

# **Tutorial: Validating Simulation Models, and Multi-Agent Systems in the Social Sciences**

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## OUTLINE

1. Introduction
2. Sufficiency and Necessity
3. Analysis in the Social Sciences
  - Simulation and Analysis
4. Validation
5. The State Similarity Measure (SSM)

See R.E. Marks, “Analysis and synthesis: multi-agent systems in the social sciences,” *The Knowledge Engineering Review*, 27(2): 123–136, 2012.

## 1. Introduction

**Computer scientists are concerned with *finding solutions* to issues such as market design, whereas —**

**social scientists in general and economists in particular have been concerned with *explaining and predicting* social phenomena.**

**(This is also true of other scientists, such as life researchers, who use simulations to model real-world phenomena (Bentley 2013).)**

**Both of these approaches demand sufficiency, but scientists (or at any rate economists) also demand necessity:**

**Not just: “This is a solution”**

**but also: “This is the set of all possible solutions.”**

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## Traditional Economic Methods

### **A certain logic:**

- **observe a real-world phenomenon**
- **identify a need to explain and understand it**
- **build a mathematical, closed-form model, with simplifying assumptions to allow its solution**
- **manipulate the model to obtain sufficient and necessary conditions for the observed phenomenon**
- **perhaps relax a simplifying assumption or two and ask how the model changes**

**This has focussed on equilibria or steady-states, precluding study of out-of-equilibrium or dynamic phenomena.**

**Simulation can overcome these restrictions, but at a cost.**

## **Judd's ideas (2006)**

**“Far better an approximate answer to the right question ... than an exact answer to the wrong question.”**

**– John Tukey, 1962.**

**That is, economists face a tradeoff between:**

**the numerical errors of computational work  
and  
the specification errors of analytically tractable models.**

**And perhaps also between: sufficiency and necessity.**

## 2. Sufficiency and Necessity

**Simulations demonstrate: existence and sufficiency,  
but not necessity.**

**Simulations can demonstrate the untruth of a proposition,  
but not provide proofs or theorems,  
simulations cannot provide generality.**

**What, never?**

**Does this matter?**

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## Formal Simulation

**Mathematical “model  $A$ ” comprises the conjunction  $(a_1 \wedge a_2 \wedge a_3 \cdots \wedge a_n)$ , where  $\wedge$  means “AND”, and the  $a_i$  denote the elements (equations, parameters, initial conditions, etc) that constitute the model.**

***Sufficiency:* If model  $A$  exhibits the desired target behaviour  $B$ , then model  $A$  is sufficient to obtain exhibited behaviour  $B$ :**

$$A \Rightarrow B$$

**Thus, any model that exhibits the desired behaviour is sufficient, and demonstrates one conjunction of conditions (or model, or solution) under which the behaviour can be simulated.**

**But if there are several such models, how can we choose among them? And what is the necessary set  $\mathcal{N}$  of all such conjunctions (models)?**

## Necessity

**Necessity:** Only those models  $A$  belonging to the set of necessary models  $\mathcal{N}$  exhibit target behaviour  $B$ .

That is,  $(A \in \mathcal{N}) \Rightarrow B$ , and  $(D \notin \mathcal{N}) \not\Rightarrow B$ .

**A difficult challenge:** determine the set of necessary models,  $\mathcal{N}$ .

Since each model is not simple:  $A = (a_1 \wedge a_2 \wedge a_3 \cdots \wedge a_n)$ , searching for the set  $\mathcal{N}$  of necessary models means searching in a high-dimensional space, with no guarantee of continuity, and a possible large number of non-linear interactions among elements.



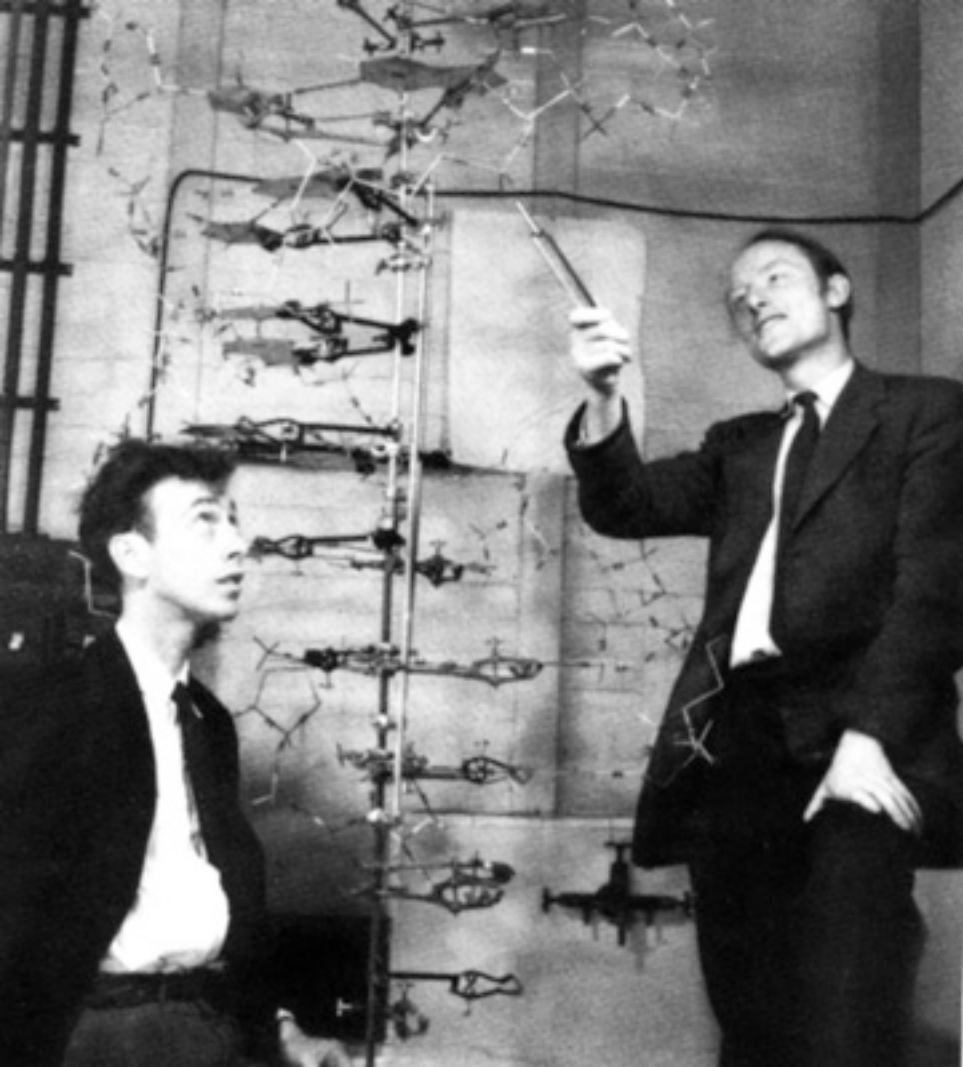
## Lack of Necessity Means ...

For instance, if  $D \not\Rightarrow B$ , it does not mean that all elements  $a_i$  of model  $D$  are invalid or wrong, only their conjunction, that is, model  $D$ .

It might be only a single element  $a_k$  that precludes model  $D$  exhibiting behaviour  $B$ .

But determining whether this is so and which is the offending element  $a_k$  is a costly exercise, in general, for the simulator.

Without clear knowledge of the boundaries of the set  $\mathcal{N}$  of necessary models, it is difficult to generalise from simulations.



## Simulation Can Sometimes Demonstrate Necessity . . .

Only when the set  $\mathcal{N}$  of necessary models is known to be small (such as in the case of DNA structure by the time Watson & Crick were searching for it) is it relatively easy to use simulation to derive necessity.

Watson & Crick had much information about the properties of DNA (from others):

when they hit on the simulation we know as the “double helix”, they knew it was right.

But still “A structure ...”, not “*The structure*” in the title of their 1953 *Nature* paper.

(And Kepler’s 1605 ellipses?)

## **Engineers and Computer Scientists' View**

**Why the fuss?**

**Having several solutions to choose from is a luxury, especially for difficult problems.**

**The question of necessity – are these the only possible solutions? – is not of concern.**

**Economists seek generality of understanding, whereas engineers seek solutions rather than generality.**

### **3. Analysis in the Social Sciences**

**In the social sciences:**

- **“positive” analysis**
- **explanation of existing phenomena, understanding**
- **prediction.**

**In engineering, crudely:**

- **“normative” analysis**
- **solving problems**
- **synthesis**
- **design.**

### 3.1 Simulation and Analysis

**The anecdote about the economist looking for his lost car keys:**

**“An accurate answer to the wrong question”? (using closed-form methods)**

**or: simulation (numerical methods)**

**“Approximate answers to the right questions”**

**Helped by the developments in computer hardware and software.**

**Meanwhile: C.S. has borrowed simulation tools from the natural world:**

**artificial neural nets, simulated annealing, genetic algorithms/programming**

**Want: dynamics, out-of-equilibrium characterisations.**

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## **Verification & Validation – “Assurance”**

**Verification (or internal validity):** is the simulation working as you want it to:

– is it “doing the thing right?”

**Validation:** is the model used in the simulation correct?

– is it “doing the right thing?”

**To Verify:** use a suite of tests, and run them every time you change the simulation code – to verify the changes have not introduced extra bugs.

**See:** D.F. Midgley, Marks R.E., and Kunchamwar D. (2007) *The Building and Assurance of Agent-Based Models: An Example and Challenge to the Field*, *Journal of Business Research*, Special Issue: Complexities in Markets.

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## 4. Validation

**For whom?**

**With regard to what?**

**A good simulation is one that achieves its goals:**

- to explore
- to predict
- to explore

**Or**

- what is?
- what could be?
- what should be?



## Validation

**To the extent that the social sciences are concerned with real-world, historical phenomena,**

**any simulations must be verified (no bugs) and validated (does the model provide behaviour which matches the stylised facts of the historical phenomenon?)**

**Midgley et al: verification + validation = *assurance***

**Back-predictions.**

**Docking.**

**Example: Consider these historical market data:**

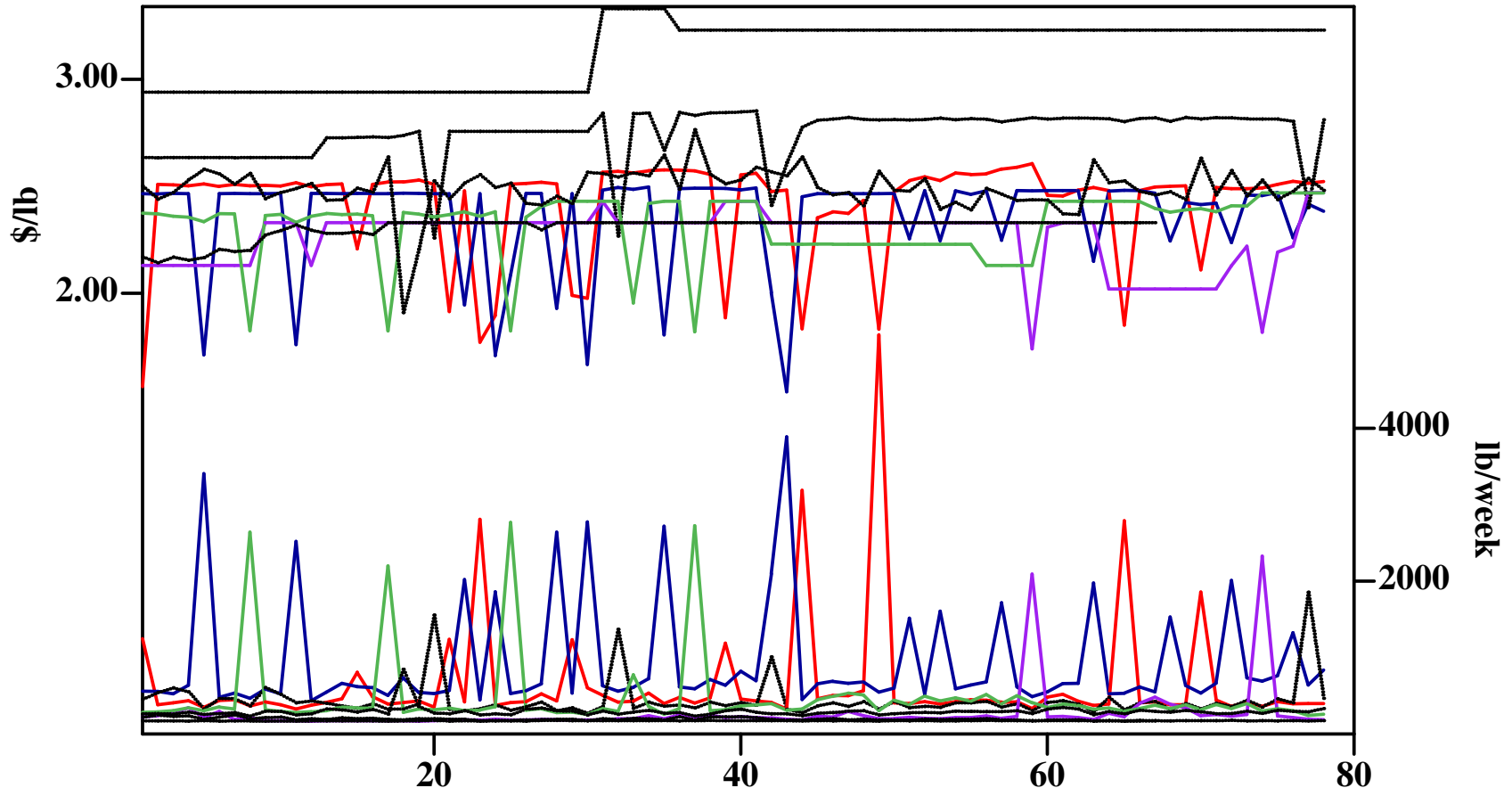


Figure 1: Weekly Sales and Prices (Source: Midgley et al. 1997)

## Real-World Rivalry

**The figure shows the historical prices (in \$/lb) and weekly quantities sold (in pounds) of nine brands of vacuum-sealed, ground coffee in a single mid-west supermarket chain over 78 weeks.**

**The four coloured lines (Folger's, Maxwell House, Chock Full O Nuts, Hills Bros) are the most "strategic" of the brands here, and we focus on their interactions.**

**Later: using GAs, we derive a model in which, constrained by the supermarket chain, the artificial brands vie to maximize their weekly profits, by changing their prices (constant for seven days) and other marketing instruments (coupons, etc.)**

**The issue: how to validate our model, which produces output similar to the figure?**

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## Stylised Facts of the Market Behaviour

- **Much movement in prices and quantities of four brands – a rivalrous dance.**
- **Pattern: high price (and low quantity) punctuated by low price (and high quantity).**
- **Other brands: (relatively) stable prices and quantities**

### Questions:

**What is the cause of these patterns?**

- **shifts in brand demand?**
- **reactions by brands?**
- **actions by the supermarket chain?**
- **unobserved marketing actions?**

**In order to explain, first grow – Epstein.**

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## **Agent-Based Models → Generative Explanation:**

**Generative explanation (Epstein 2006):**

**“If you haven’t grown it, you haven’t explained its emergence.”**

**To answer: how could the autonomous, local interactions of heterogeneous boundedly rational agents generate the observed regularity (that emerges)?**

**– Generative sufficiency is a necessary but not sufficient condition for explanation. Each realisation is a strict deduction.**

**Grüne-Yanoff (2006) argues to distinguish *functional explanations* (easier for simulators) from *causal explanations* (much less achievable for social scientists).**

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## **Explanations?**

**Interactions of profit-maximising agents, plus external or internal factors → via a model → behaviour**

**Similar (qualitatively or quantitatively) to the brands' behaviours of pricing and sales.**

**Note: assuming profit-maximising (or purposeful) agents means that we are not simply curve-fitting or description using D.E.s. Going beyond the rivalrous dance.**

## Further ...

**With a calibrated model, we can:**

**perform sensitivity analysis of endogenous with respect to exogenous variables.**

**Prediction only requires sufficiency, not necessity (“These are the *only* conditions under which the model can work.”)**

**Examine:**

- **limits of behaviour  
(Miller’s Automated Non-linear Testing System)**
  - **regime-switching**
  - **range of behaviour generated**
  - **sensitivity of the aggregate (or emergent behaviour) to a single agent’s behaviour.**
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## Validation

**Moss & Edmonds (2005): for AB models at least two stages of empirical validation.**

- 1. the micro-validation of the behaviour of the individual agents in the model, by reference to data on individual behaviour.**
- 2. macrovalidation of the model's aggregate or emergent behaviour when individual agents interact, by reference to aggregate time series.**

**with the emergence of novel behaviour, possible surprise and possible highly non-standard behaviour, difficult to verify using standard statistical methods.**

**∴ only qualitative validation judgments might be possible.**

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## Formalisation of Validation

Let set  $P$  be the possible range of observed outputs of the real-world system.

Let set  $M$  be the exhibited outputs of the model in any week.

Let set  $H$  be the specific, historical output of the real-world system in any week.

Let set  $Q$  be the intersection, if any, between the set  $M$  and the set  $H$ ,  $Q \equiv M \cap H$ .

We can characterise the model output in several cases.  
(Mankin et al. 1977).

## Five Cases for Validation

- a. no intersection between  $M$  and  $H$  ( $Q = \emptyset$ ), then the model is *useless*.
  - b. intersection  $Q$  is not null, then the model is *useful*, to some degree: will correctly exhibit some real-world system behaviours, will not exhibit other behaviours, and will exhibit some behaviours that do not historically occur. Both incomplete and inaccurate.
  - c. If  $M$  is a proper subset of  $H$  ( $M \subset H$ ) then all the model's behaviours are correct (match historical behaviours), but the model doesn't exhibit all behaviour that historically occurs: accurate but *incomplete*.
  - d. If  $H$  is a proper subset of  $M$  ( $H \subset M$ ) then all historical behaviour is exhibited, but will exhibit some behaviours that do not historically occur: complete but *inaccurate*.
  - e. If the set  $M$  is equivalent to the set  $H$  ( $M \Leftrightarrow H$ ), then (in your dreams!) the model is complete and accurate, but might be overfitted.
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## Or Graphically ...

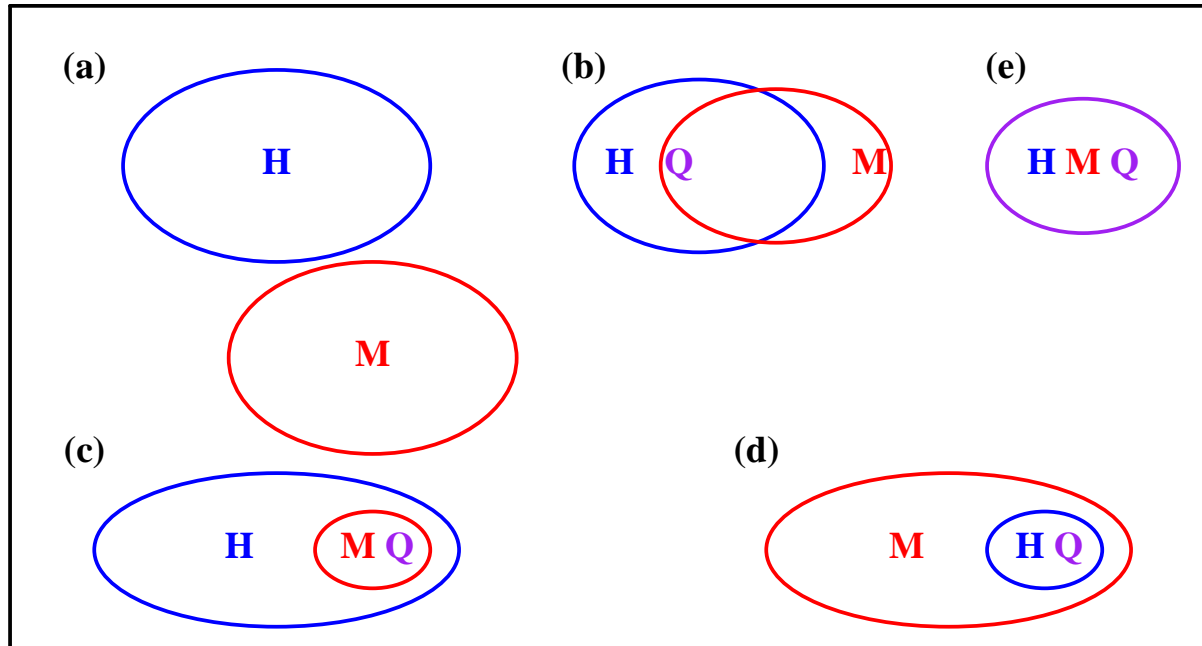


Figure 2: Validity relationships (after Haefner (2005)).

- a. useless
- b. useful, but incomplete and inaccurate
- c. accurate but incomplete
- d. complete but inaccurate ← possibly the best to aim for
- e. complete and accurate

## Modelling Goals

**One goal: to construct and calibrate the model so that**

**$M \approx Q \approx H$ : there are very few historically observed behaviours that the model does not exhibit,**

**and there are very few exhibited behaviours that do not occur historically.**

**The model is close to being both complete and accurate.**

**In practice, a modeller might be happier to achieve case d., where the model is complete (and hence provides sufficiency for all observed historical phenomena), but not accurate.**

**Not least to accommodate later real-world observations.**

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## Measures of Validity

**A measure of validity which balances the Type I error of inaccuracy with the Type II error of incompleteness.**

**Define a metric  $m()$  (a ratio scale) on the sets.**

**Define *inaccuracy*  $\alpha$  as**

$$\alpha \equiv 1 - \frac{m(Q)}{m(M)}, \quad (1)$$

**and *incompleteness*  $\gamma$  as**

$$\gamma \equiv 1 - \frac{m(Q)}{m(H)}. \quad (2)$$

**Or:  $m(|H - M|)$ , as in the SSM (see below).**

**Continued ...**

**A measure of degree of validation  $V$ : a weighted average of inaccuracy  $\alpha$  and incompleteness  $\gamma$ :**

$$V \equiv v(1 - \alpha) + (1 - v)(1 - \gamma) \quad (3)$$

$$\therefore V = v \frac{m(Q)}{m(M)} + (1 - v) \frac{m(Q)}{m(H)}$$

$$\therefore V = m(Q) \left( \frac{v}{m(M)} + \frac{1 - v}{m(H)} \right) \quad (4)$$

**The value of the weight  $v$ ,  $0 \leq v \leq 1$ , reflects the tradeoff between accuracy and completeness.**

## Trade-offs

**Possible to reduce incompleteness by generalising the model and so expanding the domain of set  $M$  until  $H$  is a proper subset of  $M$ , as in case d.**

**Or by narrowing the scope of the historical behaviour to be modelled, so reducing the domain of  $H$  (or  $P$ ).**

**Also be possible to reduce inaccuracy by restricting the model through use of narrower assumptions and so contracting the domain of  $M$ .**

**If  $M$  is sufficiently small to be a proper subset of  $H$ , as in case c., then the model will never exhibit anhistorical behaviour.**

**But not guaranteed to maintain a non-null intersection  $Q$ , and it is possible that the process results in case a., with no intersection.**

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## **Fagiolo et al. on Validation of AB Models**

**AB models can be characterized as:**

- **bottom-up models (unlike e.g. Systems Dynamics simulation models, or closed-form models)**
- **heterogeneous agents (endowments, properties, memory, rationality, etc.)**
- **boundedly rational, usually with adaptive expectations**
- **networked direct interactions.**

**Closer to dynamic, decentralized markets and economies than traditional models.**

**See: Fagiolo G., Moneta A., & Windrum P. (2007), “A critical guide to empirical validation of agent-based models in economics: methodologies, procedures, and open problems,” *Computational Economics*, 30(3): 195–226.**

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## Fagiolo 2

**And yet reluctance to use AB models. Why? Four key problems:**

- 1. no common set of the heterogeneous AB models previously developed**
- 2. (hence) lack of comparability across these models with high degrees of freedom, hence a wide range of outputs, together with lack of necessity.**
- 3. lack of standard techniques for constructing and analyzing AB models**
- 4. the “problematic” relationship between AB models and empirical data ← this is *validation*.**

## Fagiolo 3

### Some questions:

- **Is “realist” methodology appropriate?**
  - **Should empirical validation be the primary basis for accepting/rejecting a model?**
  - **Are there other tests apart from generating stylized facts?**
  - **How should we calibrate the parameters, initial conditions, stochastic variability to historical data?**
  - **How dependable are the micro and macro stylized facts anyway?**
  - **What if the “stylized facts” shed no light on the dynamics of the generating stochastic processes?**
  - **What if the “stylized facts” are too general to distinguish among models?**
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## **Fagiolo 4 – Issues with Empirical Validation**

**Comparing historical data with generated outputs.**

- 1. The world is complex: a trade-off between complexity in modelling (“concretization”) and reductionism (“isolation”): where to draw the line in modelling? Realism v. tractability.**
- 2. Friedman (1953) argued that realism was not necessary so long as the output allowed accurate prediction (“instrumentalism”), but others seek realism in the model and its assumptions as well as accuracy.**
- 3. How wedded should the modeller be to a priori assumptions (about the goals of agents, say)? Or should all aspects of the model be available (“pluralism”)?**
- 4. Importantly: how to choose which of several models is best (the “identification” or “under-determination” problem).**

## **Fagiolo 5**

**Three alternative approaches:**

- 1. the indirect calibration approach**
- 2. the Werker-Brenner approach**
- 3. the history-friendly approach, and**
- 4. the State Similarity Measure (of mine, below).**

## Judd on Validation

### Several suggestions:

**1. Search for counterexamples:**

**If found, then insights into when the proposition fails to hold.**

**If not found, then not proof, but strong evidence for the truth of the proposition.**

**2. Sampling Methods: Monte Carlo, and quasi-Monte Carlo → standard statistical tools to describe confidence of results.**

**3. Regression Methods: to find the “shape” of the proposition.**

**4. Replication & Generalisation: “docking” by replicating on a different platform or language, but lack of standard software an issue.**

**5. Synergies between Simulation and Conventional Theory.**

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## 5. Simulation

### Social Science, not Physical Science

**At the aggregate level, similar.**

**But at the micro level, the agents in social science models are people, with self-conscious motivations and actions.**

**Aggregate behaviour may be well described by differential equations, with little difference from models of inanimate agents at the micro level.**

## **A Third Way of Doing Science**

**(from Axelrod & Tesfatsion 2006)**

**Deduction + Induction + Simulation.**

- **Deduction: deriving theorems from assumptions**
- **Induction: finding patterns in empirical data**
- **Simulation: assumptions → data for inductive analysis**

**S differs from D & I in its implementation & goals.**

**S permits increased understanding of systems through controlled computer experiments**

## Emergence of self-organisation

**Examples: ice, magnetism, money, markets, civil society, prices, segregation.**

**Defn:** **emergent properties** are properties of a system that exist at a higher level of aggregation than the original description of the system.

**Not from superposition, but from interaction at the micro level.**

**Adam Smith's Invisible Hand → prices**

**Schelling's residential tipping (segregation) model:**

**People move because of a weak preference for a neighbourhood that has at least 33% of those adjoining the same (colour, race, whatever) → segregation.**

**Need models with more than one level to explore emergent phenomena.**

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## **Families of Simulation Models**

- 1. System Dynamics SD  
(from differential equations)**
- 2. Cellular Automata CA  
(from von Neumann & Ulam, related to Game Theory)**
- 3. Multi-agent Models MAM  
(from Artificial Intelligence)**
- 4. Learning Models LM  
(from Simulated Evolution and from Psychology)**

## Comparison of Simulation Techniques

**Gilbert & Troitzsch compare these (and others):**

<b>Technique</b>	<b>Number of Levels</b>	<b>Communication between agents</b>	<b>Complexity of agents</b>	<b>Number of agents</b>
<b>SD</b>	<b>1</b>	<b>No</b>	<b>Low</b>	<b>1</b>
<b>CA</b>	<b>2+</b>	<b>Maybe</b>	<b>Low</b>	<b>Many</b>
<b>MAM</b>	<b>2+</b>	<b>Yes</b>	<b>High</b>	<b>Few</b>
<b>LM</b>	<b>2+</b>	<b>Maybe</b>	<b>High</b>	<b>Many</b>

**Number of Levels: “2+” means the technique can model more than a single level (the individual, or the society) and the interaction between levels.**

**This is necessary for investigating emergent phenomena.**

**So “agent-based models” excludes Systems Dynamics models, but can include the others.**

## **Axelrod on Model Replication and “Docking”**

***Docking***: a simulation model written for one purpose is aligned or “docked” with a general purpose simulation system written for a different purpose.

**Four lessons:**

- 1. Not necessarily so hard.**
- 2. Three kinds of replication:**
  - a. numerical identity**
  - b. distributional equivalence**
  - c. relational equivalence**
- 3. Which null hypothesis? And sample size.**
- 4. Minor procedural differences (e.g. sampling with or without replacement) can block replication, even at (b).**

## Reasons for Errors in Docking

1. **Ambiguity in published model descriptions.**
2. **Gaps in published model descriptions.**
3. **Errors in published model descriptions.**
4. **Software and/or hardware subtleties.**  
**e.g. different floating-point number representation.**

**(See Axelrod 2006.)**



## ***AGENT-BASED MODELS***

**AB Models are used where the interactions are decentralised, and the autonomous agents make their own decisions (perhaps constrained).**

- .: AB models are suitable for interactions which are *bottom-up*, not *top-down*.**
  
- .: social and market interactions, rather than engineering or internal organisational interactions.**

## Using AB models

**In ABM/ACE models, a population of software objects is:**

- instantiated, and each agent is given:**
  - certain internal states (e.g., preferences, endowments) and**
  - rules of behaviour (e.g., seek utility improvements).**

**The agents are then permitted to interact directly with one another and a macrostructure emerges from these interactions.**

## Patterns Emerge

**Patterns in this macrostructure may then be (Axtell, 2005):**

- compared with empirical data,**
- to revise agent internal states and rules, and**
- the process repeated until an empirically plausible model obtains.**

**e.g. ACE stock markets have been used to model heterogeneous agents: will the stylised features of such markets emerge? Yes.**

## What is an Agent?

**An agent: a self-centred program that controls its own actions based on its perceptions of its operating environment.**

**Derived from the Distributed AI notion of a network of calculating nodes.**

**Example: the automata in Conway's Game of Life or Schelling's Segregation game or the couples in March & Lave's Sons and Daughters game..**

**Another example of an agent that won \$2,000,000 in a challenge by the U.S. Department of Defense in October 2005**

...

**Stanley here.**

## Agents and agency

**Wooldridge & Jennings (1995) would give computer agents these properties:**

- **autonomy: no others control their actions and internal state,**
- **social ability: can interact and communicate with other agents**
- **reactive: they perceive their environment and respond**
- **pro-active: they initiate goal-directed actions**
- **(intentionality: metaphors of beliefs, decisions, motives, and even emotions)**

## Further agent features:

plus (Epstein 1999):

- **heterogeneity: not “representative” but may differ**
- **local interactions: in a defined space**
- **boundedly rational (Simon): information, memory, computational capacity**
- **non-equilibrium dynamics: large-scale transitions, tipping phenomena**

## **Eight Desired Attributes of Modelled Agents (G&T)**

### **1. Knowledge & beliefs.**

**Agents act based on their knowledge of the environment (including other agents), which may be faulty – their beliefs, not true knowledge.**

### **2. Inference.**

**Given a set of beliefs, an agent might infer more information.**

### **3. Social models.**

**Agents, knowing about interrelationships between other agents, can develop a “social model”, or a topology of their environment: who’s who. etc.**

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## **Eight Desired Attributes ...**

### **4. Knowledge representation.**

**Agents need a representation of beliefs: e.g. predicate logic, semantic (hierarchical) networks, Bayesian (probabilistic) networks.**

**[Sebastian] Thrun [leader of the winning team in the 2005 DARPA Grand Challenge] had a Zen-like revelation: “A key prerequisite of true intelligence is knowledge of one’s own ignorance,” he thought. Given the inherent unpredictability of the world, robots, like humans, will always make mistakes. So Thrun pioneered what’s known as probabilistic robotics. He programs his machines to adjust their responses to incoming data based on the probability that the data are correct. — Pacella (2005).**

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## **Eight Desired Attributes ...**

### **5. Goals.**

**Agents driven by some internal goal, e.g. survival, and its subsidiary goals (food, shelter). Usually definition and management of goals imposed on the agent.**

### **6. Planning.**

**Agent must (somehow) determine what actions will attain its goal(s). Some agents modelled without teleology (simple trial-and-error), others with inference (forward-looking), or planning.**

### **7. Language.**

**For communication (of information, negotiation, threats). Modelling language is difficult. (Want to avoid inadvertent communication, e.g. through the genome of a population in the GA.)**

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- 8. Emotions.  
Emergent features? Significant in modelling agents? Or  
epiphenomenal?**

## How to Model Agent Architecture?

**Early approach to modelling cognitive abilities (symbolic paradigm) was fragile, complex, and lacked common sense.**

**Since then, five approaches:**

- 1. Production Systems**
- 2. Object Orientation**
- 3. Language Parsing & Generation**
- 4. Machine-Learning Techniques, and (most recently)**
- 5. Probabilistic Robotics – Stanley (Thrun et al. 2005).**

**Ignore 3., 4. last lecture, 5. too new.**

***Economic Journal* June 2005 Feature —**

- **focussed on Complex Adaptive Systems CAS in economics**
  - **appeared just after Leombruni & Richiardi asked, “Why are economists sceptical about agent-based simulations?” (*Physica A* 355: 103–109, 2005.)**
  - **included 4 papers: introduced by Markose, with papers by Axtell, Robson, and Durlauf**
  - **addressing, respectively,**
    - **markets as complex adaptive systems,**
    - **formal complexity issues,**
    - **the co-evolutionary “Red Queen” effect and novelty, and**
    - **the empirical and testable manifestations of CAS in economic phenomena.**
-

## Markose and the *EJ* Feature on CAS:

- many “anomalies” not understood or modelled using conventional optimisation economics:
    - innovation,
    - competitive co-evolution,
    - persistent heterogeneity,
    - increasing returns,
    - “the error-driven processes behind market equilibrium,”
    - herding,
    - crashes and extreme events such as October 1987.
  - need the “adaptive or emergent methods” of ACE simulation
-

## Moreover ...

**Axtell (2005) argues that:**

- **the decentralised market as a whole can be seen as a collective computing device**
- **the parallel distributed agent-based models of  $k$ -lateral exchange → the specific level of complexity (polynomial) in calculations of equilibrium prices and allocations.**

## 6. Economists' Uses of Multi-Agent Models

**Economic theory moved beyond the reduced forms of perfect competition and monopolies**

**to oligopolies – markets with small numbers of sellers:**

**oligopolists interact strategically (not amenable to reduced forms – such as “market clearing”)**

**strategic interactions best modelled by game theory.**

**But Nash equilibrium, although not reduced – all competitors' behaviours are inputs – but focussed on the *equilibrium*,**

**while most transactions take place off-equilibrium**

**Further: real-world firms are heterogeneous (“asymmetric”)**

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## **First Multi-Agent Systems in Economics**

**At first, to deal with the non-convexities and discontinuities of the strategy space, agent-based simulation techniques were used to search for equilibrium results, Marks and others used Genetic Algorithms (GAs)**

**to search a strategy space that was rugged, and non-stationary, when the problem was co-evolutionary, as the other firms also searched for “better” mappings from market state to action.**

**Coevolution of asymmetric firms requires separate populations in the GA, lest extra-market communication occur: collusion is generally illegal.**

**“Social” learning v. “individual” learning – Vriend**

**Coevolution was different from most engineering applications of the GA:**

**which looked for numerical optima, not the characteristics of the final generation’s population.**

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## **Agents and the Market Interact**

**Other AB simulations in economics have used agents that “learnt”,  
as opposed to the GA’s “social” learning of successive generations of agents.**

**How good do agents have to be?**

**Gode & Sunder (1993) found — not very, at least for the “double action” market, in which “zero-intelligence players” (ZIPS, who tossed coins or dice) do as well (or even better) than highly rational players (as game theory assumes).**

**We’re still not sure what it is about the structure and rules of the DA market that help the ZIPs.**

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## 7. **Synthesis in the Social Sciences**

**Designer markets are exemplified by**

- 1. derivatives markets, in general**
- 2. markets for pollution permits**
- 3. auctions for electro-magnetic spectrum**
- 4. markets for the trade of electricity, and**
- 5. on-line or automated markets**
  
- 6. Also: contract design.**

## Methods to Help Designing Markets

**Roth (1991) spoke of:**

- **traditional closed-form game-theoretic analysis**
- **human-subject experiments**
- **computer simulations**

**But Mirowski argues for a typology of market types, perhaps begun in McMillan (2002)**

## 7.1 Designing Markets

**Designing markets is complex:**

**searching for a mapping —**

- from genotype to phenotype**
- from design (structure & rules) space to performance space**
- from genotype to phenotype (in evolution)**

**In general this is complex (but see Byde 2006):  
“syntactic complexity” (Edmonds & Bryson 2003). which  
requires the performance behaviour to “emerge”, as Simon  
(1996) agreed.**

## **A Framework ...**

### **Mackie-Mason & Wellman's Marketplace Design Framework:**

#### **Three fundamental steps of a market transaction:**

- 1. the connection (search, discovery)**
- 2. the deal (negotiating, agreeing)**
- 3. the exchange (execution of the transaction).**

#### **∴ two design decisions:**

- 1. the market mechanism (mechanisms for connection, deal, and exchange)**
- 2. the agents**

**Mirowski argues that economists have focussed on agents and ignored market mechanisms, except in reduced form (“market-clearing”)**

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## Design Tradeoffs

**For simulation, tradeoffs among possible goals of modelling and simulation must be explicit.**

**LeBaron's seven basic design questions:**

- 1. the economic environment**
- 2. modelling agents' preferences ← agents**
- 3. price formation and market clearing**
- 4. the model's fitness**
- 5. information processing and communication ← agents**
- 6. learning: individual, or social ← agents**
- 7. benchmarking**

## 8. Similarities and Differences

**In 1978 I wrote a Ph.D. thesis, the title of which included the phrase: “disequilibrium dynamics”.**

**It allowed exchange to take place out of equilibrium – of course, you might say, how else is market-clearing to be attained?**

**But then it was seen as strange: although there was a short-lived literature along similar lines – Malinvaud, Barro & Grossman, and others.**

**But this focus on out-of-equilibrium dynamics is precisely what we now need as we simulate using market-based controls.**

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## **Simon's Bounded Rationality**

**Agent-based models, following Simon (1982), also assume Bounded Rationality. Indeed, in the absence of Turing machine (universal calculator), it is difficult not to.**

**But Epstein (2006) reflects:**

**“One wonders how the core concerns and history of economics would have developed if, instead of being inspired by continuum physics ... blissfully unconcerned as it is with effective computability – it had been founded on Turing. Finitistic issues of computability, learnability, attainment of equilibrium (rather than mere existence), problem complexity, and undecidability, would then have been central from the start. Their foundational importance is only now being recognized.**

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## Epstein on the virtues of boundedly rational agents ...

**“As Duncan Foley summarizes:**

**`The theory of computability and computational complexity suggest that there are two inherent limitations to the rational choice paradigm.**

**One limitation stems from the possibility that the agent’s problem is in fact undecidable, so that no computational procedure exists which for all inputs will give her the needed answer in finite time.**

**A second limitation is posed by computational complexity in that even if her problem is decidable, the computational cost of solving it may in many situations be so large as to overwhelm any possible gains from the optimal choice of action.’ (See Albin 1998, 46).”**

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## **Truth and Beauty**

**Epstein (2006): does AB simulation lack beauty?**

**Russell: Mathematics as cold, austere, supreme beauty.**

**Russell: Beauty when “the premises achieve more than would have been thought possible, by means which appear natural and inevitable.”**

**The first damns computer simulation, but the second can occur with emergence from AB models.**

**Epstein compares different schools of classical music: German v. French.**

**Truth (from agent-based modelling) can be beautiful too.**

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## Formalisation of Agent-Based Models

**Epstein (2006): every agent model is a computer program.**

**∴ Turing computable**

**But for every Turing machine,  $\exists$  a unique corresponding and equivalent *partial recursive function*.**

**They might be extremely complex and difficult to interpret, but they exist.**

**Hence: "recursive" or "effectively computable" or "constructive" or "generative" (after Chomsky) social science.**

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