Firm Size Distributions in an Industry with Constrained Resources.

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Abstract

We propose an equilibrium model for firm size distribution in an industry with a constrained essential input. The model applies when the population of firms is small and homogeneous and the supply of the necessary input factor is perfectly inelastic. We argue that although the Gibrat assumption obtains, this does not result in the lognormal distribution because of the entries, exits and mergers of firms competing for the inelastic essential resource. Using our own thirty-two-year database of firms, we test the broken-stick, or random-ordered-interval, model that we call the Whitworth distribution, successfully applied by others to a number of data sets, including the abundances of bird species. We propose the Whitworth as the basic model of the equilibrium distribution of firm sizes for such supply-constrained industries, and find it fits our thirty-one-year database best.

JEL Classification: L11.

Key words: Equilibrium/Firm Size distribution, Gibrat, Whitworth.

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Introduction

The size distribution of business firms in an industry is an important element of market structure: it affects the industry concentration, which in turn is an important element in competition. Simon and Bonini (1958, p.611) found "the empirical data on the distributions of firms by size [across industries] are numerous and monotonously similar", and this conclusion has not changed over the intervening years — the size distribution of firms has been extensively researched, and all results show that the distributions are highly skewed. From the time of Gibrat's seminal paper on the growth of firms in 1931 there has been a plethora of papers[1]. The very large literature on size distributions is comprehensively reviewed by Sutton (1997), Mitzenmacher (2003) and Audretsch et al (2004). De Wit (2005) reviewed steady-state firm size distributions, and their stabilizing forces, that had been derived by others. Unlike these earlier studies, however, we present a new argument for the stabilization of firm size distributions, namely the competition for a scarce, necessary factor of production.

One distribution has been used to describe the breakage of solids as well as firm size and the species abundance, the Whitworth distribution, which has wide applications, from the abundance of ecological species (MacArthur 1957) and the breakage of coal (Manning, 1952) and (Callcott, 1966), to the distribution of segments in the human genome (Clark, 1999), election results (Itoh and Ueda, 2000), the distribution of articles and ranking of journals (Basu 1992 and Sittig 1996) and, as we will show,

[1] See inter alia, Hart and Prais (1956), Simon and Bonini (1958), Ijiri and Simon (1964), Steindl (1965), Quandt (1966), Ijiri and Simon (1974), Ijiri and Simon (1977), Clarke 1979), Audretsch (1995), Sutton (1997), Kwasnicki (1998), Sutton (1998), Axtell (2001) and Gans and Quiggin (2003).

to firm size distributions. We examine the forces driving the development of, and changes in, the size distributions in an extractive industry from 1970 to 2001.

The paper is structured as follows; Section I presents our data; Section II tests for the Gibrat assumption; Section III describes the resource-partitioning model and applies it to our data; Section IV discusses the results; Section V concludes.

Section I: Data

Our industry is the New South Wales (NSW) black-coal industry, which is part of the Australian black-coal industry, ANZSIC code 1100. NSW coal production has ranged from 73% of Australia's total black coal production in 1970 to 42% in 2001, and, while NSW coal output is now exceeded by Queensland coal output, the NSW industry has a richer history to research.

No mining is possible in NSW without a mining lease. Mining leases are issued by the appropriate government department, which thus controls the development of new mines. Since 1976, when the first public tenders were called for the development of a new coal-mining area, the NSW State Government has restricted the issuing of new lease areas, except for extensions to existing colliery holdings. Periodically, but not frequently, new areas are released for exploration and development under competitive tender[2]. Incumbent firms can be granted new mining areas as compensation for depletion of their reserves, but firms not already operating in the industry are restricted in their access to new mines. This creates a significant barrier to entry. In general, the only way a firm can enter the industry, apart from the

[2] Tenders were called for a single mining area in 1975, and tenders were not called again until 1990, when one mining area was offered. Single mining areas were offered in 1994, 1996, 1997 and 2001.

infrequent tenders for new developmental mining areas, is to buy an incumbent firm or an operating mine. The result is that when one firm enters the industry or expands its operations, another firm exits or its proportionate size is reduced, and the total size of the industry remains constant[3].[4].

Our measure of the size of the firm is the annual quantity of coal produced by the mines controlled by the firm, but, as a normalizing procedure to correct for the common growth resulting from universal application of new technology, the analysis is conducted on the firm's share of the total industry production each year i.e. its proportionate size. Our analysis is from 1970 to 2001 and the data are gathered principally from Joint Coal Board (now Coal Services Pty. Ltd.) records. It should be noted that our data are not a sample, but provide the full enumeration of the population of the NSW coal industry.

NSW coal firms are largely homogeneous — they mine coal by similar methods, using similar technology, using a unionized labour force, and sell to the same markets. Only geological conditions have the capacity to introduce natural heterogeneity. This homogeneity is reflected in two findings: a lack of relationship between size and duration found by Lawrance and Marks (2004), and the

[3] In general, the expansion of mines by technological improvements is available to all firms, so the proportionate size only changes by the acquisition or disposal of mines (cet. par.)

[4] Although the resource (NSW black coal) is constrained (is strictly inelastic in supply), the value of NSW coal does not obey Hotelling's (1931) rule (viz. "the price of an exhaustible resource must grow at a rate equal to the rate of interest, both along an efficient extraction path and in a competitive resource industry equilibrium", Devarajan and Fisher 1981, p66). That is, its value is not determined by its (local) constrained supply, but by the existence of close substitutes: coal from Queensland and foreign suppliers, and natural gas and oil, globally traded.

compliance with both the Gibrat assumption and the Whitworth size distribution as discussed below. More details on the database may be found in Lawrance and Marks (2004).

Section II Gibrat Assumption

Simon and Bonini (1958) showed that size distributions can be generated from simple stochastic models, a common form of which is the lognormal distribution that arises when Gibrat's Law of Proportional Effect obtains. This so called "law" is characterized as: "a variate subject to a process of change is said to obey the law of proportional effect if the change in the variate at any step of the process is a random proportion of the previous value of the variate" (Chesher, 1979 p.403, see also Goddard 2006). Accordingly, proportional growth is unrelated to firm size, and all firms have an equal probability of attaining a particular growth rate over any given period, regardless of their size. This is the same as saying "that the expected value of the increment to a firm's size in each period is proportional to the current size of the firm" (Sutton 1997 p.40).

We have found no studies of growth patterns in industries (such as the extractive industry we examine) with constrained (or depleting) resources, and our prior belief was that Gibrat's Law would not to apply to firms in the NSW coal industry. As production technology is uniform across the industry, one might expect individual plants (mines) to increase their production in the same proportion, regardless of their size, as new technology becomes universally available and until the resource becomes exhausted[5]. But the result for the growth of the firm (which can own several mines) could be confounded by the acquisition of mines from other firms. One firm could experience sudden growth by acquisition of a mine while another could experience sudden reduction by disposal of a mine. We discuss this further in Section 4 below.

There are two means of growth of firms in the NSW coal-mining industry: increases in production of individual mines as new technology becomes available, and the acquisition of mines owned by other firms. In general, the former method should not increase the proportionate size of the firm for two reasons. First, proprietary mining technology is very unusual, and second, any new technology is soon diffused through the industry via the equipment manufacturers who supply to all the industry and usually hold the rights to technological advances in equipment. In practice, due to the high fixed costs of mining, each mine is rapidly developed to its maximum production capacity (taking advantage of low marginal costs), consistent with the technology available at the time and the geology of the resource. If the current owner does not have the financial ability to develop a mine to its maximum capacity, then the mine will be more valuable to another firm (with higher financial ability), which may acquire the mine (or its owner).

Changes of ownership of mines and firms are frequent, with a total of 129 mines changing hands over the period 1970 to 2001. From 1980 to 2001, an average of

[5] Mines are largely homogeneous, being made up of a aggregation of similar units. A single unit of an underground mine can be one "continuous miner" and its associated equipment, and a single mine can be comprised of one or more units.

one in ten coalmines operating in any one year in NSW were closed, or changed ownership (with a range from 1% up to 44% of all mines operating in each year). Each time a mine changes ownership, or closes, it has the potential to alter the growth of the firm, the industry concentration, and the firm size distribution. It is this change of mine ownership that we expected to confound Gibrat's assumption, although the majority of these changes resulted from a firm being wholly acquired by a newly entering firm, which would not change the concentration.

To test for Gibrat's assumption, we use the method proposed by both Mansfield (1962) and Chesher (1979) and used by (Vennet 2001), Hardwick and Adams (2002), Audretsch et al (2004) and others. This entails the regression $D_{i,t} = \beta D_{i,t-1} + \varepsilon_{i,t}$, where $D_{it} = \{[\text{mean of } \ln(S_i)] - \ln(S_i)\}_t$ at time t, and S_{it} is the size of the ith firm at time t, defined as the share of the total industry production. When β is less than unity, the larger a firm, the less likely it is to grow, and when β is greater than unity, the larger the firm, the more likely it is to grow. If $\beta_1 = 1$, it shows that the firm growth is independent of the initial size, which implies that the assumption holds. Conversely, if β is significantly different from unity, there is no evidence of Gibrat's Law. Since the the standard errors in the regression are only valid if the data are not serially correlated, we also run the regression $\varepsilon_{i,t} = \rho\varepsilon_{i,t-1} + \nu$, where $\varepsilon_{i,t}$ is the ith error term for the tth year. If $\rho = 0$, no serial correlation is indicated.

We find that only data from the years 1980, 1982,1995 and 2001 reject Gibrat at the 5% level, while the β for the pooled data for 1971 to 2001 is about 2.5 standard deviations away from unity, and the pooled 1976 to 2001 data support Gibrat. As we discuss later, from 1976 there was greater competition for mining leases, which introduced the stabilizing factor that results in the equilibrium best described by the Whitworth distribution. Table 1 summarizes the results of the regressions. Contrary to our prior beliefs, we find that the buying and selling of coalmines does not confound Gibrat’s Law. The expected lognormal firm size distribution does not, however, follow — an explanation for this is given in the Section 4.

The firm’s growth follows Gibrat's Law because of the homogeneity of the firms. The product (coal) is essentially homogeneous and the technology used is the same for all firms. Firms are made up of differing numbers of mines which comprise similar units, as discussed before, each of which can expand at the same rate per period. A large firm with mines comprising many units can therefore expand at the same rate as a small firm with a single mining unit. Despite our prior assumption, the change of ownership of individual mines does not confound Gibrat's Law. The number of firms buying or selling mines each year is a small proportion of the total number of firms and so has a small influence on the size distribution. Moreover, because many acquisitions are of existing firms in the industry by new-entry firms, only the name of the firm changes and not the size distribution.

Insert Table 1 here

Section III: Whitworth Model

To describe the relative abundances of bird species in a given locality, MacArthur (1957) hypothesized an environment of "non-overlapping niches". In this, he compared the environment to a "stick of unit length on which $n - 1$ points are thrown at random", resulting in n parts which are "proportional to the abundances of

Table 1

Regression results for Gibrat and for Serial Correlation

Gibrat: $\ln D_{it} = \beta_0 + \beta_1 \ln S_{i,t-1} + \mu_{it}$

Ser. Corr.: $\varepsilon_{i,t} = \rho \varepsilon_{i,t-1} + \nu$

Year		Gibrat	Ser. Corr.
1971	Coefficient	0.963	-0.517
	SD	0.052	0.329
1972	Coefficient	0.995	-0.375
	SD	0.056	0.328
1973	Coefficient	0.871	-0.390
	SD	0.158	0.334
1974	Coefficient	0.939	-0.424
	SD	0.151	0.302
1975	Coefficient	0.853	-0.209
	SD	0.129	0.290
1976	Coefficient	1.054	0.053
	SD	0.044	0.281
1977	Coefficient	0.988	-0.443
	SD	0.048	0.289
1978	Coefficient	0.978	-0.525
	SD	0.069	0.274
1979	Coefficient	1.045	-0.224
	SD	0.065	0.265
1980	Coefficient	***0.921***	-0.029
	SD	0.026	0.330
1981	Coefficient	0.960	-0.334
	SD	0.055	0.368
1982	Coefficient	***0.860***	***-0.743***
	SD	0.059	0.233
1983	Coefficient	0.943	-0.342
	SD	0.072	0.252
1984	Coefficient	1.005	-0.279
	SD	0.083	0.264
1985	Coefficient	0.887	-0.108
	SD	0.083	0.310
1986	Coefficient	0.976	-0.146
	SD	0.073	0.279

Year		Gibrat	Ser. Corr.
1987	Coefficient	1.005	-0.303
	SD	0.046	0.245
1988	Coefficient	1.020	-0.172
	SD	0.036	0.259
1989	Coefficient	0.969	-0.118
	SD	0.051	0.285
1990	Coefficient	0.923	0.134
	SD	0.128	0.255
1991	Coefficient	0.925	-0.006
	SD	0.043	0.355
1992	Coefficient	0.986	-0.368
	SD	0.033	0.220
1993	Coefficient	1.050	0.055
	SD	0.042	0.226
1994	Coefficient	1.033	-0.207
	SD	0.069	0.193
1995	Coefficient	***0.785***	0.148
	SD	0.081	0.435
1996	Coefficient	0.983	-0.176
	SD	0.055	0.258
1997	Coefficient	0.999	-0.212
	SD	0.039	0.237
1998	Coefficient	1.050	0.233
	SD	0.026	0.218
1999	Coefficient	1.045	-0.064
	SD	0.035	0.225
2000	Coefficient	0.996	-0.165
	SD	0.072	0.256
2001	Coefficient	***1.491***	***-0.590***
	SD	0.089	0.255

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	Combined years 1971-2001		Combined years 1971-2001	
	Gibrat	Ser. Corr	Gibrat	Ser. Corr
Coefficient	***0.970***	0.03***6***	0.979	0.086
SD	0.013	0.045	0.013	0.048

Results in ***bold italics*** show non-acceptance of Gibrat at the 5% level the species" (ibid. p.293)[6]. He gives the expected length of the *r*th shortest piece

as: $(1/n)\sum_{i=1}^{i=r}\left[1/(n-i+1) \right]$, and the expected abundance of the *r*th most rare species

as: $(m/n)\sum_{i=1}^{i=r}\left[1/(n-i+1) \right]$, where *m* is number of individuals, *n* is the number of species and $i = 1,2...n$. Whitworth (1901), cited by MacArthur, derived identical results for the expected values of the random division of a magnitude where the only constraint is that the sum of the parts will always equal the original magnitude[7]. If the magnitude *S* is divided into *n* random parts, then the expected values, arranged in ascending order of magnitude, are:

$(s/n).(1/n)$,

$(s/n).[1/n + 1/(n-1)]$,

$(s/n).[1/n + 1/(n-1) + 1/(n-2)]$,

and so on. Whitworth's simple proof of this is given in Appendix 1; Barton and David (1956) developed a more rigorous mathematical proof of the expected values; see also Moran 1947, Webb 1974 and Holst 1980.

In his empirical work, MacArthur (1957) used the (nearly) linear relationship in the Whitworth model between the expected sizes (arranged in descending order) and the natural log of the rank. He first found observed curves much steeper than predicted by the model, but when he divided the community into smaller groups and plotted their sizes this steepness was corrected. In other ecological studies, King (1964) found that MacArthur's model succeeded when the domain was small and uniform,

[6] This model results from the instantaneous division of the stick, and not from sequential breakages.

and failed when the domain was large and heterogeneously diverse. An implication of the model — that there is only one essential resource that is divided amongst the species — should be noted. As Pielou (1969) pointed out, “there is no way of randomly partitioning a space of more than one dimension into nonoverlapping parts”. May (1975) concluded that MacArthur’s Whitworth model holds for an “ecologically homogeneous group of species [that] apportion randomly amongst themselves a fixed amount of some governing resource”, whereas a lognormal pattern of relative abundance should be expected for large or heterogeneous groups of species. The Whitworth (broken-stick) model fits for “approximately small and homogeneous taxa” (Ibid). Magurran (1988) found that "good fits of the model seem to be found primarily in narrowly defined communities of taxonomically related organisms". The NSW coal industry can be seen to be just that: a small number of firms comprising a relatively homogenous community, with the domain strictly bounded as a result of geological constraints and the available mining leases.

In MacArthur’s (1957) non-overlapping niches model, the dependence among species results in the total number of all individuals across all species remaining constant, so that an increase in one species' abundance results in a decrease in one or more of the other species'. This condition also applies to firms in the NSW coal industry, using the firm share of industry annual production as the measure of size (abundance). As explained in Section I, within certain constraints, the total size of the industry is constant, and firms can only enter or increase their share by purchasing mines from other firms. This dependency is a result of an inelastic

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[7] This was first published as a pamphlet in 1898; the first edition of Whitworth's *Choice and Chance* appeared in 1901 (Manning 1952). A detailed account of Whitworth's life and work is given in Irwin (1967).

essential resource and could occur in a mature industry in which the essential resource is fully utilized, or in an industry where the resource is exogenously constrained (by regulation, for instance). The Whitworth (or broken-stick[8]) distribution requires the homogeneity of firms, which leads to the random division of the resource. The process is a dynamic adjustment that will depend on two possibilities: a change in the availability of the resource that is created by firms ready to exit or to dispose of mines, or by an increment in the total resource (by regulation or exploration).

Sugihara (1980), suggested that ecology species abundance follows the lognormal distribution and this "can reflect a broad and very simple underlying form of community structure". The log normal distribution results when "breakage occurs sequentially rather than simultaneously" (Ibid p.774)[9]. Epstein (1947) showed that successive breakage (of minerals) produced a lognormal distribution asymptotically (see also Aitchison and Brown 1957), which is consistent with Sugihara's result. It is the concept of instantaneous breakage, as distinct from sequential breakage, which

[8] The term "broken stick" is sometimes used as a model for an event with an abrupt change (Curnow 1973; Chui 1996; Guthrie and Moorehead 2002), so this can be confusing. Our term "Whitworth distribution" avoids this confusion and pays tribute to its first proponent — "it (is) lamentable that that ... Whitworth ... should be receiving no credit for the original derivation of the broken stick formula" (Ghent and Hanna 1968). David and Barton (1962, p.23) doubt that Whitworth invented the eponymous distribution: "As was customary with the textbook writer of his day, he gives no references". Whitworth's 1898 pamphlet, however, suggests that this formulation was his (Whitworth 1901).

[9] In sequential breaking the magnitude is broken once at a random point; one of the two parts is selected at random and again randomly broken into two parts and so on. In simultaneous breaking the magnitude is broken into the specified number of parts in one action.

differentiates MacArthur's work from that of Sugihara, and results in the Whitworth distribution rather than the lognormal distribution[10]. The Whitworth distribution itself is not a probability distribution but each of its parts is the expectation of a separate distribution. The frequency distributions of each part of five Whitworth parts, as produced by Monte Carlo sampling, are shown in Figure 1.

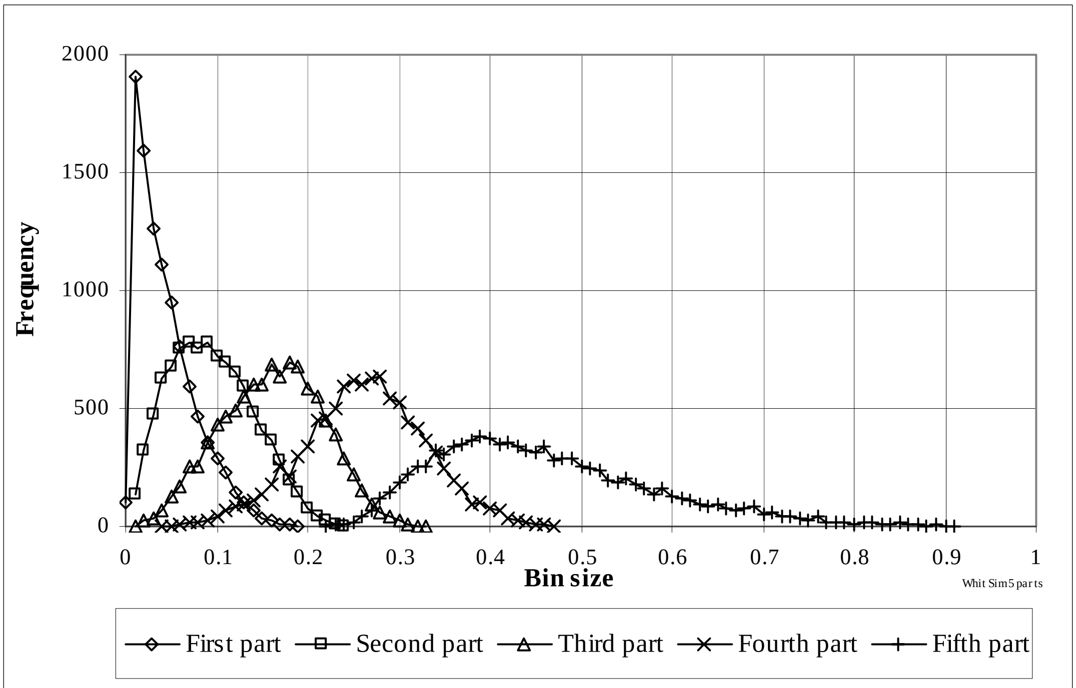
Insert figure 1 here

We can relate the results from these ecological studies to the size distributions of business firms. We propose that whenever there is a change in the number or relative sizes of firms the distribution of firm sizes diffuses to an equilibrium. For a model to achieve this, there must be an equilibrating force — Steindl (1965) speaks of a "stabilizing influence". Just as MacArthur's (1957) model is driven by the competition for resources, so our model is driven by competition for the scarce, necessary resource of mining tenements, required for the production of coal. We propose the Whitworth distribution as the model for size distribution in this industry. As Kwasnicki (1998) noted: "[one] approach to explain skewness of distributions is focused on optimal allocation of scarce factors of production".

We are not the first economists to propose the Whitworth distribution in the study of firm sizes — Cohen (1966) analyzed a number of American industries for a fit to Whitworth. He added the concept of a "threshold" to the Whitworth model, which he said could be useful in economics, where a minimum efficient size often applies. But many of his data lay outside his critical values. He explained this lack of fit by proposing that the total industry is comprised of several markets (which is similar to

[10] Tokeshi (1990), however, showed that the Whitworth (broken-stick) model can result from sequential breakage if the probability of selection for further breakage of a part is positively related to the size of that part.

Figure 1
Frequency plots of the five parts of a simulated Whitworth distribution



Theoretical Whitworth Sizes

0.04	0.09	0.16	0.26	0.46

MacArthur splitting his groups into several populations). Cohen's competition occurred in the output market, that is, competition for demand, while MacArthur's competition was for input resources, as occurs in the industry we examine. We argue that it is the constraint on the input resource that produces the Whitworth size distribution, not the general market competition.

3.1 Testing for Whitworth

We use two methods to test the fit of the Whitworth model to our data. First, we see whether each observation lies within two standard deviations (SD)[11] of the theoretical value (as used by Cohen, 1966) and, second, we use a programme that simulates the Whitworth distribution and provides a test statistic[12]. With the null hypothesis that the data conform to the Whitworth distribution, we find that our data from 1970 to 1976 do not fit the Whitworth model well, while the data from 1977 to 2001 are consistent with the Whitworth model at the 5% level (with the exception of the years 1994-1996 and 2001[13]). Table 2A presents these results and Table 2B shows all the proportionate sizes, with those outside ± 2SD marked.

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[11] The variance of the distribution of the *r*th part is given by

$$\frac{1}{n+1}\left(\frac{1}{n}\sum_{i=1}^{r}\frac{1}{(n+1-i)^2}-\frac{1}{n}\left[\sum_{i=1}^{r}\frac{1}{n+1-i}\right]^2\right)$$, where n = the number of parts (divisions)

(Barton and David 1956).

[12] We thank Glen Barnett for coding the program for the tests.

[13] In 1994 the third largest firm acquired the largest firm, which caused deviation from the Whitworth model. From 1997 on stability returned and the distribution again fitted the Whitworth model. We do not postulate the speed of equilibration, but this observation suggests a period of one to three years for a sizeable shock to be absorbed. In 2001 there was another major merger, resulting in the ninth largest firm becoming the largest.

Table 2

Results of testing for Whitworth Distribution

A. Results of Whitworth Test

Results in ***Bold Italic*** do not support the Whitworth distribution at the 5% level

Year	**1970**	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**	**1977**	**1978**
Number	18	18	18	14	17	20	19	18	18
Test Statistic	0.038	0.039	0.033	0.018	0.022	0.019	0.021	0.010	0.006
Ave *p*-value	***0.006***	***0.006***	***0.009***	0.062	***0.026***	***0.021***	***0.021***	0.086	0.220
SE	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.003

B. Testing sizes in each year against the Whitworth expected sizes

Observations below: those in ***Bold Italic*** are outside the +/- 2SD range

1970	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**	**1977**	**1978**
0.005	0.005	0.004	0.006	0.005	0.005	0.004	0.004	0.004
0.005	0.005	0.005	0.009	0.006	0.005	0.005	0.005	0.005
0.005	0.007	0.006	0.012	0.006	0.005	0.006	0.006	0.006
0.008	0.007	0.006	0.018	0.006	0.006	0.007	0.006	0.007
0.014	0.009	0.011	0.022	0.007	0.007	0.008	0.010	0.011
0.015	0.009	0.012	0.028	0.008	0.008	0.009	0.017	0.013
0.016	***0.009***	0.015	0.033	0.019	0.009	0.016	0.021	0.018
0.018	***0.013***	0.016	0.049	0.036	0.018	0.017	0.023	0.023
0.022	***0.019***	0.029	0.053	0.040	0.022	0.018	0.024	0.032
0.022	***0.020***	0.030	0.067	0.041	0.025	0.030	0.051	0.051
0.036	0.038	0.040	0.072	0.043	0.038	0.043	0.055	0.051
0.037	0.039	0.041	***0.072***	0.055	0.038	0.047	0.057	0.052
0.046	0.049	***0.042***	***0.240***	0.061	0.042	0.052	0.063	0.060
0.047	***0.049***	***0.049***	0.318	0.064	0.046	0.056	0.066	0.078
0.060	***0.049***	***0.058***		0.076	0.049	0.063	0.083	0.079
0.063	0.083	***0.071***		***0.205***	0.066	0.068	0.090	0.100
0.254	***0.253***	***0.227***		***0.321***	0.069	0.080	0.131	0.143
0.330	***0.336***	***0.336***			***0.069***	0.146	0.290	0.267
					0.171	***0.324***		
					0.301			

Table 2 continued

A. Results of Whitworth Test

Results in ***Bold Italic*** do not support the Whitworth distribution at the 5% level

Year	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**
Number	17	18	19	20	19	19	19	19	19
Test Statistic	0.010	0.007	0.006	0.004	0.004	0.006	0.003	0.002	0.006
Ave *p*-value	0.098	0.177	0.213	0.360	0.393	0.189	0.573	0.624	0.229
SE	0.002	0.003	0.003	0.004	0.004	0.003	0.004	0.004	0.003

B. Testing sizes in each year against the Whitworth expected sizes

Observations below: those in ***Bold Italic*** are outside the +/- 2SD range

1979	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**
0.004	0.005	0.003	0.003	0.003	0.003	0.003	0.003	0.002
0.004	0.005	0.003	0.003	0.004	0.004	0.003	0.003	0.003
0.004	0.005	0.004	0.004	0.005	0.007	0.010	0.008	0.007
0.009	0.005	0.004	0.004	0.006	0.010	0.014	0.013	0.017
0.010	0.012	0.006	0.008	0.017	0.011	0.015	0.018	0.018
0.014	0.014	0.014	0.015	0.023	0.018	0.018	0.020	0.020
0.020	0.024	0.016	0.017	0.025	0.023	0.025	0.022	0.020
0.023	0.028	0.021	0.019	0.029	0.024	0.028	0.024	0.024
0.034	0.030	0.027	0.023	0.030	0.028	0.031	0.027	0.028
0.038	0.038	0.035	0.027	0.033	0.031	0.034	0.039	0.038
0.048	0.040	0.035	0.031	0.039	0.041	0.040	0.042	0.044
0.053	0.050	0.047	0.036	0.047	0.044	0.041	0.052	0.047
0.082	0.053	0.057	0.053	0.051	0.053	0.042	0.057	0.051
0.085	0.074	0.058	0.060	0.069	0.069	0.058	0.062	0.051
0.094	0.087	0.066	0.062	0.074	0.069	0.060	0.073	0.066
0.208	0.092	0.090	0.075	0.075	0.073	0.101	0.080	0.090
0.271	***0.199***	0.098	0.089	0.089	0.094	0.123	0.096	0.099
	0.241	**0.197**	0.090	0.138	0.137	0.130	0.130	0.119
		0.219	***0.182***	0.241	0.260	0.225	0.231	0.255
			0.199					

Table 2 continued

A. Results of Whitworth Test

Results in ***Bold Italic*** do not support the Whitworth distribution at the 5% level

Year	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**
Number	20	23	22	23	24	24	23	23	25
Test Statistic	0.003	0.005	0.007	0.007	0.007	0.006	0.027	0.012	0.011
Ave *p*-value	0.399	0.155	0.103	0.097	0.084	0.108	***0.005***	***0.036***	***0.029***
SE	0.004	0.003	0.002	0.002	0.002	0.003	0.001	0.002	0.001

B. Testing sizes in each year against the Whitworth expected sizes

Observations below: those in ***Bold Italic*** are outside the +/- 2SD range

1988	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**
0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.003
0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.004	0.004
0.007	0.004	0.003	0.003	0.003	0.002	0.003	0.005	0.006
0.010	0.006	0.003	0.004	0.004	0.003	0.004	0.006	0.006
0.018	0.006	0.004	0.004	0.007	0.004	0.004	0.006	0.008
0.018	0.009	0.005	0.005	0.007	0.007	0.007	0.007	0.009
0.022	0.010	0.008	0.007	0.011	0.010	0.007	0.010	0.011
0.023	0.021	0.010	0.012	0.013	0.012	0.012	0.016	0.011
0.024	0.022	0.018	0.012	0.013	0.012	0.012	0.016	0.014
0.030	0.022	0.018	0.014	0.014	0.014	0.014	0.017	0.018
0.038	0.022	0.021	0.017	0.016	0.014	0.018	0.019	0.019
0.044	0.024	0.021	***0.017***	0.020	0.018	0.020	0.020	0.019
0.047	0.025	0.023	***0.020***	0.020	0.021	0.025	0.026	0.020
0.049	0.028	0.026	***0.020***	***0.021***	0.029	0.025	0.032	0.021
0.056	0.038	0.033	***0.028***	***0.022***	0.029	***0.027***	0.035	0.024
0.059	0.047	0.052	0.039	***0.028***	0.033	***0.030***	0.038	0.028
0.092	0.048	0.053	0.051	0.043	0.035	***0.035***	0.042	***0.029***
0.106	0.054	0.094	0.056	0.044	0.048	0.056	0.048	0.038
0.120	0.054	***0.130***	0.090	0.046	0.050	0.058	0.061	***0.038***
0.233	0.082	***0.136***	**0.119**	0.075	0.068	0.066	0.064	0.050
	0.101	0.136	**0.138**	***0.113***	***0.117***	0.105	0.112	0.051
	0.171	0.201	0.151	***0.143***	***0.127***	0.154	***0.165***	0.060
	0.202		0.189	0.148	0.152	***0.314***	***0.252***	0.113
				0.185	0.190			***0.184***
								0.215

Table 2 continued

A. Results of Whitworth Test

Results in ***Bold Italic*** do not support the Whitworth distribution at the 5% level

Year	**1997**	**1998**	**1999**	**2000**	**2001**
Number	24	27	25	23	20
Test Statistic	0.009	0.006	0.005	0.004	0.058
Ave *p*-value	0.057	0.076	0.133	0.205	***0.001***
SE	0.002	0.002	0.003	0.003	0.0003

B. Testing sizes in each year against the Whitworth expected sizes

Observations below: those in ***Bold Italic*** are outside the +/- 2SD range

1997	**1998**	**1999**	**2000**	**Obs 2001**
0.002	0.001	0.002	0.001	0.001
0.004	0.002	0.003	0.001	0.001
0.004	0.003	0.003	0.003	0.003
0.006	0.003	0.004	0.004	0.003
0.008	0.005	0.006	0.005	0.004
0.010	0.008	0.008	0.008	***0.004***
0.011	0.009	0.009	0.010	***0.008***
0.015	0.010	0.013	0.012	***0.010***
0.018	0.010	0.014	0.012	***0.010***
0.019	0.011	0.017	0.015	***0.012***
0.023	0.016	0.018	0.020	***0.014***
0.023	0.016	0.023	0.023	***0.018***
0.027	0.018	0.023	0.023	***0.024***
0.030	0.019	0.025	0.027	***0.024***
0.032	0.021	0.031	***0.028***	***0.026***
0.033	0.023	0.034	***0.031***	***0.026***
0.033	0.029	0.035	0.065	0.068
0.042	0.032	0.036	0.077	0.122
0.056	0.034	0.042	***0.097***	***0.300***
0.057	0.041	0.067	***0.119***	***0.320***
0.068	0.047	0.072	0.122	
0.089	0.057	0.075	0.132	
0.190	0.066	0.094	0.168	
0.200	0.070	0.131		
	0.097	0.215		
	0.167			
	0.185			

EE15c

It would be surprising to have all sizes, which represent individual distributions, lying within 2SD, and the data from 1977 to 2001 show a good fit with the Whitworth expectations. Until 1976 the supply of mining leases was not perfectly inelastic. The oil price shock of the 1970s and the rise in coal prices resulted in new firms entering the NSW coal industry, creating increasing competition for mining leases that ensured all of the available resources had been utilized by 1976. In 1976, as noted above, the NSW Government introduced strict controls on the issuing of new mining leases, from which time the supply of mining leases was perfectly inelastic.

Section IV Discussion

Despite our finding that the Gibrat assumption holds, we conclude that the size distribution is not lognormal. Using statistical tests, it is very difficult to distinguish between the lognormal (or other exponential distributions) and the Whitworth. The test becomes an extreme hypothesis test; see Savage 1954, and Ijiri and Simon 1977. Consequently, the choice of distribution should be based on the underlying theory. We argue here that the theory associated with a supply-constrained industry dictates Whitworth rather than lognormal. Moreover, since Ijiri and Simon (1977) showed that the arrival of new entrants will destroy the log-normality that results naturally from the Gibrat assumption, to achieve a lognormal equilibrium distribution requires that all the firms in the population must start their random walks at the same time. The continual entry of new firms into the NSW coal industry over the period of the study[14] ensures, therefore, that we cannot rely on the lognormal as a theoretical justification, even if the behaviour of firm sizes follows Gibrat's Law. Note that in an ecological application Williamson and Gaston (2005) pointed out that the lognormal predicts the existence of species abundance (sizes) that do not exist; and they further claimed, “it is now certain that the lognormal is unacceptable [to describe species abundance]” (Ibid p.11). Further, lognormal is continuous, while Whitworth is discrete and hence more appropriate for small numbers.

The (scarce) resource of mining tenements is allocated randomly amongst the competing firms, and whenever there is a change of industry structure — firms entering or exiting, or mergers between firms — there is an effective simultaneous division of the available mining leases. This re-division of the resource has the effect of restoring the distribution of firm sizes to the Whitworth distribution, which dominates any trend to the lognormal distribution.

The basis of the Whitworth model is that all of the necessary input resource available (in the domain considered) is allocated across homogeneous firms; that is, aggregate supply of the scarce input is fixed, or perfectly inelastic. When the input is not perfectly inelastic or there is slack (not all the available resource has been allocated) or the firms are heterogeneous, there may be departures from Whitworth. Heterogeneity could arise from organizational aspects (Gans and Quiggin, 2003), managerial and entrepreneurial ability (Lucas, 1978, Clavo and Wellisz, 1980), innovation (Kwasnicki, 1998), or decreased costs from learning by doing (Jovanovic, 1982). As Goddard et al. (2006) point out "Gibrat’s Law does not preclude the possibility that ex post, strong growth performance can be attributed to ‘systematic’ factors such as managerial talent, successful innovation, efficient organisational structure or favourable shifts in consumer demand". The lognormal

[14] From 1970 to 2001 55 new firms entered the industry.

or other distributions may better describe equilibrium firm sizes in heterogeneous circumstances.

In our model, the Whitworth stable state is reached when all the necessary industry input has been allocated — Steindl’s (1965) “stabilizing influence” being the perfect inelasticity of the vital input resource. The stabilizing (equilibrating) dynamic arises from a change in the essential input resource. This can occur when a firm offers itself or one of its mines for sale or when the total resource increases. In each of these cases there is a potential new magnitude of the resource to be divided amongst the competing firms. The result may be a new entrant to the industry or a change of size of an incumbent firm or both. With our assumption of homogeneous firms, the division is random and each division will be a stochastic element of the distribution of each Whitworth part. We expect that some of the observed sizes will fall at a distance from the expected values, but repeated divisions will converge to the Whitworth expectations. "At any moment in time a discrepancy (may) exist between the current actual size distribution of firms and the equilibrium distribution. Firms find themselves in a dynamic adjustment process toward such an equilibrium, and the evolution of the size distribution reflects the convergence towards this equilibrium" (Hashimi, 2000, p.509). Each time a change in the resource is presented to the market, a simultaneous division of the new resource magnitude produces the new equilibrium state, even if the actual adjustment to the new equilibrium takes time. Our model does not obviate the various approaches of others such as Sutton's (1998) sub-market model, and the technological and organizational model of Gans and Quiggin (2003), but it suggests that without their assumptions of differences in technology, sunk costs, and managerial ability, there will still be skewed equilibrium size distributions owing to the competition for input resources. As Basu (1999) suggested, "(because) Whitworth emerges naturally in unregulated systems, it could be used as a ‘norm’ from which to measure deviations". She pointed out that the Whitworth distribution "closely approximates the Lotka, Zipf and Bradford distributions which ... are essentially signatures of evolutionary systems"[15].

With a small number of firms, the discrete Whitworth distribution is appropriate, while for a large number the distinction between the discrete Whitworth and the continuous lognormal is less important (May, 1975). Cohen (1966) showed that independent random interactions on the Whitworth parts will tend to produce a lognormal distribution as the number of firms becomes large[16].

Section V. Conclusion

When a magnitude is randomly divided into a number of parts, or when a necessary resource is inelastic and randomly divided amongst a number of homogeneous participants, the equilibrium size distribution of the parts, or participants, will follow the Whitworth distribution of expected values. If there is a new entrant to the industry, one or more of the incumbents must necessarily lose some access to the

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[15] These distributions are applied to productivity patterns of scientists, word frequency in texts and distribution of articles in journals, respectively. Note that in an earlier paper (Basu 1992) she used the Whitworth distribution without naming it.

[16] Our analysis is consistent with that of Whittaker (1965), who distinguished between the interpretations of the relative abundance of species in nature in general and sets of interacting species in particular communities, and also with that of King (1964) and Magurran (1988), who claimed that the MacArthur (1957) model applies in narrowly defined communities of taxonomically related organisms.

input. If some resource remains unallocated after division (when the input is not fixed or is not perfectly inelastic in supply then a new entrant to the industry will not necessarily result in one of the incumbents losing some access to the input, and the equilibrium distribution may depart from the Whitworth distribution.

We suggest that the basic model of equilibrium size distribution is the Whitworth model, which applies when the population of firms is small and homogeneous and there is a principal resource to be allocated fully amongst the firms. As the number of firms increases, as heterogeneity develops, and as firms compete for multiple critical input resources, these numerous independent factors are compounded multiplicatively, which may naturally result in the lognormal or other equilibrium distribution, instead of the Whitworth.

Some might argue that firm growth is a purposeful, not a random, process. We do not disagree that owners and managers are generally trying to expand their production, revenues and profits. But the effect of this competition in the face of constrained resources is such that the resulting process appears to contain random elements, or luck, as it were. At any rate, the Whitworth distributions fit confirms the random elements of coal-mining firms' growth patterns.

An important implication of the Whitworth distribution is the presence of many small firms. While the number of firms in the industry may decrease, and concentration increase, there will always be a significant number of small firms if Whitworth holds. NSW coal industry firms are largely price takers, because of the competition from Queensland and international coal producers, and small NSW coal firms, as a group, become significant competitors with the large firms for the available markets[17].

Further research into other industries with small numbers of homogenous firms competing for an inelastic essential resource may produce the same Whitworth equilibrium distribution of firm sizes. Industries such as banking (Vennet 2001), telecommunications and the media, where entry is tightly controlled by Government regulation, can be examined in this light. In mature extractive industries, where the input resource is restricted by geological constraints, the firms should exhibit Whitworth size distributions, even without regulatory constraints.

Appendix I. Whitworth Distribution

The following is after (Whitworth, 1901). Let a body of magnitude S be divided into n parts such that:

> α is the size of the smallest part, $\alpha + \beta$ is the size of the next smallest part

and

> $\alpha + \beta + \gamma$ is the next and so on, with α, β, γ ..... being random quantities, subject only to the condition that the sum of all the parts equals S,
>
> $\alpha + (\alpha + \beta) + (\alpha + \beta + \gamma) \ldots\ldots = S$
>
> so that $n\alpha + (n-1)\beta + (n-2)\gamma \ldots\ldots\ldots = S$ equation (a).

Now if n random magnitudes $\alpha, \beta, \gamma \ldots$ are subject only to the condition that $a\alpha + b\beta + c\gamma \ldots = S$, where $a, b, c,$ .... are constants, then, as $a\alpha, b\beta, c\gamma$ are also random quantities, subject only to the condition that their sum = S, the expectations of each is the same, i.e. $E(a\alpha) = S/n$, $E(b\beta) = S/n$ etc then from equation (a)

[17] The existence of many small firms is consistent with Chuang (1999), who found a small optimal firm size for Taiwan manufacturing industries, especially if export-oriented.

i) $E[n\alpha] = S/n$ and $E[\alpha] = (S/n)(1/n)$,

ii) $E[(n-1)\beta] = S/n$ $E[\beta] = S/(n(n-1))$ and $E[\alpha + \beta] = (S/n)(1/n) + S/(n(n-1))$

$$E[\alpha + \beta] = S/n[1/n + 1/(n-1)]$$

iii) and $E[\alpha + \beta + \gamma] = S/n[1/n + 1/(n-1) + 1/(n-2)]$.

So if a magnitude S is divided into n parts, and these parts are arranged in order of magnitude beginning with the least, their respective expectations will be

$$g_1 = (S/n).(1/n)$$

$$g_1 = (S/n).[1/n + 1/(n-1)]$$

$$g_3 = (S/n).[1/n + 1/(n-1) + 1/(n-2)] \text{ and so on.}$$

As $g_r - g_{r-1} = \dfrac{1}{n(n-r)}$, where g_r is the size of the rth part and n is the number of parts, this shows that the differences between parts increase exponentially with rank, producing the highly skewed size distribution**.**

Bibliography

Aitchison, J., and J. A. C. Brown. (1957). *The lognormal distribution with special reference to its uses in economics.* Cambridge: Cambridge University Press.

Audretsch, D. B. (1995). Innovation, growth and survival. *International Journal of Industrial Organization* 13:441-457.

Audretsch, D. B., E. Santarelli, L. Klomp, and A. R. Thurik. (2004). Gibrat's law: Are the services different? *Review of Industrial Organization* 24:301-324.

Axtell, R. L. (2001). Zipf distribution of U.S. Firm sizes. *Science* 293:1818-1820.

Barton, D. E., and F. N. David. (1956). Some notes on ordered random intervals. *Journal of the Royal Statistical Society series B* 18:79-94.

Basu, A. (1992). Hierarchical Distributions and Bradford’s Law. *Journal of the American Society for Information Science.* 43 (7):494-500.

Basu, A. (1999). Inequity distributions as a signature of self organizing systems, 43d Annual Meeting of the International Society for the Systems Sciences, Pacific Grove, CA.

Callcott, T. G. (1966). Size distribution and single particle breakage. *BHP Technical Bulletin* 10 (26):26-30.

Chesher, A. (1979). Testing the law of proportional effect. *Journal of Industrial Economics* 27 (4):403-411.

Chiu, G. S. (1996). Bent-cable regression for assessing abruptness of change. Doctor of Philosophy Thesis, Statistics and Actuarial Science, University of British Columbia.

Chuang, Y.-C. (1999). Returns to scale, productive efficiency, and optimal firm size evidence from Taiwan's firm data. *Applied Economics* 31:1353-1364.

Clark, A. G. (1999). Invited editorial: The size distribution of homogeneous segments in the human genome. *American Journal of Human Genome* 65:1489-1492.

Clarke, R. (1979). On the lognormality of firm and plant size distributions: Some UK evidence. *Applied Economics* 11:415-433.

Clavo, G. A., and S. Wellisz. (1980). Technology, entrepreneurs, and firm size. *Quarterly Journal of Economics* 95 (4):663-677.

Cohen, J. E. (1966). *A model of simple competition*. Cambridge: Harvard University Press.

Curnow, R. N. (1973). A smooth population response curve based on an abrupt threshold and plateau model for individuals. *Biometrics* 29:1-10.

David, F. N., and D. E. Barton. (1962). *Combinatorial chance*. London: Griffin.

de Wit, G. (2005). Firm size distribution. An overview of steady-state distributions resulting from firm dynamics. *International Journal of Industrial Organization* 23:423-450.

Devarajan, S., and A. C. Fisher. (1981). Hotelling's "economics of exhaustible resources": Fifty years later. *Journal of Economic Literature* 19:65-73.

Epstein, B. (1947). The mathematical description of certain breakage mechanisms leading to the logarithmico-normal distribution. *Journal of the Franklin Institute* 244:471-433.

Gans, J. S., and J. Quiggin. (2003). A technological and organisational explanation for the size and distribution of firms. *Small Business Economics* 21:243-256.

Ghent, A. W., and B. P. Hanna. (1968). Applications of the "broken stick" formula to the prediction of random time intervals. *American Midland Naturalist* 79 (2):273-288.

Goddard, J., D. McMillan, and J. O. S. Wilson. (2006). Do firm sizes and profit rates converge? Evidence on Gibrat's law and the persistence of profits in the long run. *Applied Economics* 38:267-278.

Guthrie, C. G., and D. L. Moorhead. (2002). Density-dependent habitat selection: Evaluating isoleg theory with a Lotka-Volterra model. *Oikos* 97 (2):184-194.

Hardwick, P., and M. Adams. (2002). Firm size and growth in the United Kingdom life insurance industry. *Journal of Risk and Insurance* 69 (4):577-593.

Hart, P. E., and S. J. Prais. (1956). The analysis of business concentration: A statistical approach. *Journal of the Royal Statistical Society* 119 A:150-190.

Hashimi, F. (2000). An evolutionary model of the size distribution of firms. *Journal of Evolutionary Economics* 10:507-521.

Holst, L. (1980). On the Lengths of Pieces of Stick Broken at random. *Journal of Applied Probability* 17:623-634.

Hotelling, H. (1931). The economics of exhaustible resources. *Journal of Political Economy* 39 (2):137-175.

Ijiri, Y., and H. A. Simon. (1964). Business firm growth and size. *American Economic Review* 54 (2):77-89.

Ijiri, Y., and H. A. Simon. (1974). Interpretations of departures from Pareto curve firm size distributions. *Journal of Political Economy* 82 (2, Part 1,):315-331.

Ijiri, Y., and H. A. Simon. (1977). *Skew distributions and the sizes of business firms*. Edited by H. Theil. Vol. 24, *Studies in mathematical and managerial economics*. New York: North-Holland.

Itoh, Y., and S. Ueda. (2000). Observed Phase Changes and the Random Sequential Packing Model for Elections in Japan. *Physica D* 142:29-40.

Jovanovic, B. (1982). Selection and evolution of industry. *Econometrica* 50 (3):649-670.

King, C. E. (1964). Relative abundance of species and Macarthur's model. *Ecology* 45 (4):716-727.

Kwasnicki, W. (1998). Skewed distributions of firm sizes - an evolutionary perspective. *Structural Change and Economic Dynamics* 9 (1):135-158.

Lawrance, A. B., and R. E. Marks. (2004). Duration analysis in an extractive industry. *Australian Graduate School of Management Working paper* 2004-01 http://lowylib.agsm.edu.au/uhtbin/cgisirsi/SUaykv0YSt/125160017/523/253.

Lucas, R. E. (1978). On the size distribution of business firms. *Bell Journal of Economics*.

MacArthur, R. H. (1957). On the relative abundance of bird species. *Proceedings of the National Academy of Sciences USA* 43:293-295.

Magurran, A. E. (1988). *Ecological diversity and its measurement*. New Jersey: Princeton University Press.

Manning, A. B. (1952). Size distribution in a randomly fractured solid, and its application to coal. *Journal of the Institute of Fuel* 10 (22):22-39.

Mansfield, E. (1962). Entry, Gibrat's law, innovation, and the growth of firms. *American Economic Review* 52 (5):1023-1051.

May, R. M. (1975). Patterns of species abundance and diversity. In *Ecology and evolution of communities*, edited by M. L. Cody and J. M. Diamond. London: Belknap Press of Harvard University Press.

Mitzenmacher, M. (2003). A brief history of generative models for power law and lognormal distributions. *Internet Mathematics* 1 (2):226-251.

Moran, P. A. P. (1947). The Random Division of an Interval. *Supplement to the Journal of the Royal Statistical Society* 9.

Pielou, E. C. (1969). *An introduction to mathematical ecology*. New York: Wiley-Interscience.

Quandt, R. E. (1966). On the size distribution of firms. *American Economic Review* 56 (3):416-432.

Savage, L. J. (1954). *The foundations of statistics*. New York: John Wiley & Sons.

Simon, H. A., and C. P. Bonini. (1958). The size distribution of business firms. *American Economic Review* 68 (4):607-617.

Sittig, D. F. (1996). Identifying a core set of medical informatics serials: an analysis using the MEDLINE database. *Bulletin of the Medical Library Association* 84 (2).

Steindl, J. (1965). *Random processes and the growth of firms.* London: Griffin.

Sugihara, G. (1980). Minimal community structure: An explanation of species abundance patterns. *American Naturalist* 116 (6):770-787.

Sutton, J. (1997). Gibrat's legacy. *Journal of Economic Literature* 35:40-59.

Sutton, J. (1998). *Technology and market structure*. Cambridge: MIT Press.

Tokeshi, M. (1990). Niche apportionment or random assortment: Species abundance patterns revisited. *Journal of Animal Ecology* 59 (3):1129-1146.

Vennet, R. V. (2001). The law of proportionate effect and OECD bank sectors. *Applied Economics* 33:539-546.

Webb, D. J. (1974). The Statistics of Relative Abundance and Diversity. *Journal of Theoretical Biology* 43:277-291.

Whitworth, W. A. (1901). *Choice and Chance*. 5th ed. New York: Hafner.

Williamson, M., and K. Gaston. (2005). The Lognormal Distribution is not an Appropriate Null Hypothesis for the Species–Abundance Distribution. *Journal of Animal Ecology* 74 (3):409-422.

Whittaker, R. H. (1965). Dominance and Diversity in Land Plant Communities. *Science (New Series)* 147 (3655):250-260.

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