Increasing Marginal Revenue and Demand Elasticity

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ABSTRACT. If social "harm" is an inverse-U function against the degree of control of illicit drug markets, and if revenue can proxy "harm," what demand functions result in increasing marginal revenue?

1. Introduction

THIS paper explores "increasing marginal revenue" demand functions in a research program associated with "harm-minimization" illicit drug policies. The motivation is that researchers have noted that social "harm" can be described by an inverted-U function against the degree of control of illicit drug use: high levels of control correspond to high levels of harm, while low levels of control, tending to no control at all, also correspond to high levels of harm, suitably defined. An intermediate level of control results in much lower levels of harm. High levels of control correspond to low quantities consumed, and low levels to high quantities consumed. "Harm" can be proxied by revenue. Question: what are the characteristics of demand functions that result in the inverse-U-shaped curve of increasing marginal revenue?

2. Increasing Marginal Revenues

This paper explores the properties of demand functions that exhibit "increasing marginal revenue".[1] Linear demand functions exhibit decreasing marginal revenue, as revealed as inverted U-shaped revenue functions of Figure 1, Demand functions with the property of constant revenue (revealed as a horizontal revenue curve) are equivalent to:

$$PQ = \text{constant}$$

or, taking logarithms and differentiating,

$$d \ln P + d \ln Q = 0,$$

or

$$\frac{dQ/Q}{dP/P} = -1.$$

That is, constant revenue is equivalent to a unitary price elasticity of demand.

Our interest in increasing-marginal-revenue (or increasing-elasticity) curves derives from the so-called "harm" curves associated with drug policy. Observers (John Marks 1990, Figure 1, Mugford 1991, HOCBC 2005, Figure 4, and others) have noted a characteristic U-shaped curve linking "harm" with the "degree of control effect" over illicit drugs: at high levels of control effort, "harm" is high; for lower levels of control

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1. Increasing marginal revenue has been studied in relation to monopoly pricing, but not in our context here. Couglin 1984 summarises the earlier work and extends it.

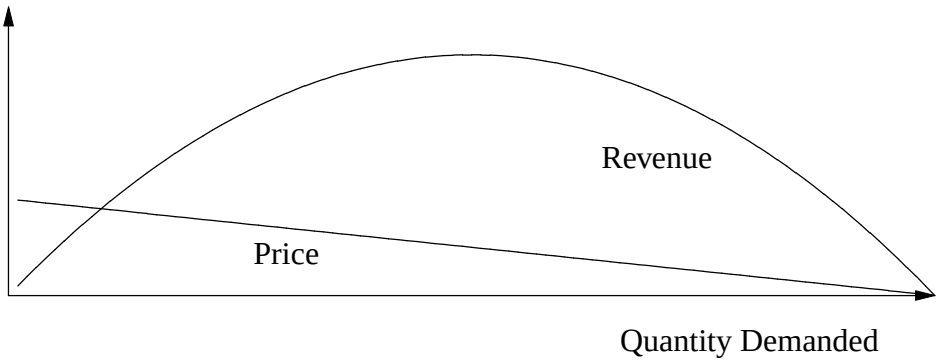

Figure 1: Linear Demand and Decreasing Marginal Revenue.

effort, harm falls; but rises again for still lower levels of control effort.[2]

This is explained by high levels of control deterring all but the most ardent consumers, those addicted to the drug, who are prepared to do what it takes, usually raising money illegally, to acquire the drug for their consumption; their demand for the drug is price-insensitive, or inelastic. At low levels of control, demand has increased as more price-sensitive consumers have entered the market (those with more elastic demand), and the social harm from their activities is high by virtue of their numbers, even though they do not need to engage in the kinds of illegal activities observed among addicts faced with high prices on the black market.

3. Absence of Estimated Demand Functions

In the absence of empirically derived demand curves for illicit drugs[3], we are reduced to finding data where we can: the U-shaped "harm" curve suggested to this author that "harm" was a proxy for total revenue associated with a quantity of, or a price for, the illicit drug, where degree of control effort was directly related to black-market price and inversely related to quantity of illicit drug exchanged (and consumed).[4]

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2. Marks (1990) lists: Prohibition, Defacto Decriminalization, Decriminalization, Prescription, Market Regulation, Legalize with few restrictions, and Legalize and promote. Note: I heard John Marks' presentation in 1990, which was the genesis for this note.

3. Three studies which provide no more than point estimates of the price elasticity of demand for heroin are Brown & Silverman (1974), who estimate a high −5, Silverman & Spruill (1977), who find an inelastic −0.17 for 1975 Detroit, and Caulkins (1995), who get the high estimate of −1.5 for the USA. Liu et al. (1999) estimate short- and long-term price elasticities of demand for opium in Taiwan 1914−1942 of −0.14 and −1.38 respectively. Van Ours (1995) estimates these at −0.7 and −1.0, respectively, for demand for opium in the Dutch East Indies, 1923−1938. Bretteville-Jensen and Biørn (2003), in a study in Sweden, distinguish dealers (resp. −0.15 and −1.51) from non-dealing users (resp. −0.17 and −1.69).

At low levels of control effort, with low price and high quantities, “harm” is high, although “harm” per user is low; at high levels of control effort, with high prices and low quantities, “harm” is high as is “harm” per user, given high price and low quantity of illicit drug. If total revenue is related to total “harm”, then an industry demand curve with increasing revenue would result in the observed behaviour.

4. Increasing-Revenue Demand Functions

Let quantity demanded $Q = P^{-\gamma}$, where γ is the price elasticity of demand. Rewriting this, price $P = Q^{-\delta}$, where $\delta = 1/\gamma$. Revenue $R = PQ = Q^{(1-\delta)}$. Differentiating, $dR/dQ = (1-\delta)Q^{-\delta} = (1-\delta)P$. This is positive iff $\delta < 1$, or iff $\gamma > 1$; negative iff $\delta > 1$, or iff $\gamma < 1$.

So, increasing marginal revenue (a U-shaped revenue curve) is equivalent to $\gamma < 1$ for low quantities demanded, rising to $\gamma > 1$ for high quantities demanded; that is, inelastic demand for low quantities (high prices) moving to elastic demand for high quantities (low prices). We are addressing long-term price elasticities of demand throughout.

Figure 2 shows a U-shaped revenue curve, exhibiting increasing marginal revenue, and its accompanying demand curve, that exhibits increasing elasticity: from low elasticity and falling revenue to high elasticity and rising revenue.[5] Note: as quantity rises, price falls.

5. Discussion

Our analysis is consistent with understanding of the market for illicit drugs, especially for narcotics, with a group of addicted users: at high prices, only the addicts, with inelastic demand, consume; at lower prices, ceteris paribus, more occasional users, with more elastic demand for the dug, enter the market. This suggests that it would be wrong, given observed increasing marginal revenue, to assume because the addicts who purchase and consumer illicit drugs in illegal markets exhibit inelastic (non-price-sensitive) demand, that a relaxation of restrictions on markets for illicit drugs, which would lower the effective price on the market, would not result in an increase in consumption.

If the U-shaped revenue curve is correct, then lower prices would attract consumers with more elastic demand, and hence increased quantities of drug would be consumed. Whether this consequence is a concern depends on one’s attitude to the purpose of the restrictions on illicit drug use: the harm-reduction advocate would argue that, so long as mortality, morbidity, crime rates, and other social impacts do not also rise, then such rises in consumption are of little concern.

6. Conclusion

The ideal level of harm for policy makers seeking to minimise harm to aim for is that

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4. Of course, the law enforcement agencies focus on their success at reducing consumption, and ignore the concomitant high prices, and the indirect costs of the more highly adulterated street drugs as supply is squeezed, in the face of an unshifting demand.

5. For the record, the demand curve in Figure 2 is given by $P = 10/Q + e^{\frac{1}{2}\log Q} + 40$.

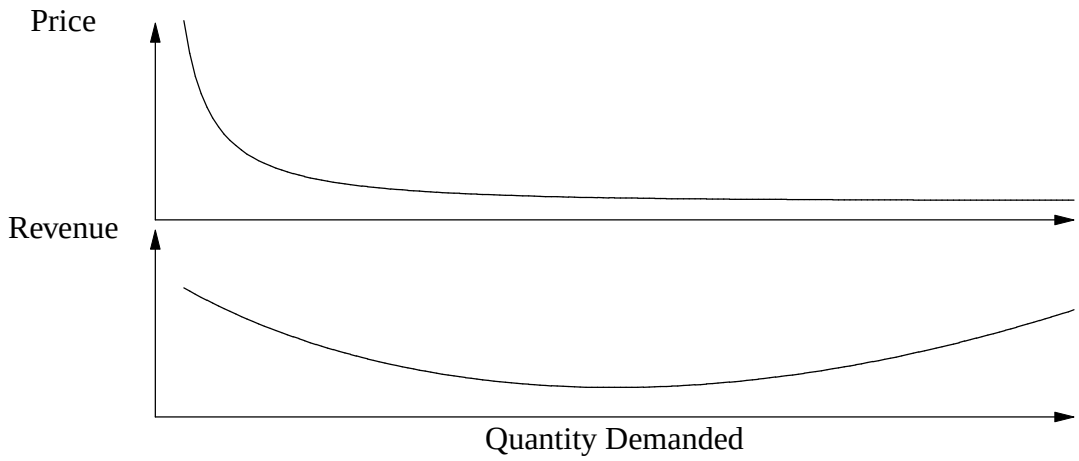

Figure 2: Increasingly Elastic Demand and Increasing Marginal Revenue.

associated with minimum total revenue. This occurs when the demand for the drug exhibits unit elasticity, at which point marginal revenue is zero, from the derivation above (Mugford's "Goldilocks" level). That is, policy should aim for that level of control where the elasticity of demand of the user population is one, at the lowest point of the U-shaped revenue curve in Figure 2.

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