Exhaustibility and the Reserves/Production Ratio

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ABSTRACT: We derive a simple equilibrium relationship, under static expectations, among the reserves/production ratio for an exhaustible resource, the price elasticity of the demand for the flow of extracted resource, and the implicit discount rate.

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1. An equilibrium relationship for exhaustible resources

WE assume an iso-elastic demand for the flow of an extracted exhaustible resource at any time t:

$$R(t) = A(t)\ P(t)^{-\eta(t)}, \qquad \eta(t) > 0, \tag{1}$$

where $R(t)$ is the flow of extracted resource produced at time t, $P(t)$ is the flow price at time t, and $\eta(t)$ is the price elasticity of demand for this flow at time t; $A(t)$ is a shift parameter which models expansions or contractions of the demand schedule through time.

We assume that the marginal cost of extraction is zero, and hence for an exhaustible resource the Hotelling Rule (Dasgupta and Heal 1979) states that the equilibrium price of the resource rises at the discount rate $r(t)$ at time t, ceteris paribus:

$$\dot{P}(t)/P(t) = r(t). \tag{2}$$

We make the assumption of *static expectations* in the three parameters, $\eta(t)$, $r(t)$, and $A(t)$; that is, for all time $\tau \geq t$,

$$A(\tau) = A(t) = \bar{A}$$
$$\eta(\tau) = \eta(t) = \bar{\eta}, \text{ and}$$
$$r(\tau) = r(t) = \bar{r}.$$

We further assume that no new discoveries are expected. These assumptions allow us to integrate equation (2) to obtain the expected price at any future time τ, given a constant discount rate $\bar{r}$:

$$P(\tau) = P(t)\ e^{\bar{r}(\tau - t)}, \qquad \tau \geq t. \tag{4}$$

$P(\tau)$ can be thought of as the *expected* price at any time τ later than t, given an expected constant discount rate $\bar{r}$ and no new discoveries.[1] From the transversality condition and equation (1), the optimum price path $\{P^*(t)\}$ can be solved from the condition that $S(t)$, the stock or reserves remaining at time t, are given by:

$$S(t) = \int_t^{infinity} A(\tau)\ P^*(\tau)^{-\bar{\eta}}\ d\tau, \tag{5}$$

with constant price elasticity of demand, $\bar{\eta}$. Substituting equation (4) into equation (5) and assuming that $A(\tau) = \bar{A}$ is constant, we obtain

$$S(t) = \bar{A}\ P(t)^{-\bar{\eta}} \int_t^{infinity} e^{-\bar{r}\,\bar{\eta}(\tau - t)} d\tau. \tag{6}$$

If expectations are not static, then we can build these expectations into equations (2) and (5), but the result is still the same: an unexpected change in one of the four parameters at any time t will result in a compensating shift in the price level $P(t)$.

From equation (1), we can eliminate the price level $P(t)$ from equation (6), to obtain

$$S(t) = R(t) \int_t^{infinity} e^{-\bar{r}\,\bar{\eta}(\tau - t)} d\tau = \frac{R(t)}{\bar{r}\,\bar{\eta}},$$

which can be rewritten as

1. It is unlikely that, over any stretch of years, there will be no new discoveries or no shifts in the discount rate, but so long as these are unanticipated then equation (4) describes the expected price trajectory. Alternatively, in a rational-expectations framework expected future discoveries would already be incorporated within the anticipated stock of resource. Banks (1986) considers a model with constant proportional growth of gross reserves, but includes no explicit demand function: he assumes instead constant proportional growth in production.

$$\bar{r}\ \bar{\eta}\ \gamma(t) = 1, \tag{7}$$

where $\gamma(t)$, the reserves/production ratio, is defined by

$$\gamma(t) \equiv S(t)/R(t).$$

Equation (7) says that the product of the instantaneous discount rate times the price elasticity of demand times the reserves/production ratio is equal to unity. This implies that if one or more of the expectations of equation (3) suddenly changes, there will have to be an adjustment to the reserves/production ratio before equilibrium is again attained.

Consider oil as the exhaustible resource. If the demand for oil suddenly (unexpectedly) becomes more elastic, then the equilibrium reserves/production ratio of oil will rise, ceteris paribus, which will result in a higher flow of oil. If there are new discoveries, increasing the known reserves unexpectedly, then the equilibrium flow of oil will rise, ceteris paribus. If the discount rate suddenly (unexpectedly) falls, then the equilibrium flow of oil will fall, ceteris paribus.

The relationship (7) does not include price $P(t)$ explicitly. But, from equation (1), a shift in the flow $R(t)$ or in the demand shift parameter $A(t)$ or in the price elasticity $\eta(t)$ will occur together with a shift in $P(t)$, and from equation (7) $R(t)$ will shift if expected discount rate $\bar{r}$, price elasticity $\bar{\eta}$, or known reserves $S(t)$ unexpectedly change. We can formalize this discussion, in the form of a theorem:

Theorem: In equilibrium, the product of the discount rate $\bar{r}$ and the price elasticity of demand $\bar{\eta}$ for the flow of an exhaustible resource equals the reciprocal of the reserves/production ratio, $\gamma(t)$:

$$\bar{r}\ \bar{\eta} = \frac{R(t)}{S(t)} = \frac{1}{\gamma(t)}\,, \tag{8}$$

Corollary: If the expected level of demand at any price is not constant, but growing at a constant proportional rate $\bar{g}$, then the four parameters are related by the equation:

$$\bar{r}\ \bar{\eta} = 1/\gamma(t) + \bar{g}. \tag{9}$$

Proof: A constant growth in demand at any price is modeled by

$$\dot{A}(t)/A(t) = \bar{g},$$

which, with constant expected growth rate $\bar{g}$, can be integrated to obtain

$$A(\tau) = A(t)e^{\bar{g}(\tau - t)}, \qquad \tau \geq t,$$

from which the demand at any time is given by

$$R(\tau) = A(t)P(t)^{-\bar{\eta}}e^{(\bar{g}\ -\ \bar{r}\ \bar{\eta})(\tau\ -\ t)}.$$

Whence,

$$S(t) = \frac{R(t)}{\bar{r}\ \bar{\eta} - \bar{g}}\,,$$

or

$$\bar{r}\ \bar{\eta} = 1/\gamma(t) + \bar{g},$$

for a growth rate $\bar{g} \leq \bar{r}\ \bar{\eta}$. □

2. References

Banks, F.E., 1986. Economic theory and the price of oil, *OPEC Review* **10**, 321–334.

Dasgupta, P.S. and G.M. Heal, 1979, *Economic Theory and Exhaustible Resources* (Welwyn, J. Nesbet).