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Working Paper 00–008
April 2000.

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ISBN 1 86274 376 2

DURATION ANALYSIS IN THE AUSTRALIAN COAL INDUSTRY[1]

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March 29, 2000

Abstract

Previous duration studies have not identified all entry and exit dates, have used some aggregated data, and have lacked complete information on mergers and acquisitions. We determine the durations of Australian coal-producing companies over a forty-year period, having constructed a clean database describing the 85 companies. We fit a Weibull distribution to the Kaplan-Meier product limit estimator results and use maximum likelihood estimators as parameters. Unlike studies of manufacturing companies, we find, first, that survival decreases with age, second, no relation between size and survival, and, third, evidence of an increase in the characteristic life over the period.

JEL Classifications: C41, L11, L71

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I *Introduction*

The use of statistical analysis of life cycles has long been established in engineering and the bio-medical fields (see J.D. Kalbfleisch and R.L. Prentice (1980) for bibliographical treatment) and was introduced into economics in the 1970s in the analysis of strikes and unemployment (Tony Lancaster 1979) and, later, in analysis of the survival of firms (David B. Audretsch 1991; Audretsch and Talat Mahmood 1995; Timothy Dunne, Mark J. Roberts, and Larry Samuelson 1989; David S. Evans 1987a; Rajshree Agarwal, 1997), although much of the work is more related to growth and the testing of Gibrat's Law of proportional growth than to the duration of firms. "Survival of firms has ... been studied as a side issue to growth of firms" (Agarwal 1997)

Audretsch and Mahmood found that "One of the most striking stylized facts regarding the dynamics of industries, that has emerged from empirical studies, is that the survival rates of businesses are positively related both to establishment size and age" (Audretsch and Mahmood 1995, p.97). This is also evidenced by the work of Dunne, Roberts, and Samuelson (1989); and Evans (1987a). The work of Agarwal "confirms Jovanovic's hypothesis [Boyan Jovanovic 1982] that survival increases with size" (Agarwal 1997, p.580).

We analyze the duration of NSW coal companies from 1960 to 1999 and test the relation of duration to size and age. Australia is the world's largest exporter of coal, overtaking the USA in 1984 (IEA/OECD 1999) and coal is Australia's largest export earner (McLennan 1999) with New South Wales (NSW) and Queensland being the exporting regions . While Queensland has now exceeded NSW in exports (JCB 1999), NSW has a longer history of substantial coal mining and exports, and our analysis is centered on NSW. From 1960 to

1998 NSW coal production rose from 16 million tonnes to 108 million tonnes and exports rose from 1 million tonnes to 76 million tonnes (Tex Report various).

Because coal companies depend on coal reserves, which decline with extraction, we hypothesize survival rates to be negatively related to age (ceteris paribus). Because large companies extract coal at higher rates than small companies (our definition of size is the annual production or extraction rate), again we expect survival rates (cet. par.) to be negatively related to size. Some aspects which would void the ceteris paribus assumption, could be the introduction of new technology, which allows economic mining at greater depths, and the acquisition of additional reserves, either by new grants by the Government or by purchase from other operators. Moreover, our definition of the size of an operator is its total production, and not the production from individual mine sites. The relation between size of operator and reduction of total reserves for a large operator will not necessarily be greater than that of a small operator, which may be reducing its reserves from a small mine at the same or a greater rate.

While there have been a number of papers on the duration of manufacturing companies, (Dunne, Roberts, and Samuelson 1988; Dunne, Roberts, and Samuelson 1989; Audretsch 1991; Audretsch and Mahmood 1995) we have found none on mining companies and certainly no published analysis of duration in the Australian coal industry.

There has been considerable interest recently in the demise of Australian companies and their failure to pay the employees their full entitlements. A notable recent example was that of the failure of the Oakdale coal company, which caused the Australian Government to call on the industry to pay the workers' entitlements from the industry long-service leave fund (SMH 1999), and so an analysis of the life times of coal companies would seem timely.

We define the "death" of a company (an "operator"), or "finish" of its life, by the date at which the company no longer has a controlling interest in a coal operation. Despite the concern of events such as Oakdale, which went into receivership, this form of finish is not the norm. Frequently, the legal corporate entity continues, but with new owners. We therefore take the ultimate majority, or managing, shareholder as the "operator" and analyze its entry into ("start"), and departure from ("finish"), the industry, in a position of controlling some coal production. Where the company shares of an operator are broadly held, with no dominant shareholder, we consider the company itself to be the operator. We also trace ownership to the ultimate controlling owner; we do necessarily not stop at the first legal entity which is a direct owner of the coal mine. A number of companies manage their mines under subsidiary companies and, failure to identify the ultimate owner, would give a completely misleading picture of the industry. We are not concerned with individual mines and, provided an operator maintains a controlling interest in at least one coal mine, its status does not alter in this duration analysis if one or more of its mines close, or are otherwise disposed of.

II *Empirical data*

In our analysis we use "single spell" or "survival" stock sampling data from the Joint Coal Board (JCB) records and other sources These data provide a list of operating companies, their controlling shareholder and the dates at which this shareholder gained or lost control of its coal operations. The method of construction of the data base, and its sources, is given in Appendix 5[2].

We define "start" as the first occasion that an operator produced coal, or the date at which the operator's first mine opened, if it achieved at least 100,000 tonnes of production in the first twelve months of operation; otherwise we use the start of the financial year (July 1 to June 30) in which 100,000 tonnes production was first achieved[3]. For "finish" date, we use the date of sale of the last coal mine owned by the operator or the date of change of control of the coal mines, or of the operator itself (in some cases this was the date of start of liquidation proceedings). Where only the month of start or finish is known, we use the first of the month. Where more than one operator has the same duration in days, we change the start date by one day—we consider this appropriate as the "correct" start date is an uncertain time and this change makes the statistical analysis easier[4]. Many analyses by other authors have been hampered by the difficulty of identifying entry and exit dates (Audretsch and Mahmood 1995), by aggregated data which does not reveal individual firms (Audretsch 1991) and by lack of data on mergers and acquisitions (Evans 1987b), but our data reveal the direct majority ownership and identify entry and exit with reasonable accuracy (allowing for the

[2] We would also like to thank Peter Murray, John Smith and Keith Ross for their help in identifying some owners.

[3] While the figure of 100,000 tonnes is arbitrary, we consider it is appropriate to eliminate the mines with very small production. In some cases, mines produce "trial samples" to send to customers and may not open for commercial production for a considerable period of time. The sample may be taken by temporary equipment, well before the major equipment is even ordered, so that the acceptability of the coal can be tested. Some very small mines are operated by family companies, with the owners and directors actually extracting the coal, and we feel that such mines do not fit into the general category of NSW coal mines, and their inclusion in the analysis could give misleading results.

[4] We recognize that the start and finish dates cannot be determined precisely as there is a question as to whether the "correct" start date is the date at which a company decides to enter the industry, the date at which a mining lease is granted, the date of commitment of development expenditure, the date of

issue of what is the "correct" date to use). We clearly recognize mergers and acquisitions and identify the controlling operator.

III *Censoring*

For some of the data we do not know the start date, because it was prior to 1960, and for some we do not know the finish date, because it was after 1999, (and for some we do not know either), so our data is said to be censored because "the value of the random variable under investigation is unobserved for some of the items in the sample" (Samuel Kotz and Norman L. Johnson 1982, p.389). Of the 85 operators in the period 1960 to 1999, as shown in table 1, forty operators did not start and finish within the time period and we call them "withdrawals". The data are "type I" censored (John P. Klein and Mervin M. Moeschberger 1997, p.60), as we only know that the true durations times for the withdrawals are greater than the observed values. The censoring depends on the dates of the start and end of the study, the date of entry (start date) and the date of exit (finish date) of the operator. A duration is Left censored if the start date is prior to the start of the study and Right censored if the finish date is after the end of the study[5]. In some cases there is duel Left and Right censoring.

[Table 1 here]

We delete the operators BHP and the State Government from the analysis because their operations are vertically integrated, the coal mines supplying their steel making and electricity

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opening of the mine at initial production or the date at which full production is achieved. We do not have all these dates.

[5] There are some unclear definitions of Left and Right censoring which could indicate that all our data is Right censored, but the *Encyclopedia of Statistical Sciences* is very precise. Right censoring is "Censoring from above, by omission of the last *r* order statistics" and Left censoring is "Censoring from below, by omission of the first *r* order statistics" (Kotz and Johnson 1982, p.396). This makes it clear that durations which have start dates prior to the start of the study are Left censored. as the "first *r* order statistics" (where *r* is the number of years from the [unknown] start date to the date of the start of the study [July 1, 1960]) have been omitted. See also Kiefer (1988 p.647) for an example of left censoring.

generation respectively, and the survival of the coal business is not as conditional on the state of the coal market as it is for coal companies selling their product on the open market[6]. We also delete Pasminco, HIH, Elders and North Broken Hill as their ownership of coal mines came about by the acquisition of holding companies for assets other than the coal assets. Each of these named companies disposed of the coal assets within a relatively short time span and so their durations as coal operators would be misleading.

The final data for analysis include 39 completed durations, which both start and finish in the 39 year study period, and 40 censored durations. Table 1 shows all the operators which were in existence in July 1 1960 through to July 1 1999 and the durations are shown graphically in chart 1 (all 85 durations) and chart 2 (completed durations only).

[Chart 1 here]

[Chart 2 here]

IV *Kaplan-Meier product-limit estimator*

Audretsch (1991), Audretsch and Mahmood (1995), Dunne, Roberts, and Samuelson (1988), Dunne, Roberts, and Samuelson (1989) and Evans (1987a) do not offer a parametric distribution model for durations in their analyses. Nicholas M. Kiefer (1988) says the exponential distribution is widely used as a model for duration analysis because it is "simple to work with and to interpret, and is often an adequate model for durations that do not exhibit much variation"[7]. He also suggests the use of Weibull and log-logistic distributions . The lognormal distribution is applicable for describing the growth of firms that follow Gibrat's

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[6] This position changed for BHP in 1982 when it purchased Utah and became a major exporter of coking coal. Similarly, the State Government mines started exporting thermal coal in the 1970s. To consider BHP and the State Government as "starting" as a coal operators when they became exporters (while their own use of their coal is still a major proportion of their output) seems to us a device of no great merit in this analysis, and so we deleted BHP and the State Government from the data.

Law of Proportional Effect (S.J. Prais 1976) but, apart from that, we do not find any demonstrated theoretical basis for the use of a particular distribution for the lives of business firms.

In the absence of a direct theoretical basis for the durations distribution, we used the nonparametric method of the Kaplan-Meier product-limit estimator (see Appendix 1) to find the survivor function, which is defined as:

$$\hat{S}(t) = \prod_{j:t_j > t} \frac{n_j - d_j}{n_j}$$

(E.L. Kaplan and P. Meier 1958) where n_j is the number operators "alive" (with no censoring withdrawals), and therefore, at risk, at time t_j, and d_j is the number of completed durations at each value of t_j . The survivor function is the probability of surviving until (at least) the determined time and is the upper tail area of the cumulative distribution function. We also find the hazard function, which is the "instantaneous rate of failure at time t given that the individual survives up till t. In particular, $h(t)\Delta t$ is the approximate probability of death in $(t, t + \Delta t)$, given survival up till t " (Jerald F. Lawless 1982, p9) and mathematically is the density function divided by the survivor function. While the hazard is a measure of the probability of failure, it is not the probability itself, and the hazard can exceed unity. The cumulative or integrated hazard function,

$$IH = \int_{t=0}^{t=T} h dt,$$

[7] Kiefer is mainly concerned with the duration of unemployment

becomes –log(survivor function) on integration, and "plots of the integrated hazard are typically smoother and therefore easier to interpret than plots of the hazard directly" (Kiefer 1988, p.659). The calculations of the hazard and integrated hazard are given in table 2.

[Table 2 here]

Chart 3 plots the integrated hazard function and chart 4 plots the hazard function. We can see the integrated hazard is close to linear while the hazard function is convex, and rising, but with such small increments that the integrated hazard appears linear (on the same scale as the integrated hazard the hazard would be approximately constant). Chart 5 shows the Kaplan-Meier survivor function, which is a step function. As stated earlier, where we have equal durations, we move the start time slightly to eliminate ties and make the analysis easier[8].

[Chart 3 here]

[Chart 4 here]

[Chart 5 here]

V *Weibull distribution*

We consider it more appropriate to use a continuous time model, instead of the discrete model of the Kaplan-Meier product-limit estimator, as the consideration of the decision to enter or exit will not be made at time intervals such as annual meetings but continuously during the normal course of business. "Inference about an underlying stochastic process that is based on interval or point sampled data may be very misleading if it is falsely assumed that the process being investigated operates in discrete time" (James J. Heckman and Burton Singer 1984, p.63) and Kiefer (1988, p.655) makes the point that "continuous-time models seem appropriate for economic settings because there is typically no natural period in which economic decisions are taken".

[8] This is a standard convention (Kalbfleisch and Prentice 1980, p.12)

In earlier work on size distributions (unpublished), we initially found that a Weibull distribution[9] model fitted quite well, and we subsequently moved to a division of parts distribution (William Allen Whitworth 1961, p.207 Proposition LV) which is close to a Weibull in shape (for shape parameters between 0 and 1). We posit there is a relation between size distribution and duration (which we will explore in later work) and so we try the fit of a Weibull model[10] for our duration analysis. The integrated hazard is approximately linear (chart 3), which is typical of the exponential distribution, but the hazard was monotonically increasing (chart 4) while the hazard of the exponential is constant, so the selection of a Weibull distribution, which is a generalization of the exponential, is not unreasonable. Other distributions are available, such as the lognormal which "like the Weibull and the exponential, has been widely used as a life time distribution model" (S.A. Shaban 1988, p.267) and (Raymond J. Lawrence 1988), but our earlier work on size distributions of firms (unpublished) showed the Weibull fitted better than the lognormal. In the Weibull distribution, the log transformation converts the distribution to an extreme value distribution with no shape parameter (Samuel S. Shapiro 1990, p.6.15). The plot of the log(completed duration time) against the loglog(1/survivor function) results in a close approximation to a straight line (see chart 6a) and gives us confidence in using the Weibull model, as suggested by (Kalbfleisch and Prentice 1980, p.24). The same plot of all the data (censored and uncensored) gives an even closer fit to a straight line as shown in chart 6b. Even though some of the data are censored, we would expect these censored durations to conform to the same distribution as would their true durations (which are unknown, but are

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[9] For details of the Weibull distribution see Appendix 2. Lancaster (1979) assumes Weibull for the distribution of unemployment durations.

[10] We also test for the Whitworth distribution but find it is not as good a fit as the Weibull

equal to, or greater than, the censored durations), as they are a random sample of the population of durations. So the inclusion of the additional information of the censored data gives a better indication of the distribution than only the completed durations. The Weibull distribution (Waloddi Weibull 1951) is widely used, with many applications in all types of duration analysis, see (Johnson 1964)and (Robert B. Abernethy 1993) for engineering applications and (Klein and Moeschberger 1997) for applications in the bio-medical field and "is by far the most frequently used parametric model" (Lancaster 1985) although Andrew C. Gorsky warns against the "Weibull euphoria" (Gorsky 1968). Coincidentally, for our work, the distribution was first described by P. Rosin and E. Rammler (1933) for size distributions for crushed coal. Although Weibull indicates "some familiarity with the work in Germany on size distribution, he refers to neither Rosin or Rammler" (Ruth Callcott 1987, p.64). A comparison of the Weibull distribution and the Rosin-Rammler distribution is given by Richter (Callcott 1987, p.64)[11]. J.G. Bennett (1936) gives a theoretical justification for the distribution for crushed coal by positing, inter alia, a random distribution of inner stresses in the coal lumps, and others have developed theoretical justifications for some applications (for example, see J.H. Gittus (1967)), but we have no theoretical justification for the use of the Weibull distribution in our analysis. As Weibull himself says, "The objection has been stated that this distribution function has no theoretical basis. But in so far as the author understands, there are—with very few exceptions—the same objections against all other distribution functions applied to real populations from natural or biological fields, at least in so far as the theoretical basis has anything to do with the population in question" (Weibull 1951, p.293).

[Chart 6 here]

The form of Weibull survivor function is:

[11] Callcott quotes from Richter (1976) *Weibull Verteilung in doppelt logarithmischen Kornungsntez nach Rosin/Rammler/Bennett*, Neue Bergbautechnik, Dresden

$$S(t) = 1 - F(t) = \exp -\left(\frac{t}{\eta}\right)^{\beta},$$

where *t* is the observed duration, η is a scale parameter (called the characteristic life) and β is the shape parameter. It can be seen that there is a linear relation between log(*t*) and loglog[1/S(*t*)].

We regress[12] the log(completed duration times) against the loglog(reciprocal of the survivor function) to find the Weibull parameters and the plot of the resulting log transformation gives, of course, the trend line in chart 6. From the regression equation, log(*t*) = *a* + *b*loglog[1/S(*t*)] + *c*, we get $a = 1/\eta$ and $b = \exp(\beta)$ with β the shape parameter and η the scale parameter of the Weibull distribution. Weibull used the chi-squared test for goodness of fit (Weibull 1951, p.294) but we find inference difficult with the ordered nature of the data and we use a modified W, test as described by Shapiro (1990, p6.16) (see Appendix 3), and find a good fit of our data with the Weibull distribution.

The product-limit estimator produces a step survivor function, as shown in chart 7a. The survivor function produced from the estimated Weibull parameters, found by the above regression, is a good visual fit to the product-limit survivor function when superimposed on it in chart 7a.

[Chart 7a here]

[12] We use the Excel 97 regression function. B.D McCullough and Berry Wilson (1999) warn against using Excel 97 for statistical analysis so we verified our regression results using Minitab, getting no variations.

The product-limit estimator gives a maximum likelihood estimator for the survivor function and is weighted for withdrawals, as the "number at risk" in the Kaplan-Meier formula counts the number of completed and incomplete durations. An assumption of some smoothness in the model is not unreasonable (David R. Cox 1972, p.190) but, in smoothing the data by imposing Weibull distribution, the likelihood estimate changes (Cox 1972, p.189). This leads us to look at the maximum likelihood estimate later.

We consider the "Life Tables" method (Lawless 1982) (see Appendix 1) loses information, due to our small number of events and the need to group data in this method, although this method produces a very similar result to that of the Kaplan-Meier method.

VI *Median ranks*

To test the applicability of another method, used in engineering applications, we calculate the median ranks of the completed durations using adjusted mean ranks to compensate for censoring, and then calculate the median value (Leonard G. Johnson 1951).

When there are data with withdrawals, the true rank of the completed durations must be different from their apparent ranks, and Johnson (1964, p.39) (see Appendix 4) gives a method of determining a corrected mean rank. The mean ranks resulting from Johnson's method are not all whole numbers and the calculation of the median ranks from tables is tedious, hence we use Benard's approximation (Abernethy 1993) (see Appendix 4) to calculate the median value.

This method of median ranks gives the ranks a weighting determined by the withdrawals (the data which are censored). If withdrawals are not considered, as one would expect, the survivor function values are too small. This is shown graphically in chart 8 with the survivor functions calculated using the completed durations only, without any weighting for the withdrawals, and by assuming all the durations (i.e. including the actual censored data) are completed (i.e. not censored) , both of which produce lower survivor functions than the survivor of the Weibull from the maximum likelihood estimates.

[Chart 8 here]

Median ranks are preferred to mean ranks because "when mean ranks are used the slope can be too low with high probability of lower extreme values falling to the left" (Johnson 1951). With median ranks the danger of underestimating the slope is eliminated because a point is just as likely to fall to the left as to the right of the maximum likelihood line. For median ranks, half the time the rank is too high and half the time the rank is too low, and so the errors cancel out. (Johnson 1951, p.1 & 5). The resulting median "ranks" are not strictly ranks, as they are not whole numbers.

We again determine the Weibull parameters for the median values by regression and the resultant survivor is shown in chart 8, which is lower than that of the Kaplan-Meier approach. The median method requires that only the data up to the last completed duration is included (Johnson 1964). As there are 7 censored operators which exhibit durations greater than the largest completed duration, their exclusion has the effect of lowering the survivor curve and could account for some of the difference from the Kaplan-Meier survivor curve (which is higher). In consequence, we reject the median method.

VII *Maximum likelihood estimators*

The Kaplan-Meier product limit estimator gives an estimate of the maximum likelihood, so we now mathematically determine the maximum likelihood estimates (MLE) of the Weibull distribution resulting from the Kaplan-Meier survivor function. We find the maximum likelihood estimates of β and η by the formulae (Abernethy 1993):

$$\frac{\sum_{i-1}^{N} t_i^{\beta} \ln t_i}{\sum_{i-1}^{N} t_i^{\beta}} - \frac{1}{r}\sum_{i=1}^{r} \ln t_i - \frac{1}{\beta} = 0 \ ,$$

and

$$\hat{\eta} = \left(\frac{\sum_{i=1}^{N} t_i^{\beta}}{r} \right)^{\frac{i}{\beta}} \ ,$$

where N is the total number of durations (t_i) and r is the number of withdrawals. We determine the maximizing value for β using the "Solver" program in Excel (we spot-check this solution by manual iterative solving of the equation).

We find that the likelihood estimate for the Weibull using the MLE of parameters is greater than the likelihood estimate for the Weibull derived from the Kaplan-Meier data (although only marginally so), and also is greater than the likelihood estimate for the Weibull from the median values. These values are shown in table 3. The survivor function of the Weibull from the maximum likelihood estimates of the parameters is greater than that of the Weibull from the Kaplan-Meier estimates and also looks a good fit by using the top of the step rather than the bottom of the step in the extremities of the distribution (see chart 7b).

[Chart 7b here]

[Table 5 here]

"Much of the maximum likelihood theory deals with the large sample (asymptotic) properties of MLE" (Kotz and Johnson 1985, p.342), but the "large sample properties of the maximum likelihood estimators still make them quite attractive for samples of 50 or more" (Alan J. Gross and Virginia A. Clark 1975, p.141). With our data set of 85 operators, we feel comfortable in using the maximum likelihood estimator of the Weibull parameters and do not expect to get the bias which would be expected from small samples.

VIII *Survivor functions*

We review all the survivor functions as shown in chart 6[13]. Because of the closeness of the survivor functions of the Weibull distributions resulting from the Kaplan-Meier and the MLE methods, we consider our rejection of the median value method is further justified. We consider the MLE method the most acceptable because of its greater likelihood, the appearance of a good fit with the Kaplan-Meier step function, and a higher survivor function than the Kaplan Meier survivor function (which indicates greater weighting of the censored data).

[13] In chart 6 the following is an explanation of the derivation of the curves

(i) "Kaplan-Meier" is the Weibull produced from parameters from regression of the Kaplan-Meier survivor function

(ii) "Medians censored" is the Weibull from the medians using with weighting for withdrawals

(iii) "MLE" is the Weibull from the maximum likelihood estimate parameters

(iv) "Assume all durations completed MLE" is the Weibull from the MLE of parameters assuming all the observations are completed durations

(v) "Only completed durations MLE" is the Weibull from the MLE of parameters ignoring the withdrawals

(vi) "No censoring (Medians)" is the Weibull from median values with no weighting for withdrawals

IX *Moments and hazard*

Using the model of the Weibull distribution from the maximum likelihood estimates, we see that the mean life is 15.8 years, which has a survivor probability of 0.41, and the median life is 13.8 years. The characteristic life (the scale parameter) is 17.6 years, which in all cases has a survivor rate of 0.36 from substitution of $t = \eta$ in the Weibull survivor rate formula:

$$S(t) = \exp -\left(t/\eta \right)^{\beta}$$

where $t = \eta$, then $S = \exp(-1) = 1/e = 0.368$.

The hazard rate for Weibull, which is approximately the conditional probability of failing in the next period (Lawless 1982, p.9), is given by:

$$H(t) = \frac{\beta t^{\beta-1}}{\eta^{\beta}}.$$

When β is < 1, the hazard functions of all Weibull distributions are monotonically declining. and when $\beta > 1$, they are monotonically increasing (for $\beta = 1$ the distribution is exponential with constant hazard). With the β for our analysis at 1.5, we get increasing hazard with duration time. Duration dependence is said to exist when $dH(t)/dt \neq 0$ (Heckman and Singer 1984, p.66) and in our case we have negative duration dependence. This is consistent our initial hypothesis of increasing hazard and increasing failures, given the depleting resource in coal companies, and compares with the results of (Dunne, Roberts, and Samuelson 1989); (Audretsch 1991); (Bronwyn H. Hall 1987); (Evans 1987a; Evans 1987b), who found that failure rates of USA manufacturing plants decline with age. Audretsch states that, "One of the most striking stylized facts regarding the dynamics of industries that has emerged from empirical studies is that the survival rates of businesses are both positively related to establishment size and age" (Audretsch and Mahmood 1995, p.97). We comment later on duration and size.

X *Change over time*

To examine the hypothesis that there had been a change over time in the durations of the operators, we divide the longitudinal data into two sections, with the division at 30 June 1979. To ensure there is no interdependence between the two sections, the "Early" section includes only those finished before July 1, 1979; and those started before July 1, 1960 and finished by July 30, 1999. The "Late" section includes those started after July 30, 1960 and not finished by June 30, 1979; those started before 1/7/60 and not finished by 30/6/99; and those started after June 30, 1979. This division is shown graphically in chart 9.

[Chart 9 here]

There are only 10 completed durations in the Early section and 29 in the Late section. On an unadjusted ranking (i.e. with no allowance for withdrawals), the characteristic life of the Early section is approximately 5 years and by the adjusted rank (by the determination of the median values, as before) is 16 years (which is approximately the value of the largest of the left section data) and clearly show that that if the withdrawals are not allowed, for a highly misleading interpretation results.

We conduct a likelihood ratio test in which the test statistic is $-2\text{Ln}\lambda$, where λ is (likelihood of the Full data)/[(likelihood of Early data)(likelihood of Late data)] and the test statistic has a χ-square distribution. The resulting test statistic is 53, so there is no evidence for the hypothesis that the Early and Late sections have the same distribution.

To determine the trend of the length of the completed durations over time, we calculate the maximum likelihood estimates of the Weibull parameters for periods commencing at 5-year intervals, diminishing by 5 years each time viz. 1st July 1965, 1st July 1970, etc., all up to June 30, 1999. This requires deleting data which falls outside the periods e.g. for the period commencing 1st July 1970 we delete all operators which fail prior to that date. The result shows increasing values of the characteristic life, as seen in table 4. We do not include the final period of 1995 to 1999, as it only contains one completed duration and gives a very large value of beta, as does the 1990 to 1999 period, with only 6 completed durations but 33 withdrawals.

Because of the interdependence of the test periods, we cannot readily test to see if the increase in characteristic life is statistically significant, but can say that there is no evidence of a reduction in life. Our analysis of the Early section and the Late section of our data shows no evidence that they are the same distribution, so we consider it is reasonable to assume that the (probable) change in distribution includes an increase in the characteristic life over time.

[Table 4 here]

XI *Relation between size and duration*

To test the hypothesis that there is a correlation between size of the operator and duration time, we run a correlation between the comparative size of operators, measured by their production as a proportion of the total NSW production in the year before the "death". We get a correlation of 0.29. This compares with the results of Dunne, Roberts, and Samuelson (1989) who found that failure rates of USA manufacturing plants decreased with plant size (although our study is based on total operator, or owner, size not plant size), and with Evans (1987a, p.568) who found that there is a positive relationship between firm size and survival for 81% of the manufacturing firms studied (which were drawn from the USA Small Business Data Base) and with Audretsch and Mahmood (1995, p.97) as noted earlier. Our hypothesis that a larger operator will have higher hazard than a smaller operator, because it extracts the resource faster, is not evidenced, and we surmise that this is because extraction at individual mines is generally at a maximum, consistent with the size of the deposit being mined, and so a "small" operator may deplete a single mine as fast as a "large" operator.

We also find there is no correlation between duration and whether the operator had a single mine or multiple mines. We do this by using a dummy variable for multiple mine sites and regressing this against duration times[14].

XII *Conclusions*

Our analysis of the duration of operators in the NSW coal industry, using the Kaplan-Meier product-limit estimator of the survivor function, shows that it is reasonable to assume that the data conforms to a Weibull distribution. We find the hazard and integrated hazard functions useful in determining the model to be used.

The Weibull distribution from the median ranks produces a survivor function lower than that of the Weibull derived from the Kaplan-Meier survivor and we conclude that this results, at least partly, from the exclusion of the last 7 censored data in calculating the mean ranks. We consider the maximum likelihood method to be the best approach, and the survivor function of the Weibull resulting from the maximum likelihood estimates of the parameters gives the highest survivor function.

The Weibull model produces a monotonic increasing hazard function, which is consistent with our general assumption of depleting resources for coal companies and is contrary to results of researchers for other industries. We posit that similar results would be obtained for other non-renewable resource industries. We find no corroboration for our hypothesis that the depleting resources of coal mines would lead to more rapid demise of coal companies when the operator's total production (extraction) rate is high (and the company large) than for small companies which, by definition, have lower production.

The analysis of the full data from 1960 to 1999 gives a mean life of 17.6 years, however our analysis reveals no evidence to indicate that the distributions of the two periods, viz. 1960 to 1975 and 1975 to 1999, are the same. The calculated characteristic life of the Early period is 27.8 years and of the Late period 21.4 years but these results reflect the manner of selecting the data for each to avoid dependence, and a different selection will change the relative characteristic lives. Analysis of the full data, progressively shortened by 5 year periods, shows no evidence of reducing life and some indication of increasing life.

We did not find any correlation between duration and size, and this result is inconsistent with findings of others for manufacturing industries, where there is a positive correlation between size and survival. We consider the lack of correlation between size and duration is due to the depleting resource nature of the coal industry and should be typical of extractive industries.

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[14]We do this to test the statement of (Dunne, Roberts, and Samuelson 1989, p.446) that "plant growth failure is influenced by ownership structure of the firms" with expected growth rate increasing with multiple enterprise firms.

APPENDIX 1 *THE KAPLAN-MEIER PRODUCT-LIMIT ESTIMATOR & LIFE TABLES*

KAPLAN-MEIER PRODUCT-LIMIT ESTIMATOR

The completed durations in years (i.e. censored, with uncompleted durations not included—called "withdrawals") are ordered by increasing size. Where there are any ties (duplicated times) one duration is increased slightly. For each duration, the number of total durations (including withdrawals) "alive" at that time (the number "at risk"(Kiefer, 1988) is determined. The hazard function is the reciprocal of the number at risk and the survivor function is the product-limit survivor function

$$\hat{S}(t) = \prod_j \left[(n_j - d_j) / (n_j) \right]$$

where n_j is the number at risk and d_j is the number of ties (for our analysis 1 in each case)

See table 2 for the analysis.

LIFE TABLES

(The following is adapted from Lawless (1982 p.54))

The life table is essentially an extension of the relative frequency table to censored data.

Divide time scale into k+1 intervals so $I_j = (a_{j-i}, a_j)$, $j = 1, \ldots k+1$ I_j is the jth interval

$a_0 = 0$ ($t_0 = 0$) $a_k = T$ ($t_k = T$) and $a_{k+1} = \infty$ where T is the upper limit of the observation

d_j is the number of lifetimes that lie in the interval I_j i.e. the frequency of the jth interval (cannot use frequency plot when there are censored data).

The data are grouped so that it is only known in which interval particular individuals die or are censored, and not the exact lifetimes and censoring times.

N_j the number of individuals at risk (alive and not censored) at time a_{j-1}

D_j = number of deaths in (i.e. number of lifetimes observed to fall into) Ij = [a_{j-i}, a_j).

W_j = number of "withdrawals" in (i.e. no. of censoring times observed to fall into) $I_j = (a_{j-i}, a_j)$.

The no. individuals know to be alive at the start of Ij is Nj and thus N_1 = n

$N_j = n_{j-1} - D_{j-1} - W_{j-1}$ $j = 2,..., k+1$

Let the distribution of lifetimes have survivor function $S(t)$ and define

$P_j = S(a_j)$ (survivor function) = probability an individual survives beyond Ij-1 with P_0 defined to be unity $P_j = p_1 p_2 ... p_j$

p_j = conditional probability that an individual survives beyond I_j having survived beyond $I_{j-1} = P_j/P_{j-1}$

q_j = conditional probability that an individual dies in Ij having survived beyond I_{j-1}

Note $P_{k+1} = 0$ and $q_{k+1} = 1$

Standard life table" estimate of q_j

$$\hat{q}_j = \frac{D_j}{N_j - W_j/2} = \frac{D_j}{N'_j}$$

this assumes $Nj > 0$, when $Nj = 0$ define

$$\hat{q}_j = 1$$

$N'_j = N_j - ½ W_j$ can be thought of as an effective number of individuals at risk for the interval I_j; this supposes that, in a sense, a withdrawal individual is at risk for half the interval (this adjustment is arbitrary, but sensible in many situations)

APPENDIX 2 *THE WEIBULL DISTRIBUTION*

The Weibull distribution function is

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

where

t is the duration time,

β is the slope or shape parameter, and

η is the scale parameter or "characteristic life" and is the length of 63.2% of the durations.

When β = 1 the distribution is exponential and η is the mean.

The probability density function is

$$f(t) - \left(\frac{\beta t^{\beta - 1}}{\eta^{\beta}}\right) \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right].$$

The survivor function is

$$S(t) = 1 - F(t) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right].$$

The hazard function is

$$H(t) = \frac{f(t)}{1 - F(t)} = \frac{\beta t^{\beta - 1}}{\eta^{\beta}}.$$

The hazard is a measure of the probability of failure in the next period, not an actual probability and can be greater than unity. Chart 10 shows the approximation calculation of the hazard at H_{12} as a rate as against the hazard function.

The integrated (cumulative) hazard function is

$$IH(t) = \int_0^{\infty} \frac{\beta t^{\beta-1}}{\eta^{\beta}} dt = \frac{\beta}{\eta^{\beta}} \int_0^{\infty} t^{\beta-1} dt = \left[\frac{\beta t^{\beta}}{\eta^{\beta}\beta}\right]_0^{\infty} = \left[\left(\frac{t}{\eta}\right)^{\beta}\right]_0^{\infty}$$

$$IH(t_J) = \left(\frac{t_j}{\eta}\right)^{\beta}.$$

The likelihood function (with no allowance for withdrawals) is (Abernethy, 1993 p.C-2)

$$L = \prod_{i=1}^{n} \left(\frac{\beta}{\eta}\right)\left(\frac{t_i}{\eta}\right)^{\beta - 1} e^{-\left(t_i/\eta\right)^{\beta}}.$$

The Maximum Likelihood Estimations (MLE) are the expression of the probabilities of β and η, given the observed data (Lawless 1982)

When a sample is censored, the Maximum Likelihood Function estimate (MLE) of β satisfies:

$$\frac{\sum_{i=1}^{N} t_i^{\beta} \ln t_i}{\sum_{i=1}^{N} t_i^{\beta}} - \frac{1}{r}\sum_{i=1}^{r} \ln t_i - \frac{1}{\beta} = 0$$

given duration times *t*, the MLE of β is found by iterative procedures, where
r = number of failures *k* = number of withdrawals
N = *r* + *k* (Abernethy, 1993).
Units censored at times *T* are assigned the values $x_{r+1} = T_i$. The second term in the equation sums the log of the failure times only.

The MLE of η is

$$\hat{\eta} = \left(\frac{\sum_{i=1}^{N} t_i^{\beta}}{r} \right)^{\frac{1}{\beta}}$$

(Abernethy, 1993).

The mean of the Weibull distribution is given by

$$\eta\, \Gamma\left[{(\beta+1)}/{\beta} \right],$$

where Γ is the gamma function .

The median is given by

$$\eta\, (\ln 2)^{1/\beta} .$$

The variance is given by

$$\eta^2 \left(\Gamma\left[\left({(\beta + 2)}/{\eta} \right) \right] - \left\{ \Gamma\left[{(\eta + 1)}/{\eta} \right] \right\}^2 \right)$$

(Merran Evans, Nicholas Hastings, and Brian Peacock 1993)

Chart 10 depicts the various aspects of the Weibull distribution for values of β = 3 and η = 10. For values of β less than 2 the density does not have the Gaussian appearance.

[Chart 10 here]

APPENDIX 3 *THE GOODNESS OF FIT TEST FOR THE WEIBULL DISTRIBUTION* (Shapiro, 1990)

The null hypothesis that H_0: (The distribution is Weibull), is rejected if the computed value of the test statistic *W* is either higher than the upper-tail critical value or lower than the lower-tail critical value.

The test statistic is *W* where:

$b = (0.6079L_2 - 0.2570L_1)/n$, with

$$L_1 = \sum_{i=1}^{n} w_i t_i$$ where n is the number of completed durations, t_i are the completed durations

$$L_2 = \sum_{i=1}^{n} w_{n+i} t_i$$

$$w_i = \ln\left(\frac{n+1}{n+1=i}\right), \quad i = 1,2,....., (n-1)$$

$$w_n = n - \sum_{i=1}^{n-1} w_i$$

$$w_{n+1} = w_i(1+\ln w_i) - 1, \quad i = 1,2,.........., (n-1)$$

$$w_{2n} = 0.4228n - \sum_{i=1}^{n-1} w_{n+1}$$

and the test statistic is

$$W = \frac{nb^2}{S^2}$$

$S^2 = \Sigma x_i^2 - 1/n(x_i)^2$

(Shapiro, 1990, p.6.16)gives a table of percentiles for the Weibull distribution test statistic *W*

The results of our test are given in table 5 which show there is no reason to reject the null hypothesis.

[Table 5 here]

APPENDIX 4 *CALCULATION OF THE MEAN AND MEDIAN RANKS*

(The following method of mean rank determination is after Johnson (1964, p39).)

The ranks of ordered uncensored data (completed durations) are determined, then the censored and uncensored values (withdrawals) are pooled and ordered. The total number of values = N and the number of withdrawals greater than the largest completed duration = m, then $n = N – m$ where n is the completed durations.

Mean rank values are determined by the original rank plus an increment. Rank increments are calculated by the formula: (n + 1) - (previous mean order number)/(1 + number of values beyond the present set of withdrawals).

The occurrence of a withdrawal before a value of a completed duration, makes the true rank of a completed duration in the full population of durations different to the rank of only eted durations. The increment, above, is an adjustment to the rank of completed durations to reflect the withdrawals. Withdrawals beyond the last uncensored value are ignored because, of course, those withdrawals will not affect the rank of the uncensored values This reduces the weighting of the withdrawals on the durations distribution of the completed durations and will lower the survivor function.

Johnson (1951) provides a formula and tables for the determination of median rank values, but, with non-integral values for the increments in the mean calculation, it becomes tedious so we used Benard's approximation (Abernethy, 1993, p.2-9) which is: Benard's median rank = $(r + 0.3)/(N + 0.4)$ where r is the adjusted mean rank.

The results for our median rank determination are shown in table 6

[Table 6 here]

APPENDIX 5 *DATA BASE*

The industry is defined as the Australian black coal export industry, which means the NSW plus Queensland black coal industries, ANZSIC code 1100. While other states produce a small amount of coal from a small number of coal mines, the coal is not exported, and is generally sub bituminous[15]. We do not consider that the inclusion of operators from states other than NSW and Queensland is useful in analysis of the whole industry and in this study we analyze NSW only.

Operators are defined as those corporate entities (or, in some cases, individual persons) which control the marketing decisions of a coal mine or a group of coal mines. In most cases, this results from greater than 50% ownership, but may result from being the largest shareholder (less than 50%, but having effective control) or by having a management agreement with other owners. We do not record the complete name of the operator, but sufficient of the name for it to be recognized.

The measure of the size of the operator is the annual quantity of coal produced by the mines controlled by the operator. We consider this is the most appropriate unit of size—assets value is not useful, as a number of coal mine owners now use contractors for a large portion of the operations and this greatly reduces the assets involved (unless it were possible to capitalize the value of the contracts, but such information is not generally available). Moreover, it is possible that many coal mines are over-capitalized. This is shown by the sale at a capital loss to other operators[16]. So asset value can very misleading in relation to effective size.

We consider that production is a reasonable measure of "size", and measure of market share, as coal is bulky and expensive to store, and frequently is liable to spontaneous combustion in storage, so, in general, all coal produced is sold without any appreciable time-lag. Also, the high cost of production and the low profit margin frequently force stock sales to provide cash flow.

We construct the data base of coal operator durations by finding the dates at which a company first becomes an "operator" by becoming a majority or controlling owner of a coal operation in NSW, and when it ceases to be a controlling owner, over the period 1960 to 1999.

We define "start" as the first occasion that an operator produces coal, or the date at which the operator's first mine opens, if it achieves at least 100,000 tonnes of production in the first twelve months of operation; otherwise, we use the start of the financial year (July 1 to June 30) in which 100,000 tonnes production is first achieved[17]. For "finish" date we use the date of sale of the last coal mine owned by the operator or the date of change of control of the

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[15] In 1995-96 South Australia and Western Australia produced 2,499,000 tonnes and 5,897,000 tonnes respectively of sub-bituminous coal, and Tasmania produced 390,000 tonnes of bituminous coal (McLennan 1998)

[16] For example, Arco purchased Cook colliery from McIlwraith McEacharn Ltd for approx. $82 million and later sold the mine (with all equipment) to Oakbridge for $15 million (company records).

[17] While the figure of 100,000 tonnes is arbitrary, we consider it is appropriate to eliminate the mines with very small production. In some cases, mines produce "trial samples" to send to customers and may not open for commercial production for a considerable period of time. The sample may be taken by temporary equipment, well before the major equipment is even ordered, so that the acceptability of the coal can be tested. Some very small mines are operated by family companies, with the owners and

company, or of the operator itself (in some cases this is the date of start of liquidation proceedings). Where only the month of start or finish is known, we use the first of the month. Where more than one operator has the same duration in days, we change the start date by one day to make the analysis easier.

We choose 1960 because the industry prior to that time exported very little, while from the 1970s the market is dominated by exports. In 1960 only 1.1 million tonnes was exported from NSW and by 1998 exports have grown to 75.9 million tonnes (JCB 1999).
The first references we use are the Joint Coal Board *Annual Reports* (JCB various-a), *Black Coal in Australia* (JCB various-b) and the *NSW Coal Industrial Profile* (Department of Mineral Resources various-b), which record coal mine ownership. The Joint Coal Board *Annual Reports* give a list of all operating mines and their owners, only from 1975-76 , and even then no indication is given of the ultimate owners. From the 1984 edition of *Black Coal in Australia*, details of ownership structure are recorded and this continues in the *NSW Coal Yearbook* (JCB 1989) until the *NSW Coal Industry Profile* takes over. Prior to 1975-76, the Joint Coal Board only comments on "major owner groups" and does not record any operator's production below 500,000 tons/a, although some changes in ownership are recorded. *MINFO Mining and Exploration Quarterly* (Department of Mineral Resources various-a) is an excellent reference on opening and closing of mines, although its first edition is only in October 1983.

Our next major reference is the Joint Coal Board workers' compensation records which record owners and mines and dates of opening and closing of mines. The "opening" date is not

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directors actually extracting the coal, and we feel that such mines do not fit into the general category of NSW coal mines and their inclusion in the analysis could give misleading results.

always accurate for the start of a mine, but gives a date at which the mine is certainly in operation, so we know that the start date was that date or some date before it.

We check the annual accounts of those companies that were listed on the stock exchange (some have since been de-listed) and, in certain cases, the microfiche records of the NSW Corporate & Business Affairs held by the AGSM. Such records frequently give the date of acquisition or disposal of mine mines. We then search *Jobson's Mining Year Book* (Dun and Bradstreet), the *Coal Manual* (Tex Report various), the *International Coal Report* (McCloskey various), and company and coal histories, *The Coal Masters* (Jay 1994), *Wallsend and Pelton Collieries* (Tonks 1990), *Coalfields and Collieries of Australia* (Power 1912), *Miners in the 1970s* (Thomas 1983), and *A History of the Miners' Federation of Australia* (Ross 1970). Newspaper records, particularly the *Australian Financial Review* and *Sydney Morning Herald*, are good sources of major events such as the purchase of a company.

The interpretation of these records is assisted greatly by the personal knowledge of one of the authors (Lawrance), who held senior management positions in the coal industry from 1975 to 1995, and was a member of the executive of the NSW Colliery Proprietors' Association and the Australian Coal Association. In a number of cases, personal knowledge and private company records are used, and discussions with associates in the industry are very helpful in determining the ownership of some mines.

When a company acquires operating coal assets, and becomes an "operator" (i.e. obtains a controlling interest in a coal operation), the "start" date is simply the date of acquisition (e.g. the acquisition of Gunnedah mine by AMI from Consolidation Coal on September 17, 1984 results in the "start" of AMI and the "finish" of Consolidation Coal. The subsequent acquisition of 47.9% of AMI by Tomen is only known to be in November 1994. While this is not a majority, it is a "controlling", shareholding and we take the start date as November 1, 1994).

When we only know that month of the acquisition, we use the first of the month, and if only the financial year of the change is known (in a few cases), we use July 1. We trace the ownership back to the ultimate owner as many operate mines through subsidiary companies.

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Chart 1
NSW Coal Operators Full data

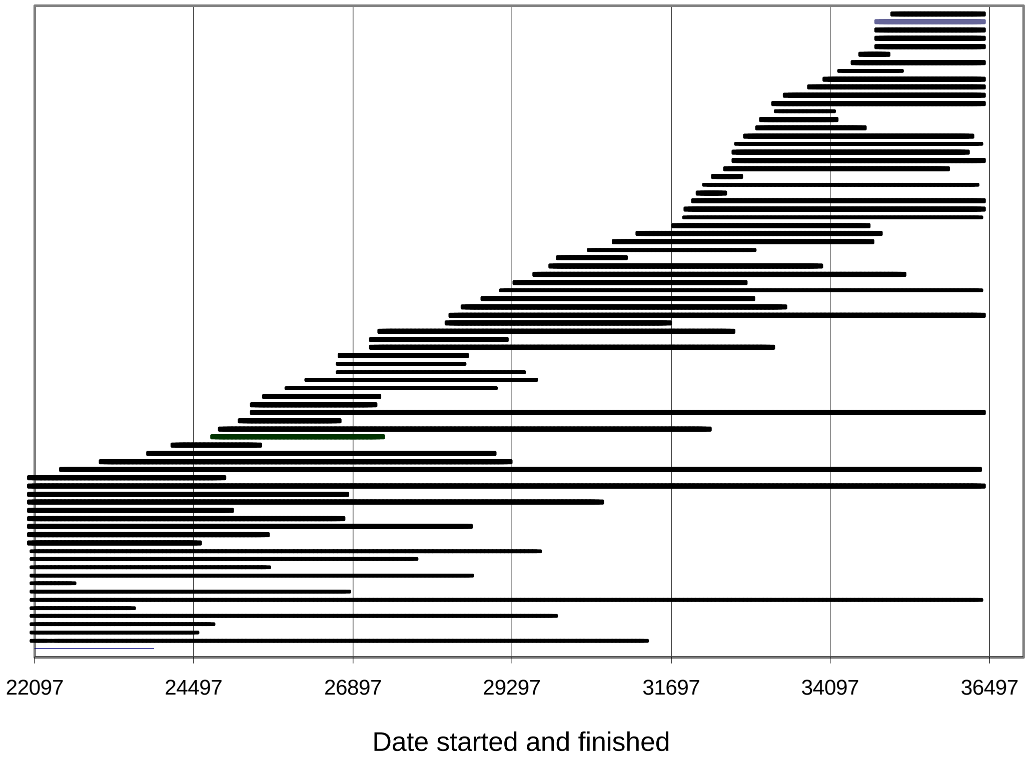

Chart 2
NSW Coal Operators Completed Durations

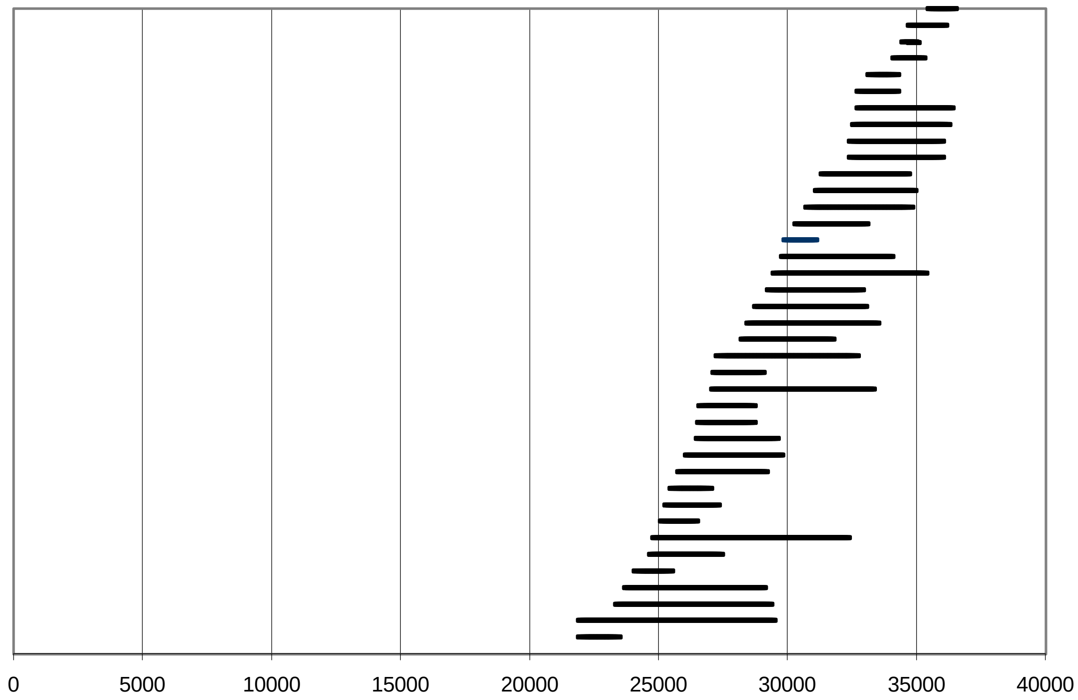

Chart 3
Integrated hazard of Kaplan-Meier product-limit estimator

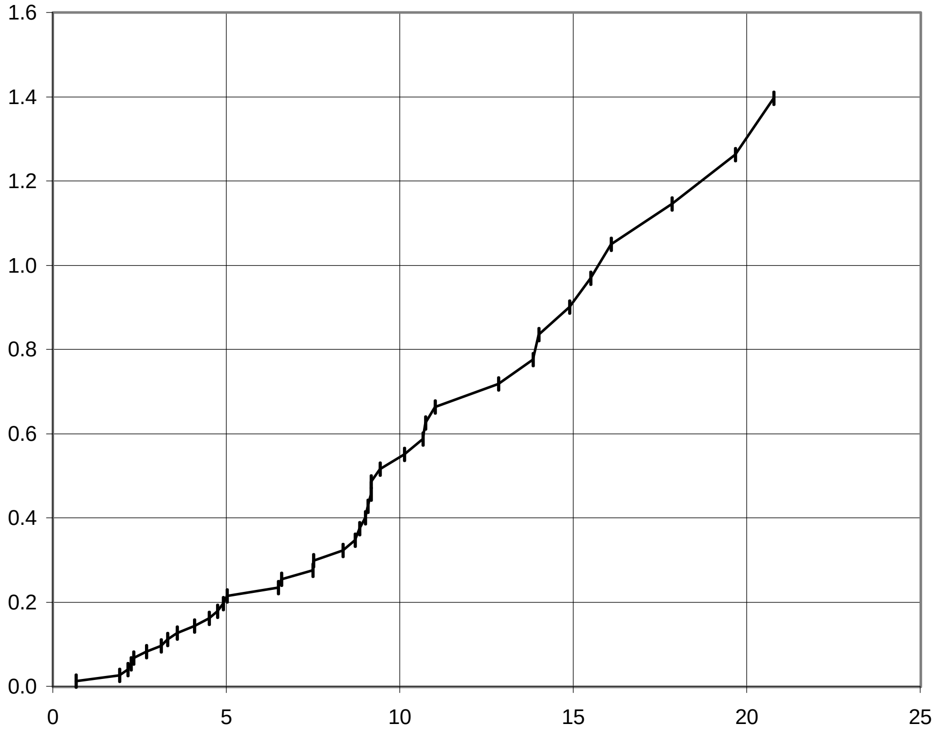

Chart 4
Hazard of the Kaplan-Meier product-limit estimator

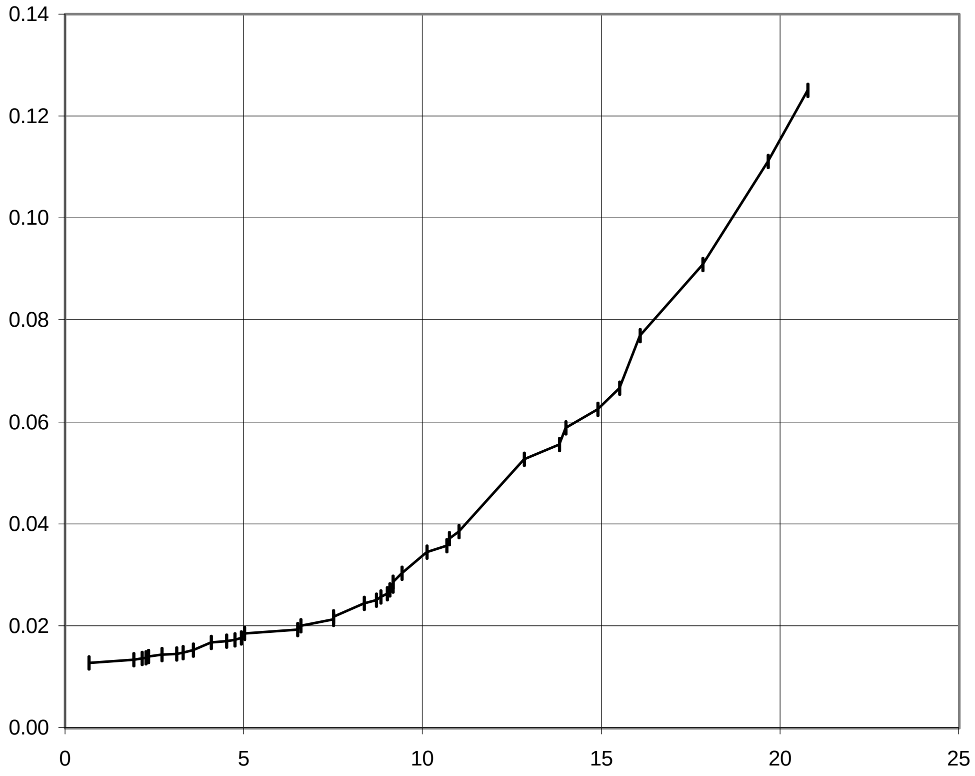

Chart 5
Survivor function of Kaplan-Meier product-limit estimator

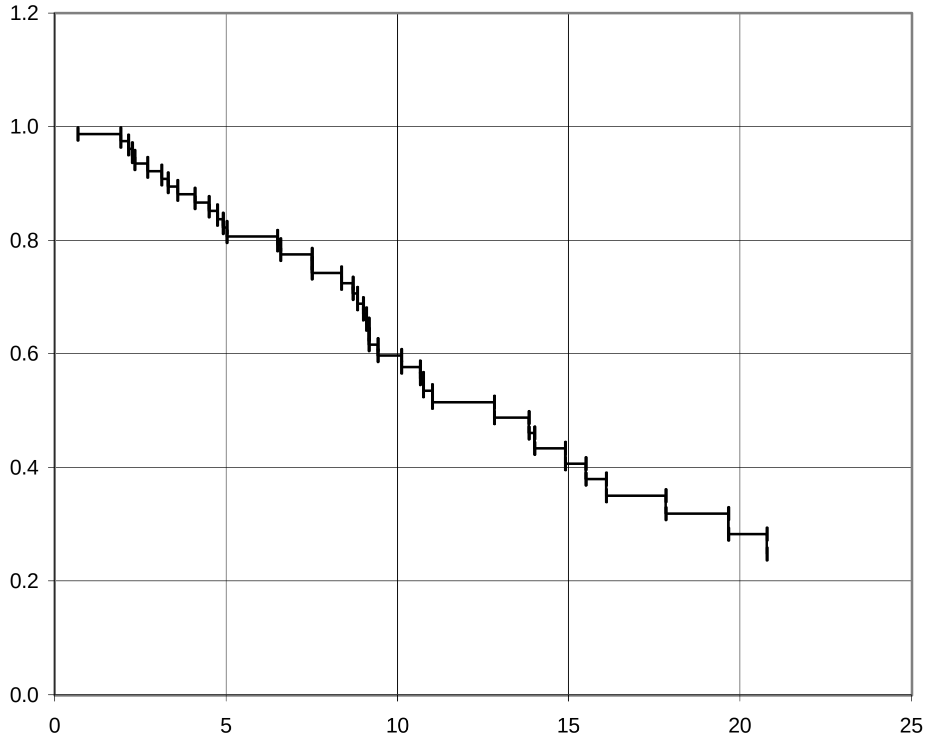

Chart 6a
Weibull plots from Kaplan-Meier product-limit estimator (uncensored data)

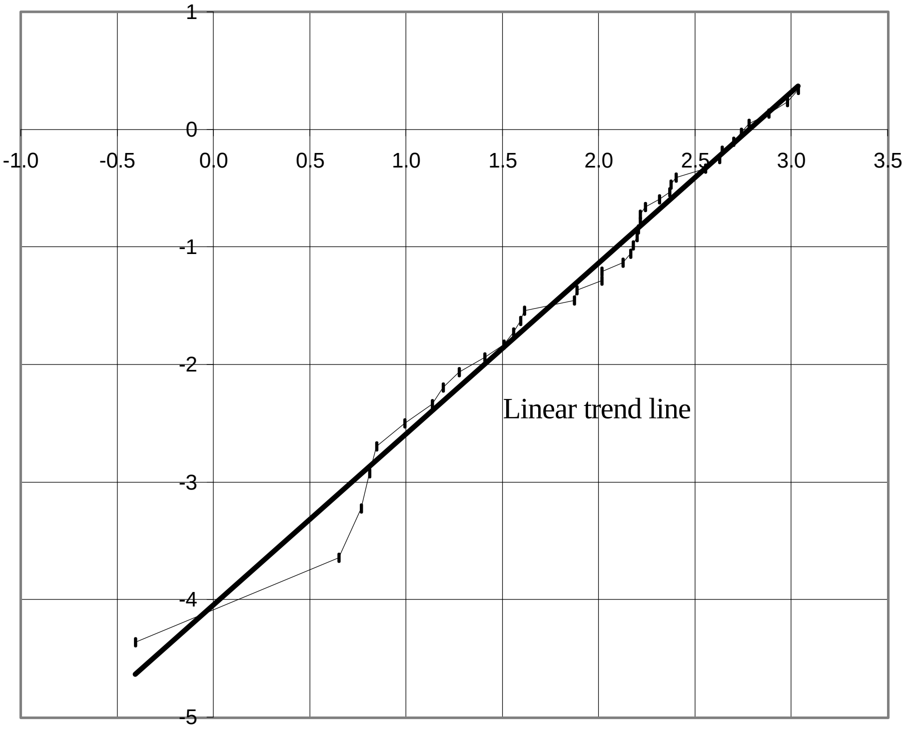

Chart 6b
Weibull plots for full data

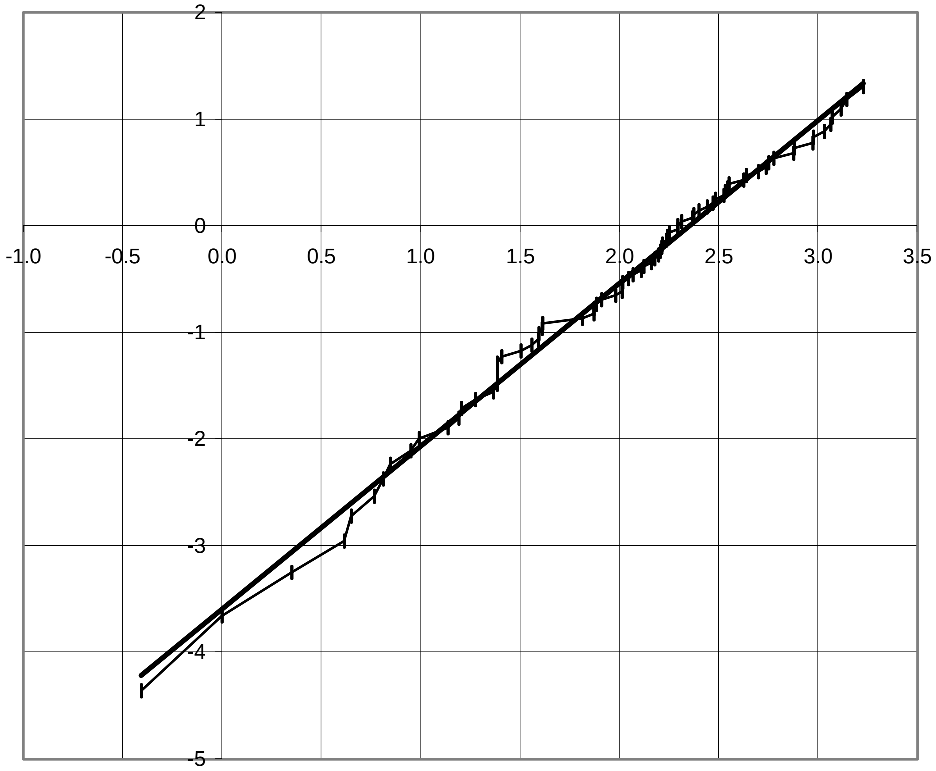

Chart 7a
Survivor functions

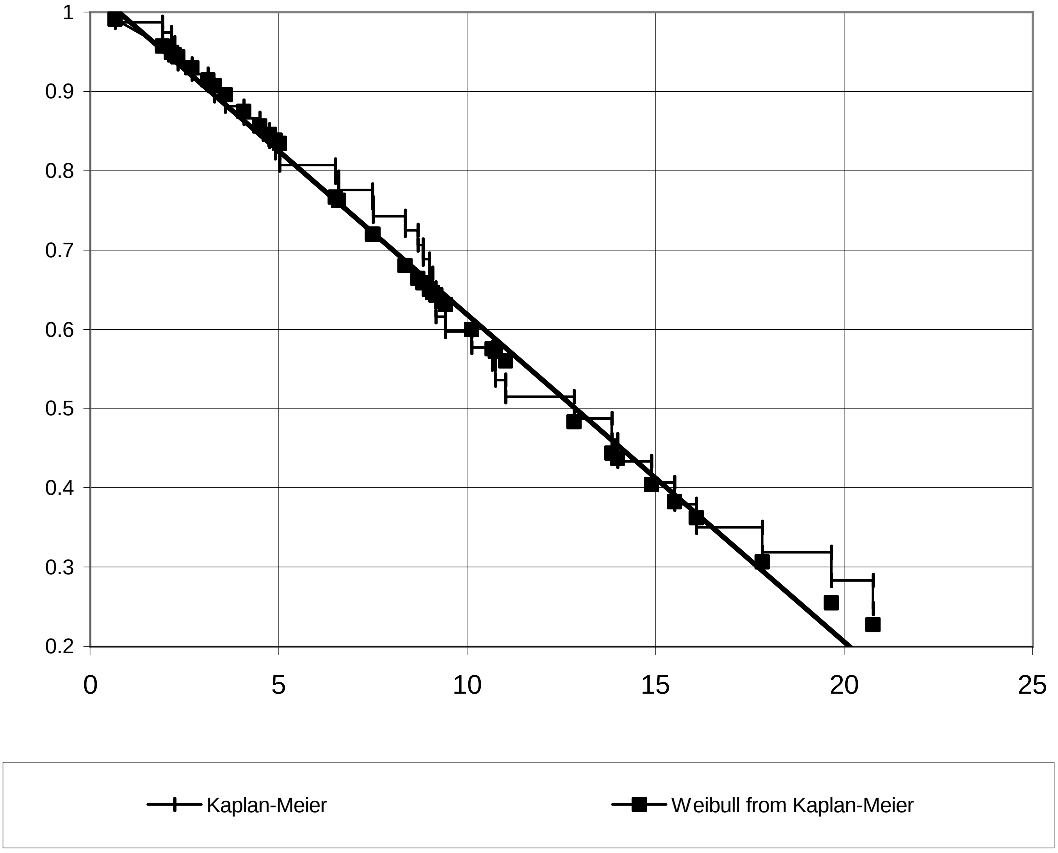

Chart 7b
Survivor functions

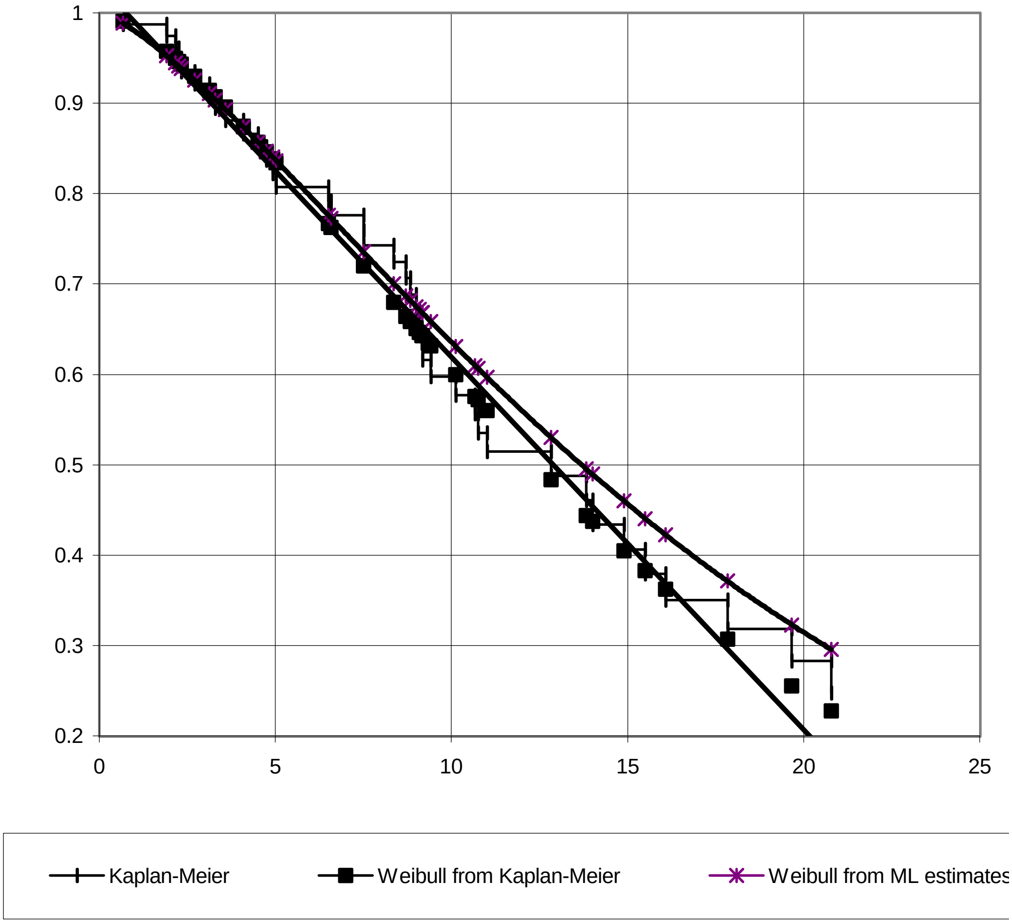

Chart 8
Comparison of all survivor functions

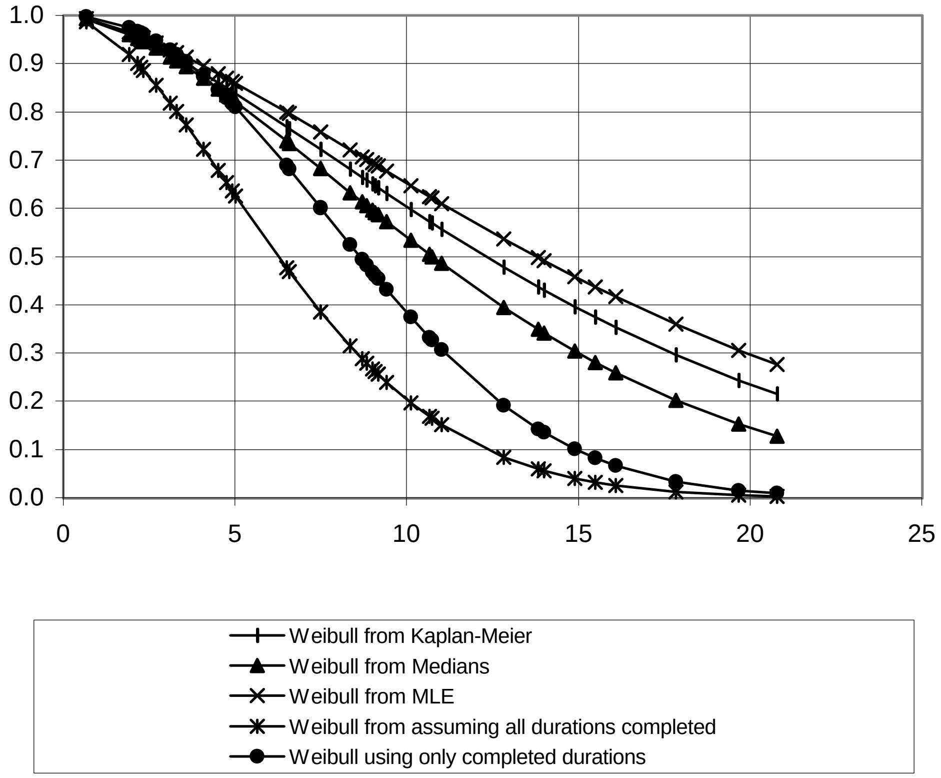

Chart 9
Division into Early and Late sections

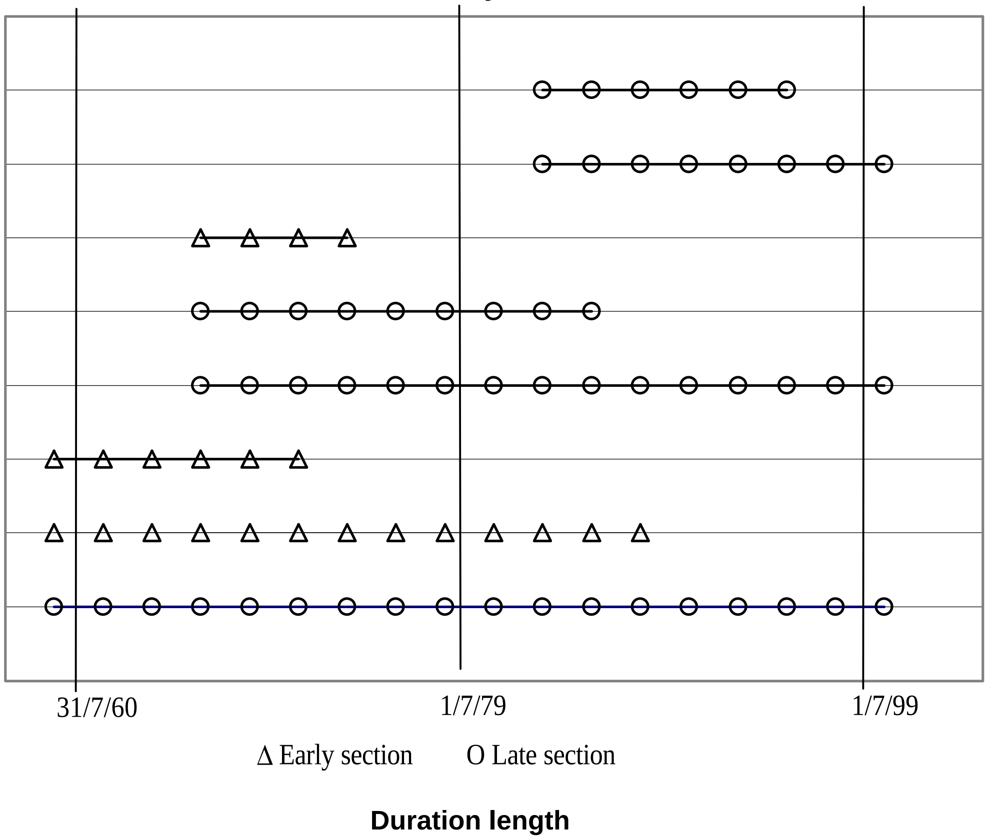

Table 1

NSW Coal Operators 1960 to 1999

Completed durations

Years	Operator	Start	Finish
0.7	Tomen	1/11/94	1/07/95
1.9	Oakdale	1/08/97	1/07/99
2.2	Coal & Allied no.2	1/03/91	27/04/93
2.3	AMP	1/12/93	1/03/96
2.3	Consolidation Coal	1/05/82	1/09/84
2.7	TNT	30/08/90	12/05/93
3.1	Genders mining Co P L	7/05/66	19/06/69
3.3	Wallamaine	16/03/70	3/07/73
3.6	Macdonald Bros P L	1/03/69	1/10/72
4.1	CIM	1/06/95	1/07/99
4.5	Bulli Main Collieries Pty Ltd	1/10/69	4/04/74
4.8	CSR	1/04/73	1/01/78
4.9	Ampol/TNT (Bulkships)	1/02/73	1/01/78
5.0	Woonona Mining & Engineering	24/09/74	1/10/79
6.5	Bond	30/06/83	1/01/90
6.6	Slater Walker	1/01/68	2/08/74
7.5	Boral	2/03/87	1/09/94
7.5	Muswellbrook 2	1/01/73	1/07/80
8.4	Eric Newham (Wallerawang)	1/01/71	11/05/79
8.7	Pacific Copper	6/10/77	18/06/86
8.8	Denehurst	1/03/89	1/01/98
9.0	FAI	1/01/90	1/01/99
9.1	Consolidated Press	1/07/80	31/07/89
9.2	Hartogen	2/11/71	1/01/81
9.2	Noette (Coalpac)	1/08/89	1/10/98
9.4	Clutha	13/09/85	14/02/95
10.1	AMI	17/09/84	1/11/94
10.7	BP Australia Holdings Ltd	1/04/79	1/12/89
10.7	Coatain	1/01/82	30/09/92
11.0	Savage	1/04/88	7/04/99
12.8	Howard Smith	1/05/78	1/03/91
13.8	D K Ludwig	1/06/65	1/04/79
14.0	White	1/01/75	1/01/89
14.9	Caltex	10/04/81	1/03/96
15.5	Consolidated Gold Fields	1/06/64	1/12/79
16.1	Oakbridge	2/08/74	3/09/90
17.8	Coal & Allied no.1	30/06/60	1/05/78
19.7	Peko-Wallsend	1/06/68	1/02/88
20.8	Goodsir & Copper P L	30/06/60	10/04/81
39	count		

Withdrawals

Years	Operator	Start	Finish
1.0	Gympie	1/07/98	1/07/99
1.4	Clinton Nattai Collieries	30/06/60	1/12/61
1.9	Allied Meridian	25/08/97	1/07/99
2.6	Austral Coal Ltd	1/12/96	1/07/99
3.3	Ingwe	1/03/96	1/07/99
3.9	Bellambi	30/06/60	1/06/64
4.0	Advance	1/07/95	1/07/99
4.0	AGIP	1/07/90	1/07/94
4.0	AMCI (Namoi)	1/07/95	1/07/99
4.0	Brimstone	1/07/95	1/07/99
4.9	Placer	29/06/60	1/06/65
5.0	Abelshore	1/07/94	1/07/99
6.1	Cyprus	12/05/93	1/07/99
6.5	Hebburn	30/06/60	1/01/67
6.8	Peabody	30/09/92	1/07/99
7.3	Barix	30/06/60	6/10/67
7.5	Wancol	30/06/60	8/01/68
7.8	Cumnock	1/10/91	1/07/99
7.9	Newcastle Wallsend Coal	30/06/60	1/06/68
8.3	Henry Walker	1/04/91	1/07/99
9.4	Maintland Mining Co P L	30/06/60	14/11/69
9.5	Gifford R A Pty Ltd	30/06/60	31/12/69
9.9	Centennial	2/08/89	1/07/99
9.9	Idemitsu	31/07/89	1/07/99
11.5	Exxon	1/01/88	1/07/99
11.8	Sumitomo	1/09/87	1/07/99
12.0	South Coast Equipment	1/07/87	1/07/99
12.5	Muswellbrook 1	30/06/60	1/01/73
12.6	R W Miller	30/06/60	1/02/73
12.8	Buchannan	30/06/60	1/04/73
15.7	Gollin	30/06/60	1/03/76
17.8	Mt Sugarloaf Colls P L	30/06/60	1/04/78
19.6	Shell	1/12/79	1/07/99
21.4	Bayswater	30/06/60	4/12/81
21.6	Hartley Valley Coal Co	11/12/77	1/07/99
22.5	Avondale	30/06/60	17/01/83
23.2	North Bulli	30/06/60	16/09/83
25.2	Austen & Butta	30/06/60	20/09/85
29.8	Big Ben Holdings	1/10/69	1/07/99
37.6	CRA	1/12/61	1/07/99
40	count		

	Deletions		
0.2	Pasminco	7/04/99	1/07/99
0.5	HIH	1/01/99	1/07/99
0.7	Elders	1/10/88	29/06/89
0.8	North Broken Hill	1/02/88	31/10/88

	Deletions		
39.0	BHP	30/06/60	1/07/99
39.0	State Govt	30/06/60	1/07/99

Table 2

Kaplan-Meier product-limit analysis

Completed duratons		Uncensored durations	Number at	Survivor function	Hazard dj/nj	Integrated hazard	Weibull analysis		Weibull distribution function	Weibull survivor function
							eta = 15.67			
							beta = 1.5			
Years		Years	risk							
tj	*d(t)*	*Tj*	*n(t)*	S(*t*)	H(*t*)	H(*t*) = -ln[S(*t*)]	Ln(*t*)	LnLn(1/*S*)	F(*t*)	1 -F(*t*)
0.67	1	0.67	79	0.987	0.013	0.013	-0.405	-4.363	0.008	0.992
1.92	1	1.00	75	0.974	0.01	0.026	0.651	-3.643	0.040	0.960
2.16	1	1.42	74	0.961	0.01	0.040	0.768	-3.225	0.048	0.952
2.25	1	1.85	73	0.948	0.01	0.054	0.811	-2.927	0.051	0.949
2.33	1	1.92	72	0.935	0.01	0.068	0.847	-2.695	0.054	0.946
2.70	1	2.16	70	0.921	0.01	0.082	0.993	-2.502	0.067	0.933
3.12	1	2.25	69	0.908	0.01	0.097	1.137	-2.338	0.082	0.918
3.30	1	2.33	68	0.895	0.01	0.111	1.193	-2.195	0.089	0.911
3.58	1	2.58	66	0.881	0.02	0.127	1.276	-2.067	0.101	0.899
4.08	1	2.70	60	0.866	0.02	0.143	1.407	-1.942	0.121	0.879
4.51	1	3.12	59	0.852	0.02	0.161	1.506	-1.829	0.140	0.860
4.75	1	3.30	58	0.837	0.02	0.178	1.558	-1.726	0.150	0.850
4.92	1	3.33	57	0.822	0.02	0.196	1.593	-1.632	0.158	0.842
5.02	1	3.58	54	0.807	0.02	0.214	1.613	-1.540	0.162	0.838
6.50	1	3.92	52	0.792	0.02	0.234	1.872	-1.454	0.231	0.769
6.59	1	4.00	50	0.776	0.02	0.254	1.885	-1.371	0.235	0.765
7.497	1	4.00	47	0.759	0.02	0.275	2.015	-1.289	0.278	0.722
7.500	1	4.00	46	0.743	0.02	0.297	2.015	-1.213	0.278	0.722
8.36	1	4.00	41	0.725	0.02	0.322	2.124	-1.133	0.319	0.681
8.70	1	4.08	40	0.707	0.03	0.347	2.163	-1.057	0.335	0.665
8.83	1	4.51	39	0.688	0.03	0.373	2.179	-0.985	0.342	0.658
9.00	1	4.75	38	0.670	0.03	0.400	2.197	-0.916	0.350	0.650
9.08	1	4.92	37	0.652	0.03	0.427	2.206	-0.850	0.354	0.646
9.16	1	4.92	36	0.634	0.03	0.456	2.215	-0.786	0.357	0.643
9.17	1	5.00	35	0.616	0.03	0.485	2.216	-0.724	0.358	0.642
9.42	1	5.02	33	0.597	0.03	0.515	2.243	-0.663	0.369	0.631
10.12	1	6.14	29	0.577	0.03	0.550	2.315	-0.597	0.402	0.598
10.67	1	6.50	28	0.556	0.04	0.587	2.367	-0.533	0.427	0.573
10.75	1	6.50	27	0.535	0.04	0.625	2.375	-0.471	0.431	0.569
11.02	1	6.59	26	0.515	0.04	0.664	2.399	-0.410	0.443	0.557
12.83	1	6.75	19	0.488	0.05	0.718	2.552	-0.331	0.522	0.478
13.83	1	7.27	18	0.461	0.06	0.775	2.627	-0.255	0.563	0.437
14.00	1	7.497	17	0.434	0.06	0.836	2.639	-0.180	0.569	0.431
14.89	1	7.500	16	0.406	0.06	0.900	2.701	-0.105	0.604	0.396
15.50	1	7.52	15	0.379	0.07	0.969	2.741	-0.031	0.626	0.374
16.09	1	7.75	13	0.350	0.08	1.049	2.778	0.048	0.647	0.353
17.84	1	7.92	11	0.318	0.09	1.145	2.881	0.135	0.704	0.296
19.67	1	8.25	9	0.283	0.11	1.262	2.979	0.233	0.757	0.243
20.78	1	8.36	8	0.248	0.13	1.396	3.034	0.334	0.785	0.215
		8.70								
		8.83	Tj (cont.)							
		9.00	12.83							
		9.08	13.83							
		9.16	14.00							
		9.17	14.89							
		9.37	15.50							
		9.42	15.67							
		9.50	16.09							
		9.91	17.75							
		9.92	17.84							
		10.12	19.58							
		10.67	19.67							
		10.75	20.78							
		11.02	21.43							
		11.50	21.56							
		11.83	22.55							
		12.00	23.21							
		12.50	25.22							
		12.59	29.75							
		12.75	37.58							
		Count =	**79**							

Table 3
Summary of results

Full data [1]	Completed durations	Withdrawals	Likelihood	Log likelihood	eta Scale	beta Shape	mean	median
Weibull from Median values	39	40	1.10E-69	-158.8	9.6	1.61	8.6	7.64
Weibull from MLE parameters	39	40	1.92E-67	-153.6	17.6	1.51	15.8	13.8
Weibull MLE	39	0	5.17E-51	-115.8	10.2	2.19	9.0	8.63
(only completed durations)								
Weibull MLE	79	0	9.34E-112	-255.7	7.7	1.77	6.8	6.26
(assume all completed)								
Weibull from Kalpan-Meier	39	40	9.71E-68	-154.3	15.7	1.52	14.1	12.31
Early section [2]								
MLE	10	19	5.77E-20	-44.299	27.8	1.17	26.4	20.33
Late section [3]								
MLE	20	30	1.3132E-36	-82.621	21.4	1.14	20.5	15.55

mean = $\eta \, \Gamma\left(\frac{\beta + 1}{\beta}\right)$ [5] [4]

median = $\eta\left(\ln 2\right)^{1/\beta}$ [5]

Max likelihood beta = [5]

$$\frac{\sum_{i=1}^{N} t_i^{\beta} \ln t_i}{\sum_{i=1}^{N} t_i^{\beta}} - \frac{1}{r}\sum_{i=1}^{r} \ln t_i - \frac{1}{\beta} = 0$$

Max likelihood eta = [5]

$$\hat{\eta} = \left(\frac{\sum_{i=1}^{N} t_i^{\beta}}{r}\right)^{\frac{1}{\beta}}$$

Likelihood function = [5]

$$L = \prod_{i=1}^{r}\left[\left(\frac{\beta}{\eta}\right)\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{-\left(t_i/\eta\right)^{\beta}}\right]\prod_{j=1}^{k}\left[e^{-\left(T_j/\eta\right)^{\beta}}\right]$$

N = number of completed durations
r = number of completed durations
t = completed durations
T = withdrawals

[1] Data from 30/6/60 to 1/7/99, completed durations and withdrawals from left and right censoring
[2] Those started before 1/7/79 and finished by 30/6/79 plus those started before 1/7/60 and finished after 30/6/79
[3] Started after 30/6/60 and not finished by 30/6/79 and all started after 30/6/79
[4] Gamma function from Beyer (1976) "Standard Mathematical Tables" and using formula $\Gamma(x+1) = x\Gamma(x)$
[5] Abernethy (1993)

Table 4

Period commencing 1st July	1960	1965	1970	1975	1980	1985	1990
Completed durations	39	35	29	21	18	12	6
Early censoring	18	19	18	20	14	13	12
Late censoring	22	22	22	21	21	21	21
Total withdrawals	40	41	40	41	35	34	33
Total	79	76	69	62	53	46	39
MLE beta	1.35	1.23	1.40	1.12	1.03	0.94	0.63
MLE eta	17.9	19.93	21.05	28.80	30.16	40.06	160.3

Table5

Rank i	Survivor function S	LnLn(1/S)	Completed durations t	Ln(t)	Ln(t)*Ln(t)	Wi	L1	W(n+i)	L2
1	0.98	-3.68	0.67	-0.41	0.16	0.03	-0.01	-1.07	0.43
2	0.95	-2.97	1.92	0.65	0.42	0.05	0.03	-1.10	-0.72
3	0.93	-2.55	2.16	0.77	0.59	0.08	0.06	-1.12	-0.86
4	0.90	-2.25	2.25	0.81	0.66	0.11	0.09	-1.13	-0.92
5	0.88	-2.01	2.33	0.85	0.72	0.13	0.11	-1.14	-0.96
6	0.85	-1.82	2.70	0.99	0.99	0.16	0.16	-1.13	-1.13
7	0.83	-1.65	3.12	1.14	1.29	0.19	0.22	-1.12	-1.28
8	0.80	-1.50	3.30	1.19	1.42	0.22	0.27	-1.11	-1.33
9	0.78	-1.37	3.58	1.28	1.63	0.25	0.33	-1.09	-1.40
10	0.75	-1.25	4.08	1.41	1.98	0.29	0.40	-1.07	-1.51
11	0.73	-1.13	4.51	1.51	2.27	0.32	0.48	-1.04	-1.57
12	0.70	-1.03	4.75	1.56	2.43	0.36	0.56	-1.01	-1.58
13	0.68	-0.93	4.92	1.59	2.54	0.39	0.63	-0.97	-1.55
14	0.65	-0.84	5.02	1.61	2.60	0.43	0.69	-0.93	-1.50
15	0.63	-0.76	6.50	1.87	3.51	0.47	0.88	-0.88	-1.66
16	0.60	-0.67	6.59	1.88	3.55	0.51	0.96	-0.83	-1.57
17	0.58	-0.59	7.50	2.01	4.06	0.55	1.11	-0.77	-1.56
18	0.55	-0.51	7.50	2.01	4.06	0.60	1.20	-0.71	-1.43
19	0.53	-0.44	8.36	2.12	4.51	0.64	1.37	-0.64	-1.36
20	0.50	-0.37	8.70	2.16	4.68	0.69	1.50	-0.56	-1.21
21	0.48	-0.30	8.83	2.18	4.75	0.74	1.62	-0.48	-1.04
22	0.45	-0.23	9.00	2.20	4.83	0.80	1.75	-0.38	-0.84
23	0.43	-0.16	9.08	2.21	4.87	0.86	1.89	-0.28	-0.61
24	0.40	-0.09	9.16	2.22	4.91	0.92	2.03	-0.16	-0.36
25	0.38	-0.02	9.17	2.22	4.91	0.98	2.17	-0.04	-0.08
26	0.35	0.05	9.42	2.24	5.03	1.05	2.35	0.10	0.23
27	0.33	0.12	10.12	2.31	5.36	1.12	2.60	0.26	0.59
28	0.30	0.19	10.67	2.37	5.60	1.20	2.85	0.43	1.01
29	0.28	0.26	10.75	2.37	5.64	1.29	3.07	0.62	1.47
30	0.25	0.33	11.02	2.40	5.76	1.39	3.33	0.84	2.01
31	0.23	0.40	12.83	2.55	6.51	1.49	3.81	1.09	2.78
32	0.20	0.48	13.83	2.63	6.90	1.61	4.23	1.38	3.61
33	0.18	0.56	14.00	2.64	6.96	1.74	4.60	1.71	4.52
34	0.15	0.64	14.89	2.70	7.29	1.90	5.12	2.11	5.70
35	0.13	0.73	15.50	2.74	7.51	2.08	5.70	2.60	7.13
36	0.10	0.83	16.09	2.78	7.72	2.30	6.40	3.22	8.95
37	0.08	0.95	17.84	2.88	8.30	2.59	7.46	4.06	11.69
38	0.05	1.10	19.67	2.98	8.87	3.00	8.92	5.28	15.74
39	0.03	1.31	20.78	3.03	9.20	5.45	16.55	13.58	41.21

NB The survivor function S is calculated using (n + 1) because the data must be assumed to be from a Weibull population which is infinite.

$$b = (0.6079L_2 - 0.2570L_1)/n$$

0.5899

$$W = nb^2/S^2$$

0.0823

Using Shapiro table 6.4 p.6.17, sample size 39 and limits 0.05 and .95 the lower and upper values are:

lower	upper
0.4403051	0.862093

As Wp lies between these two extreme value
there is no reason to reject the null hypothesis of a Weibull distribution

Table 6

Johnson method of median ranks

(n+1) = 73

Durations	Increment	Adjusted mean rank	Adjusted mean rank	Benard median rank (proportion)	Benard median rank Survivor function
0.7	0	1	1	0.01	0.99
1.0			2.04	0.02	0.98
1.4			3.09	0.04	0.96
1.9			4.13	0.05	0.95
1.9	1.04	2.04	5.17	0.07	0.93
2.2	1.04	3.09	6.23	0.08	0.92
2.3	1.04	4.13	7.29	0.10	0.90
2.3	1.04	5.17	8.35	0.11	0.89
2.6			9.43	0.13	0.87
2.7	1.06	6.23	10.61	0.14	0.86
3.1	1.06	7.29	11.79	0.16	0.84
3.3	1.06	8.35	12.96	0.17	0.83
3.3			14.14	0.19	0.81
3.6	1.08	9.43	15.37	0.21	0.79
3.9			16.62	0.23	0.77
4.0			17.90	0.24	0.76
4.0			19.24	0.26	0.74
4.0			20.59	0.28	0.72
4.0			22.09	0.30	0.70
4.1	1.18	10.61	23.58	0.32	0.68
4.5	1.18	11.79	25.08	0.34	0.66
4.8	1.18	12.96	26.58	0.36	0.64
4.9	1.18	14.14	28.08	0.38	0.62
4.9			29.57	0.40	0.60
5.0			31.07	0.43	0.57
5.0	1.23	15.37	32.62	0.45	0.55
6.1			34.38	0.47	0.53
6.5	1.25	16.62	36.13	0.49	0.51
6.5			37.89	0.52	0.48
6.6	1.28	17.90	39.65	0.54	0.46
6.8			42.21	0.58	0.42
7.3			44.78	0.61	0.39
7.5	1.34	19.24	47.34	0.65	0.35
7.5	1.34	20.59	49.91	0.69	0.31
7.5			52.47	0.72	0.28
7.8			55.41	0.76	0.24
7.9			58.93	0.81	0.19
8.3			63.62	0.87	0.13
8.4	1.50	22.09	68.31	0.94	0.06
8.7	1.50	23.58		count = 39	
8.8	1.50	25.08			
9.0	1.50	26.58			
9.1	1.50	28.08			
9.2	1.50	29.57			
9.2	1.50	31.07			
9.4					
9.4	1.55	32.62			
9.5					
9.9					
9.9					
10.1	1.76	34.38			
10.7	1.76	36.13			
10.7	1.76	37.89			
11.0	1.76	39.65			
11.5					
11.8					
12.0					
12.5					
12.6					
12.8					
12.8	2.57	42.21			
13.8	2.57	44.78			
14.0	2.57	47.34			
14.9	2.57	49.91			
15.5	2.57	52.47			

	Durations continued	Increment continued	mean rank continued
	15.7		
	16.1	2.93	55.41
	17.8		
	17.8	3.52	58.93
	19.6		
	19.7	4.69	63.62
	20.8	4.69	68.31
Exclude	***21.4***		
Exclude	***21.6***		
Exclude	***22.5***		
Exclude	***23.2***		
Exclude	***25.2***		
Exclude	***29.8***		
Exclude	***37.6***		

Count = 79

[1] Bold and italicised data are withdrawals