Software Piracy:

A Strategic Analysis and Policy Instruments

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Abstract

We examine the government's role in restricting commercial piracy in a software market. Welfare maximization may or may not result in monitoring as the socially optimal outcome. Correspondingly, either monopoly situation or market sharing between an original firm and a pirate are subgame perfect equilibria. If it is profitable for a monopolist to prevent piracy by installing a protective device, then not monitoring is the equilibrium. We also discuss the effects of network externalities, in addition to deriving the effects of changes in the reliability of the pirated software and network benefits on the policy variables, the extent of piracy, and the monopolist's incentive to prevent piracy.

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1. INTRODUCTION

The existing literature on software piracy addresses the issue from the viewpoint of piracy by end-users and the importance of network effects in protecting the software industry against piracy. Chen and Png (1999) show that it is better for a firm to deal with piracy by end-users through pricing rather than monitoring. Cheng, Sims and Teegen (1997) and Noyelle (1990) mention that the high price of software is the dominant reason for piracy. Shy and Thisse (1999) show that for strong network effects, no protection against piracy is an equilibrium for a noncooperative software industry. Earlier research by Takeyama (1994), Conner and Rumelt (1991), and Nascimento and Vanhonacker (1988) also discusses the role of network effects on the marketing of software. However, the issue of government policy towards piracy has hitherto not been addressed.

There is great variation in the piracy rates, defined as the ratio of the number of pirated copies to total installed copies, across countries. For example, in 1997, the piracy rates range from 27 percent in the United States to 98 percent in Vietnam.[1] These figures include copying by end-users and the sale of pirated software. An important implication is that the software market in some countries is very close to a monopoly while in others there is a varying degree of market sharing between an original firm and pirates, who offer unauthorized reproductions of licensed software, commercially, to compete with the original software. The existence of such a spectrum of market structures may be due to differences in the governments' stance towards piracy, and the resources required in implementing their policies.

In this paper, we examine the government's role in controlling piracy through its choice of policy instruments. We consider a market in which there is an original software firm, hereafter referred to as the monopolist, and there is also software counterfeiting by a pirate who offers unauthorized reproductions of licensed software, commercially, to compete with the monopolist's software. In such a market the monopolist, who may not be locally headquartered, may even be perceived with some hostility and the government may ignore the monopolist's incentive to innovate and place greater value on the short run benefits associated with piracy.

The domestic social-welfare maximizing policy instruments, which consist of monitoring and penalizing the pirate's illegal operations, endogenously determine whether or not there will be market sharing between the monopolist and the pirate. The optimal choices of policy variables reflect the government's attitude towards piracy. We also study the monopolist's role in preventing piracy through installing a technical protective device that prevents copying. We then discuss the effects of network externalities on the optimal policy instruments, the extent of piracy, and the monopolist's incentive to prevent piracy. A network externality means that a consumer's utility from using software increases with an increase in the number of other consumers using the same software. They benefit through the exchange of files using the same software.

In our model there is a heterogeneous group of consumers with different valuations of the software. The reliability of the pirated software is the only difference between the original software and the former. We analyze three pricing games – *Bertrand*, *leader-follower*, and *monopoly pricing*. In the monopoly pricing

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[1] See IPR (1998)

game the monopolist charges the pre-entry monopoly price ignoring the threat of entry by the pirate. We include this game to study the policy variables that may restore the monopoly outcome. Assuming that monitoring is costly, the government chooses the policy variables that maximize domestic social-welfare subject to balanced budget constraints. The latter is assumed to avoid issues of redistribution. In the case of net revenue maximization the surplus is distributed among the population through some transfer mechanism.

Welfare maximization may or may not result in monitoring as the socially optimal outcome. Correspondingly, the monopoly situation and the leader-follower outcome are the two subgame perfect equilibria. The result depends on the monitoring technology reflected in the monitoring cost, which is a deadweight loss, and other parameters such as the size of the market and the reliability factor. So the social-welfare maximizing objective determines the government's aggressiveness or passiveness towards piracy, which in turn determines the optimal market structure.

The monopolist can also prevent piracy by installing a protective device, as long as the net monopoly profits from doing so exceed profits when the market is shared. In this case not monitoring is the equilibrium. Intuitively, if the monopolist can prevent piracy, the government does not need to monitor.

The optimal monitoring rate that results in the monopoly outcome is higher when there are network externalities. It also results in an increase in the extent of piracy and the monopolist's incentive to prevent piracy. Comparative static results show that changes in the reliability of the pirated software and network benefits have positive effects on piracy, the monopolist's incentive to protect his software, and the optimal monitoring rate that results in the monopoly outcome.

The paper is arranged as follows. In section 2 we discuss the model, the different pricing games and the equilibrium government policies. In section 3 we examine the effect of network externalities. In section 4 we provide some concluding remarks.

2. THE MODEL WITHOUT NETWORK EXTERNALITIES

We consider four types of agents: the consumers, the developer of an original packaged software, referred to as the monopolist, a pirate who illegally reproduces and sells licensed software, and the government which is responsible for monitoring and penalizing the pirate. We begin our analysis by describing the monopoly situation in the absence of piracy.

There is a continuum of consumers indexed by $\theta, \theta \in [\theta_l, \theta_h]$. θ is assumed to follow a uniform distribution. We assume there is no resale market for used software. Each consumer is assumed to purchase only one unit of the software. Following Tirole (1988), the utility of a type θ consumer from purchasing a unit of the software is,

$$U(\theta) = \begin{cases} \theta - p_m & \text{if the consumer buys the software,} \\ 0 & \text{if the consumer does not buy.} \end{cases} \tag{1}$$

θ is the valuation of the consumer and p_m is the price of one unit of the software charged by the monopolist. Thus, in the model, consumers differ from one another on the basis of their valuation of the software. The heterogeneity of the consumers, represented by the different magnitudes of θ, can be interpreted as a function of factors like software usage frequency, degree of utilization, user proficiency, and so on. Higher magnitudes of each of these factors are indexed by higher values of θ.

θ_m is the marginal consumer who is indifferent between buying and not buying:

$$U(\theta_m) = \theta_m - p_m = 0 \Rightarrow \theta_m = p_m . \tag{2}$$

In the absence of piracy, the monopolist faces the demand function,

$$D_m(p_m) = \int_{\theta_m}^{\theta_h} \frac{1}{\theta_h - \theta_l} d\theta . \tag{3}$$

We treat the cost incurred by the monopolist to develop the software as a sunk cost. The cost of replicating the software after it has been developed is assumed to be zero. Hence, the expected profit of the monopolist is the total revenue, which is $\pi_m = p_m D_m$. The consumer surplus is $CS = \int_{\theta_m}^{\theta_h} (\theta - p_m) d\theta$. The equilibrium monopoly results are,

$$p_m^* = \frac{\theta_h}{2},\ \theta_m^* = \frac{\theta_h}{2},\ \pi_m^* = \frac{\theta_h^{\ 2}}{4(\theta_h - \theta_l)}, \text{ and } CS^* = \frac{\theta_h^{\ 2}}{8}. \tag{4}$$

Now, assume that a pirate exists in the market. We further assume that with the advent of digital technology, and in-built documentation and software support, identical copies can be made from the original software with a negligible loss in quality. The cost of duplicating is assumed to be zero.

The difference between the original and the pirated software lies purely in the risk of defect for a pirated copy. Since the pirate operates in the market using a makeshift arrangement, if the pirated software turns out to be defective, there is no chance of getting the defective software replaced. The pirated software is operational with a probability $q, q \in (0,1)$, where q is given exogenously.[2] With probability $(1-q)$ any particular unit turns out to be defective in which case the buyer loses the

[2] We set this bound to ensure that the profits are not indeterminate.

price because the pirated copy is not under warranty.[3] q is a measure of the reliability of the pirated version of the software and is assumed to be common knowledge. We assume that the original product receives full warranty.

The utility of a type θ consumer is,

$$U(\theta)=\begin{cases}\theta-p_m & \text{if the consumer buys the original software,}\\ q\theta-p_c & \text{if the consumer buys the pirated software,}\\ 0 & \text{if the consumer does not buy.}\end{cases} \tag{5}$$

p_c and $q\theta$ is the price and effective valuation of the pirated copy. There are two marginal consumers. The marginal buyer θ_c is indifferent between buying the original and the pirated software:

$$\theta_c-p_m=q\theta_c-p_c\Rightarrow\theta_c=\frac{p_m-p_c}{(1-q)}. \tag{6}$$

The marginal buyer θ_x is indifferent between buying from the pirate and not buying at all:

$$q\theta_x-p_c=0\Rightarrow\theta_x=\frac{p_c}{q}. \tag{7}$$

The demand faced by the monopolist and the pirate is given by (8) and (9).

$$D_m(p_m,p_c)=\int_{\theta_c}^{\theta_h}\frac{1}{\theta_h-\theta_l}d\theta=\frac{\theta_h-\dfrac{p_m-p_c}{(1-q)}}{\theta_h-\theta_l}. \tag{8}$$

$$D_c(p_m,p_c)=\int_{\theta_x}^{\theta_c}\frac{1}{\theta_h-\theta_l}d\theta=\frac{qp_m-p_c}{(\theta_h-\theta_l)(1-q)q}. \tag{9}$$

We assume that the market for software is quite large and is not fully covered, i.e., $D_m(p_m,p_c)+D_c(p_m,p_c)<1$, $D_m(p_m,p_c)>0$, $D_c(p_m,p_c)>0$, (Wauthy, 1996). The consumer surplus is,

[3] The utility from buying the pirated software is $U(\theta)=q(\theta-p_C)-(1-q)p_C=q\theta-p_C$.

$$CS = \int_{\theta_c}^{\theta_h} (\theta - p_m) d\theta + \int_{\theta_x}^{\theta_c} (q\theta - p_c) d\theta. \tag{10}$$

Let us now discuss the policy variables. The government only works through the supply side in controlling piracy. Users do not face the risk of prosecution from the use of pirated software. The government is responsible for monitoring and penalizing the pirate. Let α and G be the monitoring rate and the penalty. Let $c(\alpha)$ be the cost of monitoring. We assume $c(0) = 0, c'(\alpha) > 0, c'(0) = 0, c''(\alpha) > 0.$ The government chooses α and G to maximize domestic social-welfare subject to a balanced budget constraint. We assume this to avoid issues of redistribution that are associated with maximization of net revenue.

We assume that a firm remains in the market only if it is making nonzero profit. If the pirate's illegal operations are detected, which occurs with probability α, he has to pay the penalty G. The expected profits of the original firm and the pirate are,

$$\pi_m(p_m, p_c) = p_m D_m(p_m, p_c) = \frac{p_m}{(\theta_h - \theta_l)}(\theta_h - \frac{p_m - p_c}{(1-q)}), \tag{11}$$

$$\pi_c(p_m, p_c) = (1-\alpha) p_c D_c(p_m, p_c) - \alpha G = \frac{(1-\alpha) p_c}{(\theta_h - \theta_l)} \frac{(q p_m - p_c)}{q(1-q)} - \alpha G. \tag{12}$$

Let R be the net expected revenue of the government.

$$R = \alpha G - c(\alpha). \tag{13}$$

The balanced budget constraint means $R = 0$. This implies that the penalty equals the average cost of monitoring:

$$G = \frac{c(\alpha)}{\alpha}, \text{ for } \alpha > 0. \tag{14}$$

In the absence of monitoring, the penalty is irrelevant. So we assume $G = 0$ if $\alpha = 0$.

G is an increasing function of α. By assumption, the marginal cost of monitoring increases with monitoring. So the average cost of monitoring also increases with monitoring. The social welfare is the sum of the profits of the original developer and the pirate, and consumer surplus. Using (14) the social welfare can be written as,

$$SW(\alpha) = \pi_m(p_m, p_c) + \pi_c(p_m, p_c) + CS =$$

$$\frac{p_m((1-q)\theta_h - (p_m - p_c))}{(1-q)(\theta_h - \theta_l)} + \frac{p_c(1-\alpha)(qp_m - p_c)}{q(1-q)(\theta_h - \theta_l)} - c(\alpha) + CS\,. \tag{15}$$

From (15) it is evident that the monitoring cost is a deadweight loss.

The game played between the government, the monopolist and the pirate is specified in extensive form as follows.

Stage 1: Government announces α and G that maximizes social-welfare subject to balanced budget constraint.

Stage 2: The monopolist observes the policy variables. He then decides whether to choose his price simultaneously with the pirate (*Bertrand* game), or to move first and choose a price. In the latter case he can charge the monopoly price, which we call the *monopoly pricing game*, thus ignoring the fact that a pirate may enter the market, or to set a price taking into consideration that a pirate may enter the market, which we call the *leader-follower game.* The monopolist can choose to move first being the original developer of the software. The pirate acts accordingly.

In the next subsection we only discuss the monopoly pricing and the leader-follower subgames.[4] We then determine the equilibrium policy variables and the subgame perfect equilibrium pricing game in subsection 2.2.

[4] We relegate the discussion of the Bertrand game to Appendix A. Later we show that the monopolist will always choose to move first as he earns a higher profit by moving first rather than simultaneously.

2.1. PRICING SUBGAMES

In this section we analyze the leader-follower (*lf*), and the monopoly pricing (*mp*) subgames. We derive the equilibrium prices, the equilibrium market share of both players, equilibrium profits, and the consumer surplus in each game.

LEADER-FOLLOWER GAME

In the *lf* game, the monopolist takes into consideration that a pirate will enter the market, and, therefore, incorporates the reaction function[5] of the pirate, into its profit function and then chooses the profit-maximizing price. The results of this game are summarized in Proposition 1. The proof is provided in Appendix A. Before discussing the results of the *lf* game let us mention an important property concerning the monitoring rate, which is used in Proposition 1.

Lemma 1

$x = \dfrac{c(\alpha)}{1-\alpha}$ is an increasing function of α, $0 \le \alpha < 1$.

The proof of Lemma 1 follows from the fact that $\dfrac{dx}{d\alpha} > 0$.

Proposition 1

(i) The no piracy condition is $\alpha \ge \alpha_1$ where $\dfrac{\alpha_1 G_1}{(1-\alpha_1)} = \dfrac{c(\alpha_1)}{1-\alpha_1} = \dfrac{(1-q)q\theta_h{}^2}{4(2-q)^2(\theta_h - \theta_l)}$.

The equilibrium results are, $p_m^{lf*} = \dfrac{(1-q)\theta_h}{(2-q)}$, $\theta_c^{lf*} = \dfrac{\theta_h}{2}$, $\pi_m^{lf*} = \dfrac{(1-q)\theta_h{}^2}{2(2-q)(\theta_h - \theta_l)}$,

and $\pi_c^{lf*} = 0$.

(ii) A necessary condition for piracy is $\alpha < \alpha_1$. The equilibrium prices and the marginal consumers are, $p_m^{lf*} = \dfrac{(1-q)\theta_h}{(2-q)}$, $p_c^{lf*} = \dfrac{(1-q)q\theta_h}{2(2-q)}$, $\theta_c^{lf*} = \dfrac{\theta_h}{2}$, and

$\theta_x^{lf*} = \dfrac{(1-q)\theta_h}{2(2-q)}$. The market is uncovered at the equilibrium if $\dfrac{\theta_h}{\theta_l} > \dfrac{(4-2q)}{(1-q)}$. The profits of the monopolist and the pirate are:

$$\pi_m^{lf*} = \frac{(1-q)\theta_h^{\ 2}}{2(2-q)(\theta_h - \theta_l)}, \tag{16}$$

$$\pi_c^{lf*} = \frac{(1-\alpha)(1-q)q\theta_h^2}{4(2-q)^2(\theta_h - \theta_l)} - \alpha G. \tag{17}$$

The pirate's profit is $\pi_c^{lf*} = \dfrac{(1-\alpha)(1-q)q\theta_h^{\ 2}}{4(2-q)^2(\theta_h - \theta_l)} - \alpha G$. α_1 and G_1 are the policy variables that satisfy $\pi_c^{lf*} = 0$ in which case the pirate cannot enter the market. These can be interpreted as the "minimalist,, policy variables that deter the pirate's entry. Using $\pi_c^{lf*} = 0$ we get,

$$\frac{(1-q)q\theta_h^{\ 2}}{4(2-q)^2(\theta_h - \theta_l)} = \frac{\alpha_1 G_1}{(1-\alpha_1)}. \tag{18}$$

From the balanced budget constraint we know that $\dfrac{\alpha G}{(1-\alpha)} = \dfrac{c(\alpha)}{1-\alpha}$. The pirate cannot enter if $\dfrac{c(\alpha)}{1-\alpha} \geq \dfrac{c(\alpha_1)}{1-\alpha_1}$. From Lemma 1 we know that $\dfrac{c(\alpha)}{1-\alpha}$ is an increasing function of α. So the pirate cannot enter if $\alpha \geq \alpha_1$. The pirate enters if $\alpha < \alpha_1$ and there is piracy in equilibrium because $\theta_c^{lf*} - \theta_x^{lf*} = \dfrac{\theta_h}{2(2-q)} > 0$.

Using (10) the consumer surplus is,

$$CS^{lf*} = \begin{cases} \dfrac{(4+q-q^2)\theta_h^{\ 2}}{8(2-q)^2} & \text{if } \dfrac{\alpha G}{1-\alpha} < \dfrac{\alpha_1 G_1}{1-\alpha_1}, \\ \dfrac{(2+q)\theta_h^{\ 2}}{8(2-q)} & \text{otherwise.} \end{cases} \tag{19}$$

[5] See equation (A2) in the Appendix.

The pirate cannot enter if $\alpha \geq \alpha_1$ and only the monopolist serves the market. This explains the consumer surplus described in (19).

<u>MONOPOLY PRICING GAME</u>

Let us analyze the case where the original firm charges the pre-entry monopoly price, $p_m^{mp*} = p_m^* = \frac{\theta_h}{2}$, ignoring the possibility of a pirate's entry. We discuss this case in order to examine the policy variables that deter the pirate's entry and maintain the monopoly outcome described in (4). The pirate observes this price and incorporates it into its reaction function. We introduce the concepts of *partial crowding-out* and *complete crowding-out.* Partial crowding-out means that the pirate partially captures the market of the monopolist. Some of the consumers switch from buying the original product to buying the pirated product. Complete crowding-out means that the pirate captures the entire market. The results of this game are summarized in Proposition 2. The proof is contained in Appendix A.

<u>Proposition 2</u>

(i) If $q < \frac{2}{3}$, a necessary condition for partial crowding-out is $\alpha < \alpha_2$,

α_2 satisfies $\frac{\alpha_2 G_2}{1-\alpha_2} = \frac{c(\alpha_2)}{1-\alpha_2} = \frac{q\theta_h^2}{16(1-q)(\theta_h - \theta_l)}$. The market is uncovered at the equilibrium when $\frac{\theta_h}{\theta_l} > 4$. The equilibrium is characterized by, $p_m^{mp*} = p_m^* = \frac{\theta_h}{2}$,

$p_c^{mp*} = \frac{q\theta_h}{4}$, $\theta_c^{mp*} = \frac{(2-q)\theta_h}{4(1-q)} > \theta_{\mathrm{m}}^*, \theta_c^{mp*} > \theta_x^{mp*} = \frac{\theta_h}{4}$,

$$\begin{aligned} \pi_m^{mp*} &= \frac{(2 - 3\mathrm{q})\theta_{\mathrm{h}}^2}{8(1 - \mathrm{q})(\theta_{\mathrm{h}} - \theta_l)}, \\ \pi_c^{mp*} &= \frac{(1-\alpha)q\theta_h^2}{16(1-q)(\theta_h - \theta_l)} - \alpha G. \end{aligned} \tag{20}$$

(ii) If $q \geq \frac{2}{3}$, a necessary condition for complete crowding-out is $\alpha < \alpha_3$,

α_3 satisfies $\frac{\alpha_3 G_3}{1-\alpha_3} = \frac{c(\alpha_3)}{1-\alpha_3} = \frac{3q\theta_h{}^2}{16(\theta_h - \theta_l)}$. The market is uncovered at the

equilibrium when $\frac{\theta_h}{\theta_l} > 4$. The equilibrium is characterized by, $p_m^{mp*} = p_m^* = \frac{\theta_h}{2}$,

$p_c^{mp*} = \frac{q\theta_h}{4}$, $\theta_c^{mp*} = \theta_h, \theta_x^{mp*} = \frac{\theta_h}{4}$,

$$\pi_m^{mp*} = 0,$$
$$\pi_c^{mp*} = \frac{(1-\alpha)3q\theta_h^2}{16(\theta_h - \theta_l)} - \alpha G. \tag{21}$$

(iii) The monopoly results hold if $q < \frac{2}{3}$ and $\alpha \geq \alpha_2$ or if $q \geq \frac{2}{3}$ and $\alpha \geq \alpha_3$.

α_2 and G_2 (or α_3 and G_3) are the "minimalist„ policy variables that deter the pirate's entry and lead to the monopoly outcome if $q < \frac{2}{3}$ (or $q \geq \frac{2}{3}$). The necessary conditions in Proposition 2 (i) and (ii) are obtained using the properties of α in Lemma 1.

From (20) we see that the monopolist earns positive profit if $q < \frac{2}{3}$. He earns monopoly profits only if $\alpha \geq \alpha_2$, otherwise the market is shared. From the equilibrium values of the marginal consumers in Proposition 2 (i) we see that some of the consumers switch from buying the original software to buying the pirated software, $\theta_c^{mp*} > \theta_{\mathrm{m}}^*$. There is piracy because $\theta_c^{mp*} > \theta_x^{mp*}$. From (20) we further see that the monopolist does not exist in the market if $q \geq \frac{2}{3}$ and $\alpha < \alpha_3$. The pirate captures the entire market.

The consumer surplus in this game is,

$$CS^{mp*} = \begin{cases} \dfrac{\theta_h^2}{8} & \text{if } q < \dfrac{2}{3} \text{ and } \alpha \geq \alpha_2, \\ \dfrac{(4-3q)\theta_h^2}{32(1-q)} & \text{if } q < \dfrac{2}{3} \text{ and } \alpha < \alpha_2, \\ \dfrac{\theta_h^2}{8} & \text{if } q \geq \dfrac{2}{3} \text{ and } \alpha \geq \alpha_3, \\ \dfrac{9q\theta_h^2}{32} & \text{if } q \geq \dfrac{2}{3} \text{ and } \alpha < \alpha_3. \end{cases} \tag{22}$$

In Proposition 3 we show that the Bertrand game is never a subgame perfect equilibrium. To do this we need to compare α_1, α_2, and α_3, which is provided in Lemma 2. The proofs of Lemma 2 and Proposition 3 are given in Appendix A.

Lemma 2

$\alpha_3 > \alpha_2 > \alpha_1 > 0$.

$\alpha_3 > \alpha_2 > \alpha_1 \Rightarrow c(\alpha_3) > c(\alpha_2) > c(\alpha_1)$**.** This means that it is relatively costly to implement the monopoly situation compared to the *lf* situation with or without the pirate's entry. Since G is an increasing function of α,

$\alpha_3 > \alpha_2 > \alpha_1 \Rightarrow G_3 > G_2 > G_1$.

Proposition 3

The monopolist always moves first and chooses his price, for any values of the policy variables. Either the monopoly situation or the *lf* equilibrium with or without the pirate's entry will result.

The first part of Proposition 3 results from the fact that the monopolist earns a higher profit by moving first rather than simultaneously. The second part of Proposition 3 implies that complete crowding-out or partial crowding-out will not occur. If $\alpha \geq \alpha_3$ for $q \geq \dfrac{2}{3}$ (or $\alpha \geq \alpha_2$ for $q < \dfrac{2}{3}$), the monopolist charges the monopoly price and the monopoly outcome results because the pirate cannot enter.

Otherwise, the monopolist will always charge the equilibrium price in the *lf* game because his profit in the *lf* game exceeds that in the Bertrand game. If he charges the monopoly price the pirate enters and there will be complete or partial crowding-out. The monopolist's profit in the *lf* game exceeds that in the *mp* game with partial or complete crowding-out.

In the situation where the monopolist charges the equilibrium price in the *lf* game, the pirate's entry depends upon the optimal monitoring rate. This we discuss in the next subsection.

2.2 OPTIMAL CHOICE OF POLICY INSTRUMENTS

The government chooses α and G in stage 1 of the extensive form game to maximize social-welfare subject to the balanced budget constraint. We assume $\frac{\theta_h}{\theta_l} > \frac{4-q}{1-q}$ to guarantee that the market is uncovered for all the pricing games discussed earlier. From (15) we know that the social welfare function is,

$$SW(\alpha) = \frac{p_m}{(\theta_h - \theta_l)}(\theta_h - \frac{p_m - p_c}{(1-q)}) + \frac{(1-\alpha)p_c}{(\theta_h - \theta_l)}\frac{(qp_m - p_c)}{q(1-q)} - c(\alpha) + CS\ .$$

<u>Lemma 3</u>

$SW(\alpha)$ is a decreasing function of α .

Intuitively, as monitoring increases, the pirate's profit decreases and the cost of monitoring increases thus increasing the deadweight loss. So social welfare decreases. Let α^* be the government's equilibrium monitoring rate.

From Proposition 3 we see that either the monopoly situation, or the *lf* equilibrium with or without the pirate's entry, is the possible outcome. From Lemma 3 we know that the social welfare decreases as monitoring increases. So to maximize social-welfare the government will choose from among the "minimalist,, monitoring rates that lead to the above outcomes. $\alpha = \alpha_3$, (or $\alpha = \alpha_2$) are the “minimalist„ monitoring rates that lead to the monopoly outcome for $q \geq \frac{2}{3}$, (or $q < \frac{2}{3}$). $\alpha = \alpha_1$ is the “minimalist„ monitoring rate that results in the *lf* equilibrium without the pirate’s entry. The *lf* equilibrium with the pirate’s entry results if $\alpha < \alpha_1$. So $\alpha = 0$ is the “minimalist„ monitoring rate that leads to this result. This analysis gives us the equilibrium set of pairs of monitoring rates and penalty, $\{(\alpha^*, G^*)\}$, which is discussed in Lemma 4.

Lemma 4

$$(\alpha^*, G^*) \in \begin{cases} \{(\alpha_3, G_3), (\alpha_1, G_1), (0,0)\} \text{ if } q \geq \frac{2}{3}, \\ \{(\alpha_2, G_2), (\alpha_1, G_1), (0,0)\} \text{ if } q < \frac{2}{3}. \end{cases}$$

$$G_i = \frac{c(\alpha_i)}{\alpha_i},\ i = 1,2,3.$$

Using (15) we determine the social-welfare functions for each of the above values of α. We then compare the different social-welfare functions to determine the equilibrium monitoring rate and the penalty.

$$SW(\alpha_3) = \frac{\theta_h^2}{4(\theta_h - \theta_l)} + \frac{\theta_h^2}{8} - c(\alpha_3). \tag{23}$$

$$SW(\alpha_2) = \frac{\theta_h^2}{4(\theta_h - \theta_l)} + \frac{\theta_h^2}{8} - c(\alpha_2). \tag{24}$$

$$SW(\alpha_1) = \frac{(1-q)\theta_h^2}{2(2-q)(\theta_h - \theta_l)} + \frac{(2+q)\theta_h^2}{8(2-q)} - c(\alpha_1). \tag{25}$$

$$SW(0) = \frac{(1-q)\theta_h^2}{2(2-q)(\theta_h - \theta_l)} + \frac{q(1-q)\theta_h^2}{4(2-q)^2(\theta_h - \theta_l)} + \frac{(4+q-q^2)\theta_h^2}{8(2-q)^2}. \tag{26}$$

The second term in the RHS of (28) is the pirate’s profit in the *lf* game with $\alpha = 0$.

From (25) and (26) it is clear that $SW(0) > SW(\alpha_1)$. This is because in the *lf* game, with the pirate's entry, social welfare includes the pirate's profit, which is not included in (25), and the consumer surplus is higher because some of the consumers switch from not buying to buying from the pirate. The monopolist's profit is the same in (25) and (26). So $\alpha = 0$ strongly dominates $\alpha = \alpha_1$. Hence, the government will choose between $\alpha = 0$ and $\alpha = \alpha_2$ (or α_3) for $q < \frac{2}{3}$ (or $q \geq \frac{2}{3}$).

$$SW(0) - SW(\alpha_2) = \frac{((5-2q)(\theta_h - \theta_l) - 2)q\theta_h^2}{8(2-q)^2(\theta_h - \theta_l)} + c(\alpha_2). \tag{27}$$

$$SW(0) - SW(\alpha_3) = \frac{((5-2q)(\theta_h - \theta_l) - 2)q\theta_h^2}{8(2-q)^2(\theta_h - \theta_l)} + c(\alpha_3). \tag{28}$$

Using (27) and (28) we summarize the optimal policy variables, (α^*, G^*), in Proposition 4.

<u>Proposition 4</u>

(i) For $q < \frac{2}{3}, (\alpha^*, G^*) = \begin{cases} (0, 0), & \text{if } SW(0) - SW(\alpha_2) \geq 0, \\ \left(\alpha_2, \dfrac{c(\alpha_2)}{\alpha_2}\right) & \text{if } SW(0) - SW(\alpha_2) < 0. \end{cases}$

If $\alpha^* = 0$, then the *lf* game with the pirate's entry is the subgame perfect equilibrium.

If $\alpha^* = \alpha_2$ then the monopoly situation is the subgame perfect equilibrium.

(ii) For $q \geq \frac{2}{3}, (\alpha^*, G^*) = \begin{cases} (0,0) & \text{if } SW(0) - SW(\alpha_3) \geq 0, \\ \left(\alpha_3, \dfrac{c(\alpha_3)}{\alpha_3}\right) & \text{if } SW(0) - SW(\alpha_3) < 0. \end{cases}$

If á$^* = 0$, then the *lf* game with the pirate's entry is the subgame perfect equilibrium.

If $\alpha^* = \alpha_3$ then the monopoly situation is the subgame perfect equilibrium.

The proof of Proposition 4 follows from (27) and (28). Intuitively, if the monitoring cost, which reflects the monitoring technology, is very low then the monopoly situation may be the subgame perfect equilibrium. The result also depends on the market size, $(\theta_h - \theta_l)$, and the reliability factor, q. The *lf* game is the unique subgame perfect equilibrium outcome if $\theta_h - \theta_l \geq \frac{2}{5-2q}$. This also follows from (27) and (28). Each of the expressions are positive if

$(\grave{e}_h - \grave{e}_l)(5-2q) \geq 2 \Rightarrow \grave{e}_h - \grave{e}_l \geq \frac{2}{5-2q}$.

It is interesting to note that there are some parameter values for which monitoring is the optimal outcome. Let us consider a numerical example that supports this finding. Since anecdotal evidence suggests that q is generally very high, we take $q = 0.9$ in our numerical example. We assume $\theta_h = 0.1$ and $\theta_l = .001$. These support values satisfies the condition for uncovered market $\frac{\theta_h}{\theta_l} = 100 > \frac{4-q}{1-q} = 31$.

Suppose $c(\alpha) = c\alpha^2$. Assume c = 1. Now, $\frac{c(\alpha_3)}{1-\alpha_3} = \frac{3q\theta_h{}^2}{16(\theta_h - \theta_l)}$. Substituting the values of the parameters and using the above form of the monitoring cost function we get $\alpha_3^2 = 0.014960566$. Substituting this and the parameter values in (28) we get

$$SW(0) - SW(\alpha_3) = \frac{((5-2q)(\theta_h - \theta_l) - 2)q\theta_h^2}{8(2-q)^2(\theta_h - \theta_l)} + \alpha_3^2 = -0.000847097 < 0$$

. If c =0.1, then, $SW(0) - SW(\alpha_3) = -0.004495263 < 0$. This numerical example shows that for some values of the parameters monitoring is the optimal policy. Further, a decrease in the value of c increases the possibility of monitoring to be the optimal policy.

The domestic government's social-welfare maximizing objective determines its aggressiveness or passiveness towards piracy, which in turn endogenously determines whether there will be market sharing between the monopolist and the pirate or the monopoly outcome will result. In Proposition 5 we discuss the effects of an increase in the reliability of the pirated software. The proof is in Appendix A.

Proposition 5

(i) An increase in q increases piracy.

(ii) An increase in q increases the optimal monitoring rate that results in the monopoly outcome.

A change in the reliability factor does not affect the market in the case of the monopoly outcome. However, it affects the market when the *lf* game is the subgame perfect equilibrium. Any change in q does not affect the monopolist's market because, $\theta_c^{lf*} = \frac{\theta_h}{2}$, does not depend on q. Intuitively, as the reliability of the pirated software increases, the monopolist maintains the same market as in the monopoly case, by lowering its price, $\frac{dp_m^{lf*}}{dq} = \frac{-\theta_h}{(2-q)^2} < 0$. The profit of the monopolist also goes down in response to an increase in q. Since $\frac{d\theta_x^{lf*}}{dq} = \frac{-2\theta_h}{4(2-q)^2} < 0$, it means that the pirate's market share and piracy increases with an increase in q. Part (ii) of Proposition 2 implies that since piracy in the monopoly pricing game increases with an increase in q, the optimal monitoring rate that restores the monopoly outcome also increases. Let us discuss the implications of the balanced budget assumption. To do this we need to study the effects on government policies if the government's net revenue is not binding and positive.

Suppose the government employs a monitoring agent, which is a government agency, and provides it with a budget B to monitor commercial piracy. So the government's net revenue is now $R = \alpha G - B$ and the agent's net revenue is $B - c(\alpha)$. In this formulation one can expect the monitoring agent's budget constraint to be binding so that $B = c(\alpha)$. Hence, the government's net revenue becomes $R = \alpha G - c(\alpha)$. Suppose the government's net revenue does not bind and is positive, that is, $R = \alpha G - c(\alpha) > 0$. The social welfare function is the same as in (15) and is a decreasing function of α.

Let us now examine the effects of positive government net revenue on α_1, α_2, and α_3, which are the "minimalist" monitoring rates in the *lf* and *mp* subgames. The pirate's profit is $\pi_c = (1-\alpha)p_c D_c - \alpha G$. So the "minimalist" monitoring rates must satisfy $\pi_c = (1-\alpha)p_c D_c - \alpha G = 0 \Rightarrow p_c D_c = \frac{\alpha G}{1-\alpha}$. Since $R = \alpha G - c(\alpha) > 0$, we have $\frac{\alpha G}{1-\alpha} > \frac{c(\alpha)}{1-\alpha}$. From Lemma 1 we know that $\frac{\alpha G}{1-\alpha}$ is an increasing function of α. Hence, the result, $\alpha_3 > \alpha_2 > \alpha_1$, continues to hold. Combining this with the fact that the social welfare is a decreasing function of α, the equilibrium monitoring rates remains the same as discussed in Proposition 4. However, since $R > 0$, we cannot determine the penalty G endogenously from the model. In this case, G becomes an exogenously given parameter.

In general, the main implication of the balanced budget constraint is that the penalty is derived endogenously. If we assume that the government's net revenue is positive, then the equilibrium monitoring rates are the same as summarised in Proposition 4. However, we cannot determine the penalty endogenously from the model. The penalty corresponding to the different equilibrium monitoring rates (as summarised in Proposition 4) must be such that the government's net revenue from monitoring is positive.[6] Let us now discuss the monopolist's role in deterring piracy.

2.2 MONOPOLIST'S ROLE IN PREVENTING PIRACY

The original firm always favors the monopoly outcome. However, if the optimal monitoring rate is 0 then the pirate enters and the monopolist's profit goes down by $\pi_m^* - \pi_m^{lf*} = \dfrac{q\theta_h^2}{4(2-q)(\theta_h - \theta_l)}$. The question, therefore, is whether the monopolist can do anything to prevent piracy.

Let us suppose that the monopolist can install a technical protection device in the software to prevent copying. For simplicity we assume that this protective device does not cause any inconvenience to the user. So the users of the original software continue buying the original software.[7] Let F be the fixed cost of installing the technical protection device. This is common knowledge.

Proposition 6

(i) If $F < \dfrac{q\theta_h^2}{4(2-q)(\theta_h - \theta_l)}$, then the monopolist installs the protective technical device and $\alpha^* = 0$. Otherwise, the results are the same as in Proposition 4.

(ii) An increase in the reliability factor increases the monopolist's incentive to protect his software against piracy.

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[6] Introducing a parameter that measures the marginal social value of revenue can permit the study of the policy issues in a more general form. Let λ be a measure of the marginal social value of an additional dollar of revenue to the government. So the social welfare function becomes:

$$\begin{aligned} SW(\alpha) &= p_m D_m + (1-\alpha)p_c D_c - \alpha G + CS + \lambda\alpha G - c(\alpha) \\ &= p_m D_m + (1-\alpha)p_c D_c + CS + (\lambda - 1)\alpha G - c(\alpha). \end{aligned}$$

Social welfare can now be an increasing or a decreasing function of α. If it is a decreasing function, then the results for the equilibrium monitoring rates are the same as those in Proposition 4. However, if the social welfare is an increasing function of α then the results are different.

[7] Gurnsey (1995) cites that software with protection, though it prevents copying, tends to be user-

The monopolist installs the protective device, for all monitoring rates, if the net monopoly profit from the installation exceeds his profit in the leader-follower case. So if the monopolist prevents the pirate's entry, the government does not need to monitor. However, if $F \geq \frac{q\theta_h^2}{4(2-q)(\theta_h - \theta_l)}$, then the monopolist has no incentive to install the protective device and the results are the same as described in Proposition 4. The monopolist's incentive to prevent piracy depends on the cost of doing so. For high costs, the monopolist is better off allowing piracy if the optimal monitoring rate is zero. So for high costs of installing a protective device, piracy depends on the optimal policy variables and the monopolist plays no role in it.

As seen earlier, an increase in q lowers the equilibrium price of the monopolist in the leader-follower game, which reduces his profit. Thus the difference between the monopoly profit without the protective device and the leader's profit increases with an increase in q.[8] Intuitively, the greater the difference between the monopoly profit without the protective device and the leader's profit, the higher will be the monopolist's incentive to protect his software. It allows the monopolist to spend more in installing the protective device.

3. EFFECTS OF NETWORK EXTERNALITIES (NE)

In this section we explore the network effects on the policy variables and on the monopolist's incentive to prevent piracy through installing a costly technical protective device. We compare the results with those in the absence of NE. The

unfriendly. This reduces the consumers' valuation of the software and some of the buyers may not buy. So the original firm's profit may fall and self-protection becomes questionable.

algebraic analysis and the proofs are given in Appendix B.

A network externality means that a consumer's utility increases with an increase in the number of other consumers using the same software, legally or illegally. Consumers benefit through exchange of files using the same software. This feature is captured in the utility function. We begin our discussion with the no piracy case.

The utility of a type θ consumer from buying one unit of the software is,

$$U(\theta) = \begin{cases} \theta - p'_m + \beta D'_{\mathrm{m}} & \text{if the consumer buys the software,} \\ 0 & \text{if the consumer does not buy.} \end{cases} \tag{29}$$

β is the coefficient measuring the degree of network externalities. We assume that $\beta \in (0, (\theta_h - \theta_l))$ to avoid indeterminate results. D'_m is the demand for the original software and p'_m is the price charged by the monopolist.

Comparison of the monopoly results in the presence and in the absence of NE shows that the equilibrium price is the same in both the situations. However, the monopolist's market share increases in the presence of NE. Consequently, with network effects the monopolist's profit is higher.

Let us introduce the pirate in the model. q is the probability that the pirated software is operational. The consumer buying the original software enjoys the benefit θ and the network externality generated by those who buy the original and the pirated software, only if the latter is operational. The consumer buying the pirated software enjoys the benefit and the network, only if the pirated software is operational. So the utility function is,

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8 $\dfrac{d(\pi_m^* - \pi_m^{lf*})}{dq} = \dfrac{4(2-q)(\theta_h - \theta_l)\theta_h^2 + 4(\theta_h - \theta_l)q\theta_h^2}{(4(2-q)(\theta_h - \theta_l))^2} > 0$.

$$U(\theta) = \begin{cases} \theta - p'_m + \beta D'_m + q\beta D'_c & \text{if the consumer buys the original software,} \\ q(\theta + \beta D'_m + \beta D'_c) - p'_c & \text{if the consumer buys the pirated software,} \\ 0 & \text{if the consumer does not buy.} \end{cases} \quad (30)$$

p'_c and D'_c are the price charged by the pirate and the demand of the pirated software. α' and G' are the policy variables in this model with NE.

The government, in equilibrium, may or may not monitor. In the first case the monopoly outcome is the subgame perfect equilibrium. In the second case the *lf* outcome, with the market being shared by the monopolist and the pirate, is the subgame perfect equilibrium. To avoid repetition, we just provide a discussion of the comparative analysis of the results in the presence and absence of NE. The important aspects of the comparative analysis are summarized in Propositions 7 and 8.

Proposition 7

(i) Piracy in the *lf* game with NE is higher than that in the game without NE. Piracy increases as network benefits increase.

(ii) The optimal monitoring rates that result in the monopoly outcome in the presence of NE exceed those in the absence of NE.

(iii) The optimal monitoring rates that result in the monopoly outcome increases with an increase in network benefits.

Due to network effects, the pirate's market is larger in the *lf* game with NE compared to that without it. Since the price is the same in both the situations but the market size of each player is greater in the *lf* game with NE, the profit of each player is also higher in the latter case. The pirate's market share increases as the benefit from network effects, which is captured by the coefficient measuring NE, increases.

The pirate's expected profit in the *mp* game with NE is higher than that without NE. In the partial and the complete crowding-out cases, with NE, the pirate's market is larger than the same without NE. Therefore, it requires a higher monitoring rate to detect the pirate. For the same reason, an increase in the coefficient measuring network benefits causes an increase in the monitoring rates that restore the monopoly outcome. Thus our model shows that the higher is the level of piracy, greater is the monitoring rate and the higher is the cost of implementing the monopoly outcome. Correspondingly, the penalty is also higher. This follows from the fact that the penalty is an increasing function of monitoring rate as discussed in section 2.

Let us now analyze the monopolist's role in preventing piracy and compare it with the results in the absence of NE. Suppose the monopolist incurs a fixed cost F' in installing a technical protective device to prevent copying. He will install the device only if the net monopoly profit from doing so exceeds his profit in the *lf* game. So the monopolist will install the protective device for any monitoring rate if $F' < \pi_m'^* - \pi_m^{lf'*} = \dfrac{q\theta_h^2}{4(2-q)(\theta_h - \theta_l - \beta)}$.[9] Therefore, the government will not monitor and $\alpha'^* = 0$. Otherwise, the monopolist will not install the protective device and the prevention or existence of piracy depends on the government's optimal policy. Without NE the condition for installing the protective device is $F < \dfrac{q\theta_h^2}{4(2-q)(\theta_h - \theta_l)}$. Now $\dfrac{q\theta_h^2}{4(2-q)(\theta_h - \theta_l - \beta)} > \dfrac{q\theta_h^2}{4(2-q)(\theta_h - \theta_l)}$.

Proposition 8

(i) In the presence of NE, the monopolist has greater incentive to prevent piracy.

(ii) With NE, the monopolist's incentive to prevent piracy increases with an increase in q or an increase in β.

In presence of NE the original firm's profit in the monopoly and in the *lf* cases are higher than the same without NE. However, due to network effects the difference between the monopoly profits and the original firm's profit in the *lf* game is higher than that in the absence of NE. This allows the monopolist in the case of NE to spend more in installing the protective device. Also, since the monopolist's profit is higher in the presence of NE, he has all the more incentive to prevent piracy.

The result stated in Proposition 8 (ii) follows from the fact that

$$\frac{d(\pi'^{*}_{m} - \pi^{lf'*}_{m})}{dq} = \frac{4(2-q)(\theta_h - \theta_l - \beta)\theta_h^2 + 4(\theta_h - \theta_l - \beta)q\theta_h^2}{(4(2-q)(\theta_h - \theta_l - \beta))^2} > 0, \text{ and}$$

$$\frac{d(\pi'^{*}_{m} - \pi^{lf'*}_{m})}{d\beta} = \frac{4(2-q)q\theta_h^2}{(4(2-q)(\theta_h - \theta_l - \beta))^2} > 0.$$ Intuitively, with an increase in q or an increase in β, the difference between the monopoly profit, without protection, and the leader's profit increases. This increase in the difference between the two allows the monopolist to spend more in installing the protective device and the monopolist has more incentive to protect the software.

We see that piracy in the presence of NE is larger than that in its absence. Changes in network benefits have a positive effect on piracy.[10] With network effects, the optimal monitoring rate that results in the monopoly outcome is higher compared to that without it. The monopolist has more incentive to prevent piracy, due to higher profits, in the presence of NE. Changes in the reliability factor and the network benefits have positive effects on the monopolist's incentive to prevent piracy.

4. CONCLUSION

The focus of the previous research on software piracy has been on piracy by end-

[9] Refer to equations B2 and B8 in Appendix B.

[10] Unfortunately, the comparative static analysis with respect to the reliability factor becomes algebraically intractable.

users and effects of network externalities in protecting the software industry. In this paper, we analyzed the government's role, through monitoring and penalizing the illegal operation of a software pirate, in restricting piracy in a software market where the original firm is not headquartered. The government's social-welfare maximizing policy endogenously determined the market outcome.

We found that due to a first-mover advantage, the monopolist always moves first to choose his price and the Bertrand game is never a subgame perfect equilibrium. If not monitoring is the optimal policy, then the monopolist and the pirate shared the market. In this case the pirate's market was larger in the presence of network externalities than in its absence. If monitoring is the optimal policy, then the monopoly outcome resulted. However, the monitoring rate is higher when network externalities are present. Due to network externalities the monopolist's and the pirate's market share increased. However, the increase in the pirate's market share is much more pronounced than that of the monopolist. This explains the higher degree of piracy in the leader-follower outcome and the higher monitoring rate that results in the monopoly outcome when there are network externalities.

We also found that the monopolist has a greater incentive to prevent piracy through installing a protective device when network externalities are present. The monopoly profits are higher when there are network effects and so the incentive to prevent piracy is also higher.

The comparative static analysis showed that changes in the reliability of the pirated software and the coefficient measuring network benefits have positive effects on piracy and on the optimal monitoring rate that results in the monopoly outcome. Their effect on the monopolist's incentive to prevent piracy is also positive. The reliability factor and the coefficient measuring network benefits are demand-shifting parameters. So the sale of original and pirated software increases with an increase in these two factors. As a result the monopolist's and the pirate's profit increases.

The government can also control piracy through education and awareness campaigns, informing the buyers on the hazards and hidden costs of using pirated software. Such measures, if successful, devalue the quality of the pirated software. We could proxy this change by a fall in the reliability factor, which shrinks the pirate's market. However, such campaigns require resources. So the prevention of piracy through campaigns may or may not enhance social welfare.

REFERENCES

Chen, Y. and Png, I., "Software Pricing and Copyright: Enforcement Against End-Users„, mimeo, 1999.

Cheng, H.K., Sims, R.R. and Teegen, H., "To Purchase or Pirate Software: An Empirical Study„, *Journal of Management Information Systems*, Vol. 13, No. 4, Spring 1997, 49-60.

Conner, K.R. and Rumelt, R.P., "Software Piracy: An Analysis of Protection Strategies„, *Management Science*, Vol. 37, No. 2, 1991, 125-139.

Correa, C.M., "The Legal Protection of Software„, in United Nations Industrial Development Organization (UNIDO), *Software Industry: Current Trends and Implications for Developing Countries, UNIDO,Vienna*, 1993.

Gurnsey, J., *Copyright Theft*, Aslib Gower, England, 1995.

International Planning and Research Corporation (IPR) (1998) for Business Software Alliance (BSA) and Software Publishers Association (SPA) 1997 Global Software Piracy Report, BSA, Washington D.C., June.

Nascimento, F. and Vanhonacker, W.R., "Optimal Strategic Pricing of Reproducible Consumer Products„, *Management Science*, Vol. 34, No. 8, August 1988, 921-937.

Noyelle, T., "Computer Software and Computer Services in Five Asian Countries„, in United Nations Conference on Trade and Development/ United Nations Development Programme (UNCTAD/UNDP), *Services in Asia and the Pacific: Selected Papers*, Vol 1, United Nations, New York, 1990.

Oz. S. and Thisse, J.F., "A Strategic Approach to Software Protection„, *Journal of Economics and Management Strategy*, vol. 8, no. 2, Summer 1999, 163-190.

Takeyama, L., "The Welfare Implications of Unauthorized Reproduction of Intellectual Property in the Presence of Network Externalities„, *Journal of Industrial Economics*, Vol. 62, No. 2, June 1994, 155-166.

Tirole, J., "The Theory of Industrial Organization„, The MIT Press, 1988.

Wauthy, X., "Quality Choice in Models of Vertical Differentiation„, *The Journal of Industrial Economics*, Vol XLIV, No. 3, September, 1996.

APPENDIX A

BERTRAND GAME

In this game each player competes in price simultaneously. From the first order conditions we get the following reaction functions:

$$p_m^b(p_c^b) = \frac{(1-q)\theta_h + p_c^b}{2}, \tag{A1}$$

$$p_c^b(p_m^b) = \frac{qp_m^b}{2}. \tag{A2}$$

The Nash Equilibrium prices are $p_m^{b*} = \frac{2(1-q)\theta_h}{(4-q)}$ and $p_c^{b*} = \frac{q(1-q)\theta_h}{(4-q)}$. The marginal customers are $\theta_c^{b*} = \frac{(2-q)\theta_h}{(4-q)} < \theta_m^* = \frac{\theta_h}{2}$ and $\theta_x^{b*} = \frac{(1-q)\theta_h}{(4-q)}$. Clearly, $\theta_c^{b*} > \theta_x^{b*}$. Now, $\frac{\theta_h}{\theta_l} > \frac{(4-q)}{(1-q)} \Rightarrow \theta_x^{b*} > \theta_l$. The market is uncovered at the equilibrium if $\frac{\theta_h}{\theta_l} > \frac{4-q}{1-q}$. The monopolist expands its market compared to the monopoly case.

Each player's profit in equilibrium is,

$$\pi_m^{b*} = \frac{4(1-q)\theta_h^{\ 2}}{(\theta_h - \theta_l)(4-q)^2} < \pi_m^* = \frac{\theta_h^{\ 2}}{4(\theta_h - \theta_l)}, \tag{A3}$$

$$\pi_c^{b*} = \frac{(1-\alpha)(1-q)q\theta_h^{\ 2}}{(\theta_h - \theta_l)(4-q)^2} - \alpha G. \tag{A4}$$

The pirate enters the market only if $\pi_c^{b*} > 0$ i.e. $\frac{(1-\alpha)(1-q)q\theta_h^{\ 2}}{(\theta_h - \theta_l)(4-q)^2} > \alpha G$.

<u>*Proof of Proposition 1*</u>

We substitute the reaction function of the pirate, (A2), into the profit function of the monopolist, (A1). From the first-order conditions the equilibrium prices and the marginal consumers are as follows:

$$p_m^{lf*} = \frac{(1-q)\theta_h}{2-q} > p_m^{b*} = \frac{2(1-q)\theta_h}{(4-q)}, \tag{A5}$$

$$p_c^{lf*} = \frac{(1-q)q\theta_h}{2(2-q)} > p_c^{b*} = \frac{q(1-q)\theta_h}{(4-q)}, \tag{A6}$$

$$\theta_c^{lf*} = \frac{\theta_h}{2} = \theta_m^*, \text{ and } \theta_x^{lf*} = \frac{(1-q)\theta_h}{2(2-q)}. \tag{A7}$$

In equilibrium the market is uncovered if $\theta_x^{lf*} = \frac{(1-q)\theta_h}{2(2-q)} > \theta_l \Rightarrow \frac{\theta_h}{\theta_l} > \frac{(4-2q)}{(1-q)}$.

In equilibrium the profits of the firms are:

$$\pi_m^{lf*} = \frac{(1-q)\theta_h^{\ 2}}{2(2-q)(\theta_h - \theta_l)} > \pi_m^{b*}, \tag{A8}$$

$$\pi_c^{lf*} = \frac{(1-\alpha)(1-q)q\theta_h^{\ 2}}{4(2-q)^2(\theta_h - \theta_l)} - \alpha G. \tag{A9}$$

The pirate enters if $\pi_c^{lf*} > 0$. Let α_1 and G_1 be the policy variables such that $\pi_c^{lf*} = 0$. Using $\pi_c^{lf*} = 0$ we get $\frac{(1-q)q\theta_h^{\ 2}}{4(2-q)^2(\theta_h - \theta_l)} = \frac{\alpha_1 G_1}{(1-\alpha_1)} = \frac{c(\alpha_1)}{1-\alpha_1}$. The pirate enters and his profit is given by (A9) if $\frac{c(\alpha)}{1-\alpha} < \frac{c(\alpha_1)}{1-\alpha_1}$ From Lemma 1, $\frac{c(\alpha)}{1-\alpha} < \frac{c(\alpha_1)}{1-\alpha_1} \Rightarrow \alpha < \alpha_1$. The pirate cannot enter the market and his profit is 0 if $\alpha \geq \alpha_1$. ■

The overall market covered in the *lf* game is less than in the Bertrand game because $\theta_x^{lf*} > \theta_x^{b*}$. The pirate's market if he enters is $\theta_c^{lf*} - \theta_x^{lf*} = \frac{\theta_h}{2(2-q)}$. However, the size of the market covered by the pirate in the *lf* game is higher than that in the Bertrand game since $(\theta_c^{lf*} - \theta_x^{lf*}) = \frac{\theta_h}{2(2-q)} > (\theta_c^{b*} - \theta_x^{b*}) = \frac{\theta_h}{4-q}$

<u>*Proof of Proposition 2*</u>

The market is uncovered in equilibrium if $\theta_x^{mp*} = \frac{\theta_h}{4} > \theta_l \Rightarrow \frac{\theta_h}{\theta_l} > 4$. Substituting the monopoly price in the reaction function of the pirate (A2), we get the equilibrium prices, marginal consumers and the profits.

$$\pi_m^{mp*} = \frac{(2-3q)\theta_h^{\ 2}}{8(1-q)(\theta_h - \theta_l)}. \tag{A10}$$

$$\pi_c^{mp*} = \frac{(1-\alpha)q\theta_h^{\ 2}}{16(1-q)(\theta_h - \theta_l)} - \alpha G. \tag{A11}$$

α_2 and G_2 satisfies $\pi_c^{mp*}=0 \Rightarrow \dfrac{\alpha_2 G_2}{1-\alpha_2}=\dfrac{c(\alpha_2)}{1-\alpha_2}=\dfrac{q\theta_h^2}{16(1-q)(\theta_h-\theta_l)}$. If $q<\dfrac{2}{3}$ and

$\dfrac{c(\alpha)}{1-\alpha}\geq\dfrac{c(\alpha_2)}{1-\alpha_2}\Rightarrow\alpha\geq\alpha_2$, then the pirate cannot enter and the monopoly results hold.

However, if $q<\dfrac{2}{3}$ and $\alpha<\alpha_2$ then the market is shared. The profits of the two players are given in A(10) and A(11). There is piracy in equilibrium because $\theta_c^{mp*}-\theta_m^*=\dfrac{(2-q)\theta_h}{4(1-q)}-\dfrac{\theta_h}{2}=\dfrac{q}{4(1-q)}>0$. This means that some of the consumers switch from buying the original software to buying the pirated version. Let us consider the situation where $q\geq\dfrac{2}{3}$. In this case from (A10) we see that $\pi_m^{mp*}\leq 0$ which implies that the pirate captures the entire market if he enters. We call this the complete crowding-out situation. In this case $\theta_c^{mp*}=\theta_h$. The pirate's profit is,

$$\pi_c^{mp*}=\int_{\frac{\theta_h}{4}}^{\theta_h}\frac{p_c^{mp*}}{\theta_h-\theta_l}d\theta=\frac{3(1-\alpha)q\theta_h^{\ 2}}{16(\theta_h-\theta_l)}-\alpha G. \tag{A12}$$

α_3 and G_3 satisfies $\dfrac{\alpha_3 G_3}{1-\alpha_3}=\dfrac{c(\alpha_3)}{1-\alpha_3}=\dfrac{3q\theta_h^{\ 2}}{16(\theta_h-\theta_l)}$. If, $\dfrac{c(\alpha)}{1-\alpha}<\dfrac{c(\alpha_3)}{1-\alpha_3}\Rightarrow\alpha<\alpha_3$, then the pirate captures the entire market (complete crowding-out situation). The pirate's profit in this case is given by (A12). The monopoly results hold if $\alpha\geq\alpha_3$.

■

Proof of Lemma 2

From Lemma 1 we know $x=\dfrac{\alpha G}{1-\alpha}$ is increasing in α. Let $x_i=\dfrac{c(\alpha_i)}{1-\alpha_i}$, $i=1,2,3$.

$$x_2-x_1=\frac{q\theta_h^2}{16(1-q)(\theta_h-\theta_l)}-\frac{q(1-q)\theta_h^2}{4(2-q)^2(\theta_h-\theta_l)}=\frac{(4-3q)q^2\theta_h^2}{16(1-q)(2-q)^2(\theta_h-\theta_l)}>0.$$

$$x_3 - x_2 = \frac{3q\theta_h^2}{16(\theta_h - \theta_l)} - \frac{q\theta_h^2}{16(1-q)(\theta_h - \theta_l)} = \frac{3(\frac{2}{3}+\varepsilon)\theta_h^2}{16(\theta_h - \theta_l)} - \frac{q\theta_h^2}{16(1-q)(\theta_h - \theta_l)}.$$

Since $q \geq \frac{2}{3}$ in $\frac{3q\theta_{\mathrm{h}}^2}{16(\theta_{\mathrm{h}} - \theta_l)}$, we write q as $q = \frac{2}{3} + \varepsilon, \varepsilon \in [0, \frac{1}{3})$.

Therefore, $x_3 - x_2 = \frac{5(1-q) + 9\varepsilon(1-q) + 1}{16(1-q)(\theta_h - \theta_l)} > 0$. So $x_3 > x_2 > x_1 \Rightarrow \alpha_3 > \alpha_2 > \alpha_1$. ■

Proof of Proposition 3

If $q \geq \frac{2}{3}$ and $\alpha \geq \alpha_3$, the monopolist charges the monopoly price because the pirate cannot enter. Further, the monopolist earns the highest profit by charging the monopoly price. If $q \geq \frac{2}{3}$ and $\alpha_1 \leq \alpha < \alpha_3$, then there is complete crowding-out if the monopolist charges the monopoly price. The monopolist charges the equilibrium price in the *lf* game because his profit exceeds that in the Bertrand game.

$$\pi_m^{lf*} = \frac{(1-q)\theta_h{}^2}{2(2-q)(\theta_h - \theta_l)} - \pi_m^{b*} = \frac{4(1-q)\theta_h{}^2}{(\theta_h - \theta_l)(4-q)^2} = \frac{(1-q)q^2\theta_h^2}{2(2-q)(4-q)^2(\theta_h - \theta_l)} > 0.$$

The pirate cannot enter. If $q \geq \frac{2}{3}$ and $\alpha < \alpha_1$ then also the monopolist charges the equilibrium price in the *lf* game because of the same reason. However, in this case the pirate enters. If $q < \frac{2}{3}$ and $\alpha \geq \alpha_2$ then the monopolist charges the monopoly price because the pirate cannot enter. If $q < \frac{2}{3}$ and $\alpha_1 \leq \alpha < \alpha_2$ then there is partial crowding-out if the monopolist charges the monopoly price. In this case the monopolist charges the equilibrium price in the *lf* game because,

$$\pi_m^{lf*}=\frac{(1-q)\theta_h^{\ 2}}{2(2-q)(\theta_h-\theta_l)}-\pi_m^{mp*}=\frac{(2-3q)\theta_h^{\ 2}}{8(1-q)(\theta_h-\theta_l)}=\frac{q^2}{8(1-q)(2-q)(\theta_h-\theta_l)}>0$$. The pirate cannot enter. If $q<\frac{2}{3}$ and $\alpha<\alpha_1$ then also the monopolist charges the equilibrium price in the *lf* game but in this case the pirate enters. ■

Proof of Lemma 3

$SW'(\alpha)=-\frac{p_c(qp_m-p_c)}{q(1-q)(\theta_h-\theta_l)}-c'(\alpha)<0$ because $c'(\alpha)>0$ by assumption. ■

Proof of Proposition 5

(i) $\frac{d\theta_x^{lf*}}{dq}=\frac{-2\theta_h}{4(2-q)^2}<0$.

(ii) $x_2=\frac{c(\alpha_2)}{1-\alpha_2}=\frac{q\theta_h^2}{16(1-q)(\theta_h-\theta_l)}$ and $x_3=\frac{c(\alpha_3)}{1-\alpha_3}=\frac{3q\theta_h^2}{16(\theta_h-\theta_l)}$.

$\frac{dx_2}{dq}=\frac{16(1-q)(\theta_h-\theta_l)\theta_h^2+16(\theta_h-\theta_l)q\theta_h^2}{(16(1-q)(\theta_h-\theta_l))^2}>0$ and

$\frac{dx_2}{d\alpha_2}=\frac{(1-\alpha_2)c'(\alpha_2)+c(\alpha_2)}{(1-\alpha_2)^2}>0$. Therefore, $\frac{d\alpha_2}{dq}>0$. Similarly, $\frac{d\alpha_3}{dq}>0$. ■

<u>APPENDIX B</u>

In Appendix B we provide the complete algebraic analysis with NE.

<u>MONOPOLY RESULTS</u>

Let θ'_m be the marginal consumer who is indifferent between the buying the original software and not buying. Setting $\theta-p'_m+\beta D'_m=0$, we get $\theta'_m=p'_m-\beta D'_m$.

The demand for the software is,

$$D'_m(p'_m)=\int_{\theta'_m}^{\theta_h}\frac{1}{\theta_h-\theta_l}d\theta=\frac{\theta_h-p'_m}{\theta_h-\theta_l-\beta}. \tag{B1}$$

The equilibrium price, market share, and the profit are,

$$p_m'^* = \frac{\theta_h}{2},\ \theta_m'^* = \frac{\theta_h}{2} - \frac{\beta\theta_h}{2(\theta_h - \theta_l - \beta)},\ \pi_m'^* = \frac{\theta_h^2}{4(\theta_h - \theta_l - \beta)}. \tag{B2}$$

In the presence of the pirate there are two marginal consumers. The marginal consumers indifferent between buying the original and the pirated software, and indifferent between buying the pirated software and not buying at all are,

$$\begin{aligned} \theta_c' &= \frac{(\theta_h - \theta_l)(p_m' - p_c')}{(1-q)(\theta_h - \theta_l - \beta)} - \frac{\beta\theta_h}{(\theta_h - \theta_l - \beta)}, \\ \theta_x' &= \frac{p_c'(\theta_h - \theta_l) - \beta\theta_h}{q(\theta_h - \theta_l - \beta)}. \end{aligned} \tag{B3}$$

The demand for the original and the pirated software are,

$$\begin{aligned} D_m'(p_m', p_c') &= \frac{\theta_h}{(\theta_h - \theta_l - \beta)} - \frac{(p_m' - p_c')}{(1-q)(\theta_h - \theta_l - \beta)}, \\ D_c'(p_m', p_c') &= \frac{(qp_m' - p_c')}{q(1-q)(\theta_h - \theta_l - \beta)}. \end{aligned} \tag{B4}$$

The profit functions of the monopolist and the pirate are,

$$\begin{aligned} \pi_m'(p_m', p_c') &= p_m' D_m'(p_m', p_c'), \\ \pi_c'(p_m', p_c') &= (1-\alpha')p_c' D_c'(p_m', p_c') - \alpha' G'. \end{aligned} \tag{B5}$$

<u>*Results of the Leader-Follower Game*</u>

$$p_m^{lf'*} = \frac{(1-q)\theta_h}{(2-q)} \text{ and } p_c^{lf'*} = \frac{q(1-q)\theta_h}{2(2-q)}. \tag{B6}$$

$$\theta_c^{lf'*} = \frac{(\theta_h - \theta_l - 2\beta)\theta_h}{2(\theta_h - \theta_l - \beta)} \text{ and } \theta_x^{lf'*} = \frac{(q(1-q)(\theta_h - \theta_l) - 2(2-q)\beta)\theta_h}{2q(2-q)(\theta_h - \theta_l - \beta)}. \tag{B7}$$

$$\pi_m^{lf'*} = \frac{(1-q)\theta_h^2}{2(2-q)(\theta_h - \theta_l - \beta)}. \tag{B8}$$

$$\pi_c^{lf'*} = \begin{cases} (1-\alpha')\dfrac{q(1-q)\theta_h^2}{4(2-q)^2(\theta_h - \theta_l - \beta} - \alpha' G' & \text{if } \dfrac{\alpha' G'}{1-\alpha'} < \dfrac{\alpha_1' G_1'}{1-\alpha_1'}, \\ 0 & \text{otherwise.} \end{cases} \tag{B9}$$

(α_1', G_1') are the minimum policy variables that deter the pirate's entry in the *lf* game. Therefore, (α_1', G_1') satisfies,

$$\pi_c^{lf'*} = 0 \Rightarrow \frac{\alpha_1' G_1'}{1-\alpha_1'} = \frac{q(1-q)\theta_h^2}{4(2-q)^2(\theta_h - \theta_l - \beta)}. \tag{B10}$$

The pirate enters only if $\frac{\alpha' G'}{1-\alpha'} < \frac{\alpha_1' G_1'}{1-\alpha_1'}$. Otherwise, he earns zero profit. Let us compare the size of the market served by the monopolist and the pirate in the two situations. Comparing the marginal consumers does this.

$$\theta_c^{lf*} - \theta_c^{lf'*} = \frac{\beta\theta_h}{2(\theta_h - \theta_l - \beta)} > 0. \tag{B11}$$

So the monopolist's market is larger in the *lf* game with NE. The pirate's market is $\theta_c - \theta_x$. We compare this for the two situations and get,

$$(\theta_c^{lf'*} - \theta_x^{lf'*}) - (\theta_c^{lf*} - \theta_x^{lf*}) = \frac{(2(q-1)(q-2)+q)\beta\theta_h}{2q(2-q)(\theta_h - \theta_l - \beta)} > 0, \text{ (since } q < 1). \tag{B12}$$

From (A20) we see that $(\theta_c^{lf'*} - \theta_x^{lf'*}) - (\theta_c^{lf*} - \theta_x^{lf*})$ is an increasing function of β.

<u>*RESULTS OF THE MONOPOLY PRICING GAME*</u>

In this case the monopolist charges the monopoly price, $p_m^{mp'*} = \frac{\theta_h}{2}$. The equilibrium price of the pirated software is $p_c^{mp'*} = \frac{q\theta_h}{4}$. The marginal consumers are;

$$\theta_c^{mp'*} = \frac{\theta_h((2-q)(\theta_h - \theta_l) - 4(1-q)\beta)}{4(1-q)(\theta_h - \theta_l - \beta)} \text{ and } \theta_{\mathrm{x}}^{\mathrm{mp}'*} = \frac{\theta_h(q(\theta_h - \theta_l) - 4\beta)}{4q(\theta_h - \theta_l - \beta)}.$$

In equilibrium the profits of the monopolist and the pirate are,

$$\pi_m^{mp'*} = \frac{(2-3q)\theta_h^2}{8(1-q)(\theta_h - \theta_l - \beta)}, \tag{B13}$$

$$\pi_c^{mp'*} = \frac{(1-\alpha')q\theta_h^2}{16(1-q)(\theta_h-\theta_l-\beta)} - \alpha' G'. \qquad \text{(B14)}$$

There is partial crowding-out if $q < \frac{2}{3}$ and $\frac{\alpha' \mathrm{G}'}{1\text{-}\alpha'} < \frac{\alpha_2' \mathrm{G}_2'}{1\text{-}\alpha_2'} = \frac{q\theta_h^2}{16(1-q)(\theta_h-\theta_l-\beta)}$.

$\theta_m'^{*} < \theta_c^{mp'*}$ implies that some of the buyers of the original software buy the pirated software. The monopoly results hold if $q < \frac{2}{3}$ and $\frac{\alpha' \mathrm{G}'}{1\text{-}\alpha'} \geq \frac{\alpha_2' \mathrm{G}_2'}{1\text{-}\alpha_2'}$. Suppose $q \geq \frac{2}{3}$.

In this case the pirate's profit is,

$$\pi_c^{mp'*} = \frac{(1-\alpha')\theta_h^2(3q(\theta_h-\theta_l)+4\beta(1-q))}{16(\theta_h-\theta_l-\beta)(\theta_h-\theta_l)} - \alpha' G'. \qquad \text{(B15)}$$

There will be complete crowding-out if $q \geq \frac{2}{3}$ and $\frac{\alpha' G'}{1-\alpha'} < \frac{\alpha_3' G_3'}{1-\alpha_3'}$, where

$\frac{\alpha_3' G_3'}{1-\alpha_3'} = \frac{\theta_h^2(3q(\theta_h-\theta_l)+4\beta(1-q))}{16(\theta_h-\theta_l-\beta)(\theta_h-\theta_l)}$. The monopolist's profit is 0 and the pirate serves the entire market. The monopoly results hold if $q \geq \frac{2}{3}$ and $\frac{\alpha' G'}{1-\alpha'} \geq \frac{\alpha_3' G_3'}{1-\alpha_3'}$.

(α_2', G_2') and (α_3', G_3') are the minimum enforcement variable that deters the pirate's entry for $q < \frac{2}{3}$ and $q \geq \frac{2}{3}$.

<u>Proof of Proposition 7</u>

(i) The proof follows from (B12). Piracy in the network externality case, $(\theta_c^{lf'*} - \theta_x^{lf'*}) - (\theta_c^{lf*} - \theta_x^{lf*})$, is an increasing function of β.

(ii) $\frac{\alpha_2' G_2'}{1-\alpha_2'} - \frac{\alpha_2 G_2}{1-\alpha_2} = \frac{\beta q\theta_h^2}{16(1-q)(\theta_h-\theta_l)(\theta_h-\theta_l-\beta)} > 0$ and

$\frac{\alpha_3' G_3'}{1-\alpha_3'} - \frac{\alpha_3 G_3}{1-\alpha_3} = \frac{3\beta q\theta_h^2 + 4\beta(1-q)\theta_h^2}{16(\theta_h-\theta_l)(\theta_h-\theta_l-\beta)} > 0$. Now $\frac{\alpha' G'}{1-\alpha'} = \frac{c(\alpha')}{1-\alpha'}$ is an increasing function of α'. So $\alpha_2' > \alpha_2 \Rightarrow \mathrm{c}(\alpha_2') > \mathrm{c}(\alpha_2)$ and $\alpha_3' > \alpha_3 \Rightarrow c(\alpha_3') > c(\alpha_3)$.

(iii) Let $x_2 = \dfrac{c(\alpha_2')}{1-\alpha_2'} = \dfrac{q\theta_h^2}{16(1-q)(\theta_h - \theta_l - \beta)}$ and

$x_3 = \dfrac{c(\alpha_3')}{1-\alpha_3'} = \dfrac{\theta_h^2(3q(\theta_h - \theta_l) + 4\beta(1-q))}{16(\theta_h - \theta_l)(\theta_h - \theta_l - \beta)}$. x_2 and x_3 are increasing in β and x is an increasing function of α'. So α_2' and α_3' are increasing functions of β. ■