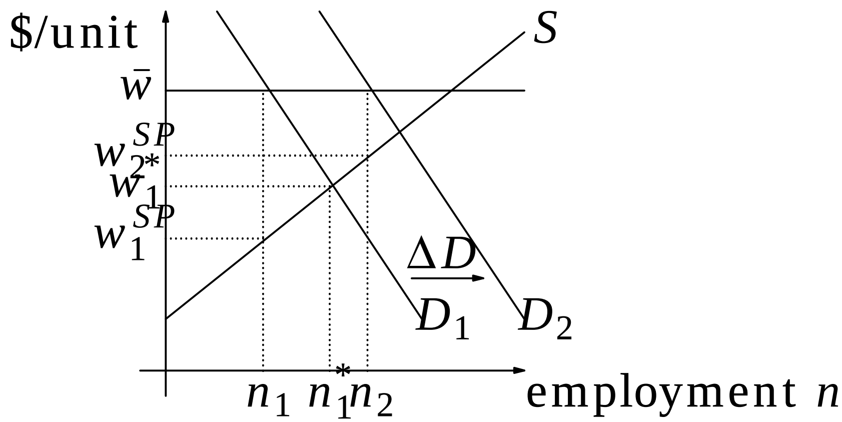
3.1.4 *Example 4*: **Shadow pricing of labour** [FP Ch. 6.4.1, Ch. 10; DoF 3.9]

- In a properly functioning labour market the shadow price of labour is simply the market wage:



- But the project shifts the demand for labour from D_1 to D_2
- more jobs are “created” $(n_2^* - n_1^*)$
- workers move from lower-paid to higher-paid jobs
- and there is no “involuntary” unemployment

Labour (cont.) — Suppose there is a minimum wage $\bar{w}$ for labour set by the IRT. Then the employment level n_1 will be less than the competitive level, n_1^*.

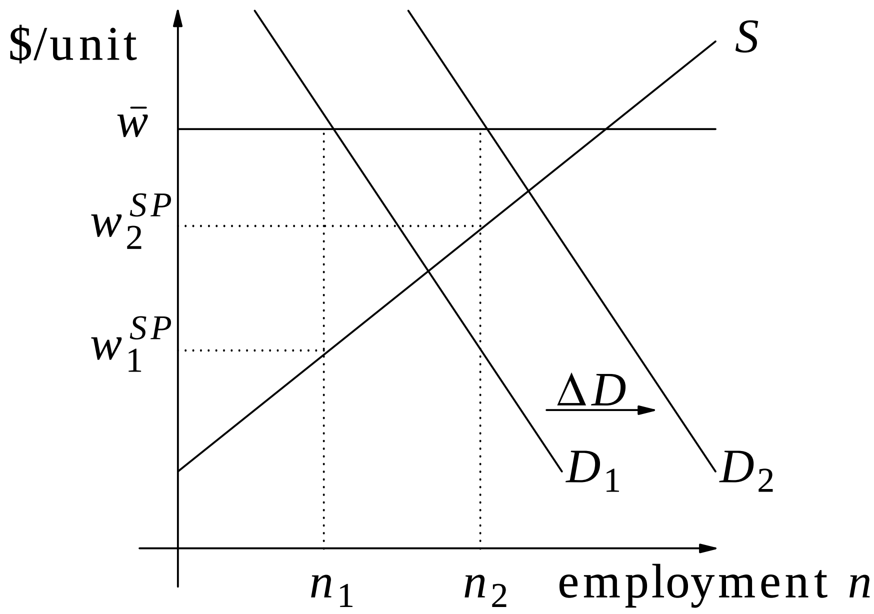


- The projects shifts out demand for labour from D_1 to D_2
- If the additional workers who receive jobs value leisure at w_1^{SP}, then w_1^{SP} is the shadow wage
- New employed workers may have a higher value of leisure w_2^{SP} than w_1^{SP}

∴ this higher average value w_2^{SP} should be used

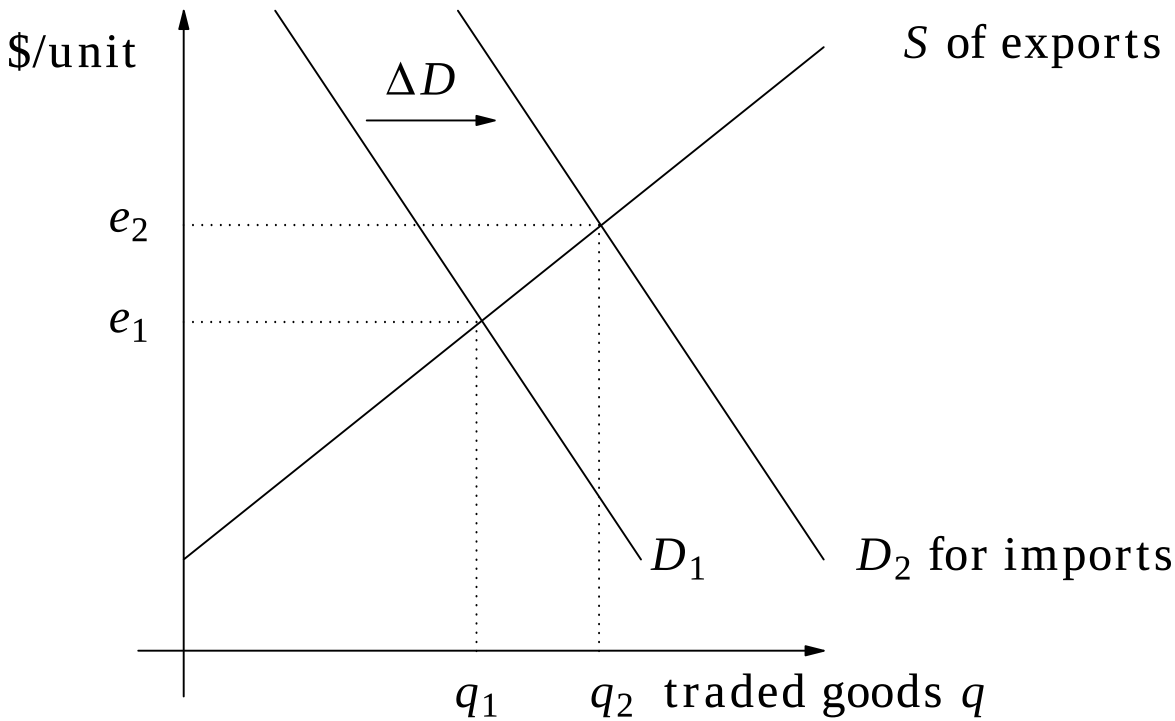
- The social cost (shadow price) is lower than the market price $\bar{w}$
- Why? Because there is *unemployment* at minimum wage $\bar{w}$.

Labour (cont.)



- The change in the wage bill = $\bar{w} \times (n_2 - n_1)$; in F.A. it was the "rectangle" (brown + green).
- The change in the social cost = $\dfrac{w_1^{SP} + w_2^{SP}}{2}(n_2 - n_1)$.
- The opportunity cost of getting a job is less than $\bar{w}$, which is reflected in the supply curve.

3.1.5 *Example 5:* **Foreign exchange** [FP Ch. 9.2; DoF 3.8]



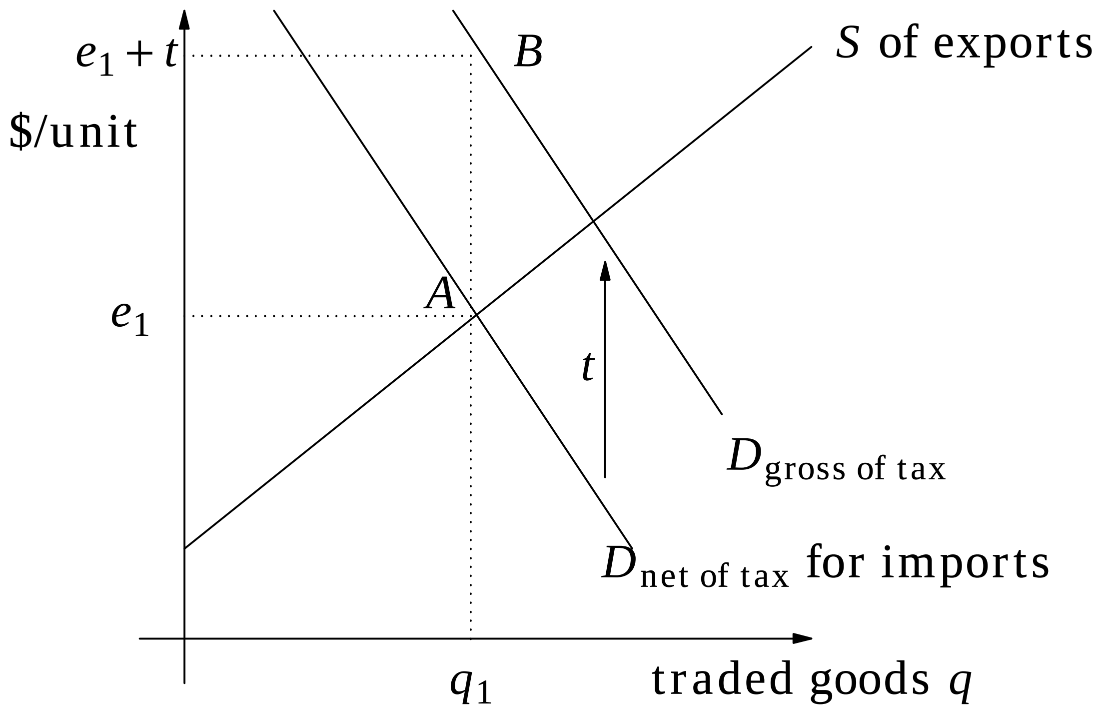
Foreign Exchange (cont.)

In the figure:

- the vertical axis shows the real price of traded goods = the inverse of the exchange rate.
- the supply and demand for foreign currency is initially in balance at q_1, e_1.
- if the demand for imports by Australians goes up by ΔD, the real price of traded goods will cost more
- the $A will devalue in terms of foreign currency as the real price of imports rises (and e rises)
- Australian exporters will gain more revenue in $A terms, to encourage additional exports

∴ an upwards sloping supply curve S.

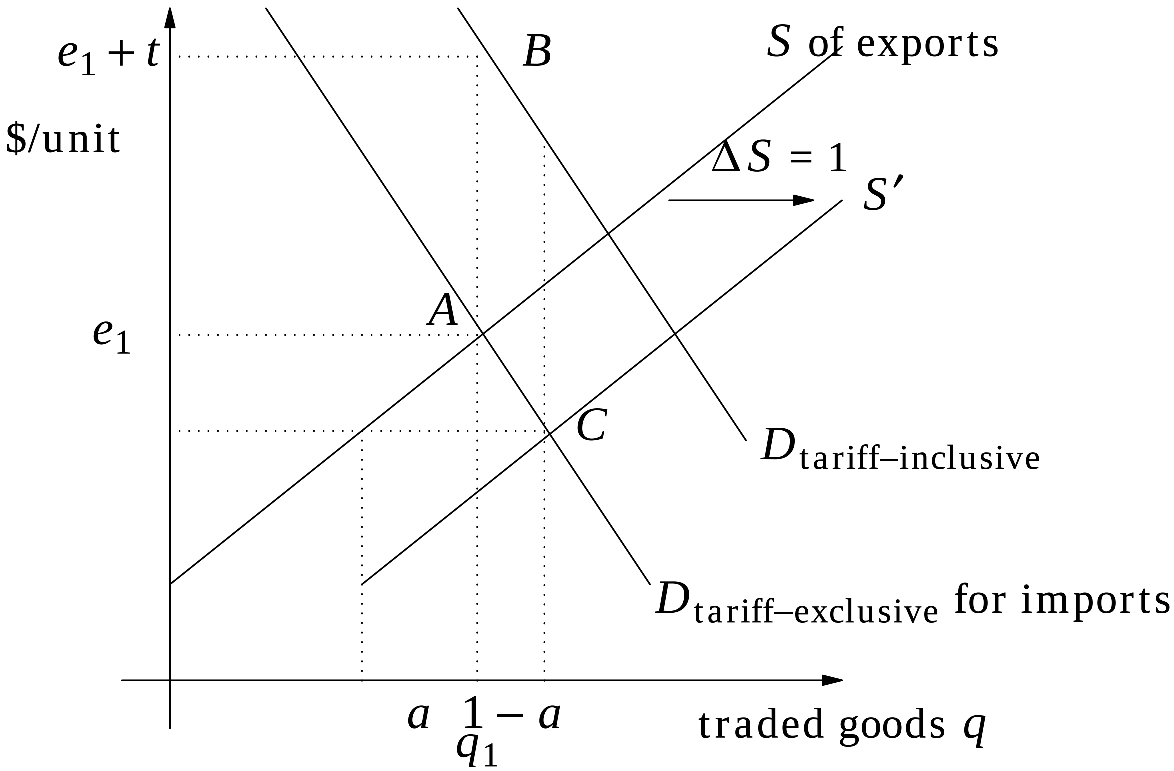
Foreign Exchange (cont.)



A tariff (tax) of t is now imposed on imports

- A represents the equilibrium value of exports
- B is the tariff-inclusive value of imports
- measured in terms of foreign currency, the value of imports and exports is equal.

Foreign Exchange (cont.) —
Project now increases our supply of foreign exchange ΔS via additional exports.
Let ΔS = 1.



- this situation is similar to the tax example (p.5-12)

Foreign Exchange (cont.) —

- increased exports facilitates increased imports valued at $e_1 + t$ (area base $1 - a$)
- increased new exports displaces traditional exports a valued at net-tax price of e_1
- shadow price = a (post-tax price of traded goods) + $(1 - a)$(pre-tax price of traded goods)
- shadow exchange rate will exceed market exchange rate, since exports are under-valued by the market exchange rate. — The Gregory Thesis or Dutch Disease.

Summary of Lecture 5

This lecture introduced the use of market prices — suitably adjusted to become *shadow prices* which accurately reflect the opportunity cost of the goods and services used by the project, whether produced in response to the project's demand (*incremental*) or bid away from existing uses (*displaced*) — in CBA studies..

- How to adjust market prices for taxes (which are *transfers*, by and large).
- How to adjust market prices for price changes caused by the project.
- How to adjust market prices for regulated prices, such as minimum wages with unemployment among the workers the project will hire.
  Shadow wages.