Zero Intelligence in Economics and Finance

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Abstract

This paper reviews the Zero Intelligence methodology for investigating markets. This approach models individual traders, operating within a market mechanism, who behave without strategy in order to determine the impact of the market mechanism and consequently the effect of trader behaviour. The paper considers the major contributions and models within this area from both the economics and finance communities before going on to examine the strengths and weaknesses of this methodology.

1 Introduction

The idea behind Zero Intelligence is very simple. By accurately modelling the market mechanism whilst assuming individuals have no strategy and behave at random the effect of the market mechanism may be observed. Consequently any effects that are not observed cannot solely be due to the market mechanism and so must require the interaction of the myriad of subtle strategies employed by market participants. As such it is possible to separate out the effects of the market mechanism and trader strategy and to determine the driving forces within markets.

In general analysing and explaining the effect of the market mechanism and certain aspects of market behaviour has proven to be a very difficult task. Two problems in particular arise. The first being that in order to accurately capture the effect of the market mechanism it is frequently necessary to model markets at a low level, often the level of individual bids and offers. This is difficult to do analytically and often requires very limiting assumption e.g. Parlour (1998) who models an order book with just four prices (ticks). One solution to this is the application of agent-based techniques. As Tesfatsion and Judd (2006) state in the preface to their Handbook, agent based computational economics models stress the economic processes and local interactions amongst economic agents rather than equilibrium properties. As such one of their strengths is that they can capture the subtleties of the environment within which the agents are interacting without having to worry about maintaining analytical tractability. Sunder (2006) notes that agent-based approaches can then be used to explore the link between the micro and macro behaviours. Zero Intelligence models follow this approach in order to facilitate their analysis of the market. It should be noted, however, that these models differ from much agent-based research in the simplicity of the agents (Tesfatsion, 2006). In the majority of agent-based computational economics models the agents learn or adapt to their economic environment[1], however, the explanatory power of Zero Intelligence models comes from their complete inability to do this.

The second problem in explaining the effect of the market mechanism has been that determining the effects of the interactions between, and even the types of, strategies has proved troublesome. Several studies have taken the approach of interviewing traders within financial markets regarding the strategies they employ (e.g. Dominguez, 1986; Oberlechner, 2001, to name but two). As would be expected the rational "fundamentalist" strategy is frequently cited in addition to the well-known chartist strategy. Unfortunately neither of these strategies is clearly defined. In the case of the fundamentalist strategy, the idea that the price will move back towards the fundamental is straightforward, however, how is the fundamental determined? Moving averages? Analysis of company profile? Stock dividends? Private information? Much comes down to the choice of the individual. The chartist strategy is even more complex. Chartists typically look for patterns in financial time series and from these patterns predict the future behaviour of the asset. The nature of these patterns can take many forms and frequently change with time scales. Traders frequently use a combination of these strategies coupled with gut feelings in order to make their trading decisions (Oberlechner, 2001). As a result understanding how an individual strategy behaves can be a difficult task, understanding how groups of these strategies interact and drawing out general conclusions is truly daunting.

The Zero Intelligence approach avoids this problem by removing all the

[1]see the many examples in Tesfatsion and Judd (2006)

complexity of strategies from the model resulting in a method for examining the market mechanism in isolation from the traders who populate the market. Consequence it is possible to see the effects of the market mechanism on trade and therefore to deduce the effect of trader strategy. These models have proven one of the most successful applications of agent based computational economics with notable pieces of work appearing within high quality economics (e.g. The Journal of Political Economy, The Quarterly Journal of Economics) and physical science journals (e.g. Proceedings of the National Academy of Science).

This paper will review the major contributions and models within both economics and finance. It will attempt to capture different variants of the Zero Intelligence approach rather than every application and will be influenced in particular by my own work in finance (Ladley and Schenk-Hoppé, 2007). The paper will finish by examining the strengths and weaknesses of this approach.

2 Economic Origins

The Zero Intelligence approach can trace its roots to prior to the advent of agent based computational economics. The Nobel Laureate Gary Becker employed this technique in an early paper (Becker, 1962) to analyse a model of markets in which participants behaved irrationally and in some areas at random. Using this model he found that features such as the downward slope of market demand curves and the upward slope of market supply curves typically associated with rational trader behaviour arose without any individual rationality. As a consequence he was able to deduce that these features were a result of the market mechanism that governed the interaction of the traders. In effect the market was creating system level rationality.

Although Becker managed to demonstrate certain results of market structure using this model, little further progress was made using this approach, as even without trader strategy the model is still very difficult to analyse. It was not until individual based computational simulation techniques became available that this research area made rapid progress.

2.1 Gode and Sunder

The first application of agent-based computational economics techniques to this area was the seminal work by Gode and Sunder (1993a,b) who examined the continuous double auction market mechanism. In their paper Gode and Sunder compared the performance of markets populated by human traders (graduate students of business) and markets populated by Zero Intelligence agents. In both cases a number of traders, N, ($N = 12$ in all experiments reported in Gode and Sunder (1993a)) were separated into two groups: buyers and sellers. Buyers were each given a number of reservation prices v_i, $i : 1...n$ each of which entitled the trader to buy one unit of a nameless commodity for the price p_i and receive the profit $v_i - p_i$. Sellers were allocated the right to sell one or more units of the commodity at a cost of c_i, $i : 1...n$ where c_i is the cost to the seller for selling unit i and $p_i - c_i$ is the profit. Traders were compelled to trade unit i prior to trading unit $i + 1$. Each trader only knew their personal demand (supply) function for the commodity, which was defined by his or her set of reservation prices (units for sale). They were not aware of other individual or the global demand or supply functions.

Traders interacted though the continuous double auction mechanism. Under this market mechanism all traders are able to submit shouts (bids in the case of buyers and offers in the case of sellers) at any time. These shouts consist of a price/quantity combination. In this experiment the quantity was constrained to be one so traders only submitted a price. A buyer could submit a bid at any price, subsequently other buyers could choose to improve on that bid by placing a higher priced bid. Simultaneously sellers could enter offers of decreasing value. If the prices of a bid and offer were equal or crossed a trade was executed at the price of the first shout. A successful trade cancelled all existing bids and offers.

In the case of the human experiment traders were permitted to make shouts at any time they wished during the four-minute experiment. In the computerised case traders were selected at random to make shouts throughout the thirty-second experimental duration. The decisions of human traders regarding how to trade and choice of price were governed by their own strategies. In contrast it was necessary to explicitly specify the strategies of the computational traders. Rather than attempting to copy the complex and often ill defined strategies employed by real traders this approach gains its power from assuming that traders have no strategy.

In order to separate the effect of strategy from that of the market mechanism two types of Zero Intelligence computational traders were defined. Both types of traders generate prices with uniform probability from the integer distribution $[1, 200]$. Neither type of trader learns, strategieses or examines the market, they behave at random, justifying the label Zero Intelligence. The first type of trader, labelled Zero Intelligence Unconstrained, behaves exactly in this manner. As a consequence they are able to submit bids and offers at prices that are loss making relative to their reservation prices. The second type of trader, labelled Zero Intelligence Constrained, operate under an additional constraint preventing them from shouting prices that would result in them making a loss. Comparison between the human and Zero Intelligence constrained traders was designed to provide insight into the effect of trader strategy and rationality. Comparison between the performance of the two types of Zero Intelligence traders highlighted the effect of the market mechanism.

Gode and Sunder (1993a) compared these three types of traders under five different supply and demand regimes over a number of simulated trading days. They found that as predicted by previous experiments (Smith, 1962) markets populated by human traders rapidly converged to the equilibrium price. Strikingly they found that markets populated by Zero Intelligence constrained traders behaved very similarly to those populated by human traders, both markets converged and achieved allocative efficiencies close to 99% [2]. In contrast markets populated by Zero Intelligence Unconstrained traders did not converge and achieved efficiencies of approximately 90%. Gode and Sunder also considered the distribution of profits within markets. They measured the profit dispersion[3] within the markets and found that Zero Intelligence unconstrained traders had a significantly higher profit dispersion than both Zero Intelligence constrained and human traders. Human markets also evidenced a weak tendedncy for a decrease in profit dispersion over experiemental days which was not shown in Zero Intelligence markets. Hence it was postulated that memory or strategy must be responsible for this effect.

These results demonstrated that the market mechanism and not trader strategy was the dominant factor governing the high allocative efficiency and convergence associated with continuous double auctions. As Gode and Sunder (1994) state "the ability of traders to observe, remember, and learn does not seem to affect the efficiency of simple double auctions". The results also suggested that although individual participants within a market may behave irrationally the aggregating effect of the market mechanism produces rational behaviour at the market level. This both contradicted and supported traditional economic thinking. Economic theory had hypothesised that the high performance of markets in allocating goods was due to the ability of traders to rationally maximise their expected returns. This result shifted some of the focus from the perfectly rational participant towards the market environment. By demonstrating that the market itself was efficient Gode and Sunder were able to argue the validity of models which assumed rational market behaviour as even without individual rationality the market at an

[2] In this context allocative efficiency is the ratio of the actual gains from trade to the potential gains from trade.

[3] $\sqrt{\frac{\sum_i (a_i - \pi_i)^2}{n}}$ where a_i is the actual profit achieved and π_i the theoretical equilibrium profit of trader i

aggregate level behaved in an efficient manner. As Sunder (2004) states "markets can exhibit elements of rationality absent in economic agents".

Gode and Sunder (1997) extended their work to examine exactly which market rules resulted in continuous double auctions having high allocative efficiency. In their paper they state three ways in which efficiency can be reduced in markets: traders make unprofitable trades, fail to negotiate profitable trades or extra marginal traders displace intra-marginal traders[4]. By performing experiments in which they changed the rules of the market in which the Zero Intelligence traders participated they quantitatively demonstrated the impact of each individual rule. They showed that if traders have the judgement and opportunity to turn down loss making trades the first source of inefficiency is avoided. The opportunity for traders to negotiate over several rounds reduces the second form of inefficiency. The third form of inefficiency is governed by several factors: the shape of the extra marginal supply and demand schedules, the use of the double auction rather than sealed bid system, public bids and offers and a price system (binding contracts and price priority). In addition if the market is synchronised rather than continuous i.e. all bids and offers are collected prior to trade rather than a trade being executed as soon as the best bids and offers cross efficiency was increased though at the cost of slower price discovery.

2.2 Extensions

The original Gode and Sunder (1993a) work has been extended by other authors to investigate related market mechanism questions. Bollerslev and Domowitz (1993) employ the Zero Intelligence approach to analyse the effect of an order book. The order book market mechanism differs from that of the standard double auction in that rather than forcing traders to improve on the best quotes (best bid and ask), the order book stores all bids and ask made by traders as limit orders - price quantity combinations entered into a book. The orders within the book are sorted by price, the most competitively priced being at the front of the queue with ties being resolved by quantity or time of submission, larger, older orders usually being favoured. Orders remain in the book until either the submitting trader cancels them or a trade satisfies them. When a shout is made it is checked against the best price on the opposite side of the book if the price of the bid is greater than or equal to the price of the offer a trade is executed at the price of the existing limit

[4]In order to be maximally efficient it is necessary for trades to be conducted between intra-marginal traders (those who value the commodity at prices that allow them to trade at the equilibrium price). If extra marginal traders are involved in trades then intra-marginal traders lose out and the total gain achieved is reduced.

order, if the quantities differ a limit order may be left for the remaining quantity. If the prices do not cross the shout is entered into the book as a limit order. Importantly the execution of a trade does not cause all previous limit orders to be cancelled, instead they remain in the book.

The order book mechanism results in additional properties that are desirable in some market situations. For instance, the book provides higher liquidity than a continuous double auction, the limit orders within the book provide the potential for large volumes of trade without having to wait for all traders to submit new bids and offers after every trade. The power of this market mechanism has led it being employed in many of the worlds financial exchanges both in its pure form such as the Paris Bourse and variants such as the New York Stock Exchange (with the addition of specialists).

Bollerslev and Domowitz (1993) focused on the effect of varying or restricting the size of the order book. Order books of size 1 (equivalent to a continuous double auction) to 6 were investigated. It was found that as order book size increased the trade price within the market became less volatile, average spread and deviation in the spread reduced, liquidity increased and orders executed more rapidly. By storing shouts the order book provided a form of system memory that stabilised the market and aided convergence resulting in better performance for the traders involved and the market as a whole.

Cason and Friedman (1996) investigated the Zero Intelligence behavioural model along with two others, Bayesian Game Against Nature (BGAN) and Waiting Game Double Auction (WGDA) in the context of the price formation process of double auctions. These models vary in their rationality, Zero Intelligence being completely irrational, BGAN being boundedly rational and WGDA being completely rational. Laboratory experiments were conducted with students and their results compared with those predicted by the models. Each model was found to explain some aspects better than others, BGAN did a good job of explaining the bid ask sequence, WGDA gives a better account of the certain effects in markets with unequal numbers of expert traders whilst Zero Intelligence does a good job of explaining the order of transactions and the outcomes in sessions with inexperienced traders. None of the models, however, could be considered to adequately explain price formation in double auction markets (Cason and Friedman, 1996).

Jamal and Sunder (1996, 2001) examine the case of markets with imperfect information and uncertainty of state. They used three variants of the Zero Intelligence traders in which the limits of the price range are dependant on biased heuristic, Bayesian and empirical Bayesian rules respectively. In all cases they showed that the Bayesian equilibrium could be obtained without profit maximisation, arbitrage or selection of strategies. Bosch-Domenech and Sunder (2000) demonstrated that market efficiency drops if traders are allowed to participate in multiple interlinked markets. Whilst Gode et al. (2004) extend earlier results to the general equilibrium setting of Edgeworth's box.

In a recent paper Gode and Sunder (2004) examined the effect of non binding price controls on market behaviour. In this work Gode and Sunder replicated with Zero Intelligence simulations experiments originally performed with humans by Smith and Williams (1981). These experiments used several supply and demand schedules to investigate the effect of imposing floors or ceilings on the prices of bids and offers which traders could submit. Traditional competitive equilibrium theory predicts that as these price ceilings and floors did not prevent traders trading at the equilibrium price they should have no effect on the behaviour of the market. However, experimental results (Isaac and Plott, 1981; Smith and Williams, 1981) clearly demonstrate that this is not the case. Gode and Sunder (2004) replicate five of seven effects highlighted in previous work which had been attributed to humans modifying their strategy based on the presence of the price control. Due to the lack of strategy in this model Gode and Sunder assign a different explanation to these effects: namely that traders do not adjust their strategy sufficiently to deal with price controls. This view is supported by the observation that non binding price controls have no effect on transaction prices if they are introduced after the market has settled to equilibrium. In this case traders have already learned about the market and so are able to adapt their strategies accordingly. In addition they again demonstrate the role of extra marginal buyers and sellers in determining market efficiency although classical competitive equilibrium theory predicts they should have no effect.

The Zero Intelligence model has also been employed in analysing bubbles and crashes in asset markets. Duffy and Unver (2006) investigate whether data from experiments performed on human market participants which resulted in bubbles can be qualitatively and quantitatively matched to those generated by simulations using Zero Intelligence traders. Within these experiments a dividend paying asset is bought and sold by the traders over a series of trading periods. The asset pays at the end of each period. At the end of the final trading period the asset is bought back from the traders at a predetermined price. Consequently the fundamental value of the asset decreases over the length of the experiment. This, however, is frequently not reflected in experimental time series. Instead a hump shaped distribution is observed in which trade prices tend to be higher than the fundamental value early in the experiment and then crash below the fundamental towards the end.

In order to capture this behaviour three departures were made from the standard Zero Intelligence set up. Firstly, whether a trader is a buyer or seller is dependant on the period of the market. Initially there is a 50% chance of being a buyer within the market, however, this probability decreases linearly to 0% as time passes. Secondly, rather than reservation prices, traders are given a money account, they may make trades as long as they maintain a non-negative balance in this account. Finally the shouted price is no longer completely random. Instead it is a factor of two components, an anchoring component and a random component drawn from a uniform distribution which decrease in size as time progresses. As a result of these modifications these traders are termed near Zero Intelligence (Duffy and Unver, 2006). Values of the model parameters were estimated form experimental data. This model was used to explain and replicate the characteristic shape of bubbles and crashes observed in experimental asset markets in addition to features such as changes in the relationship between price movements and the relative numbers of bids and offers submitted. In some cases these replications are quantitative rather than qualitative. This model provides useful insights into the phenomena of bubbles and crashes though due to the additional processes involved in trader decision making it is not possible to say the shape of these bubbles is due to the market structure. Instead the model demonstrates that very little strategy is required in order to generate these patterns and that the anchoring hypothesis may be an important feature in their occurrence.

Crockett (2008) employed Zero Intelligence traders amongst other types along with human subjects to further investigate the results of Crockett et al. (Forthcoming) regarding the learning of competitive equilibrium in exchange economies. The evidence for this is mixed, however, he does find that heterogeneity amongst individuals plays a large part in this process in particular the willingness of individuals to give up immediate gains when they expect the market to move in a favourable direction.

2.3 Concerns about Zero Intelligence

Concerns have been raised about the models appropriateness in analysing market behaviour. Notable amongst these criticisms are those of Cliff and Bruten (1997). By re-implementing the simulations presented in Gode and Sunder (1993a) Cliff and Bruten were able to demonstrate that the accuracy with which the model captures the behaviour of real markets is dependant on the supply and demand functions. When the supply and demand functions are approximately symmetric, as is the case in Gode and Sunder (1993a), the model accurately reflects experimental results. However, as supply and demand become less similar the Zero Intelligence model does an increasingly poor job of replicating reality. As a consequence it can be argued that the model only serves as a viable baseline for the effect of the market mechanism under relatively balanced supply and demand functions.

Cliff and Bruten (1997) attempted to improve on this model by adding a simple learning mechanism resulting in "Zero Intelligence Plus" (ZIP) traders which are only slightly more intelligent than Zero Intelligence traders. The learning rule uses information gained from competitors' shouts to maximise the profit of the individual. Each trader maintains a profit margin, m, that reflects that individuals' belief of the profit which it can obtain from a successful transaction. The price, p, a trader will shout or accept for the commodity is given by $p = \lambda(1 + m)$ where λ is the traders reservation price. Where $m \geq 0$ for sellers and $-1 \leq m \leq 0$ for buyers. All traders follow rules to adapt their profit margin every time another trader shouts in order to maximise gains.

The profit margin is adjusted using the Widrow-Hoff learning rule with momentum (Widrow and Hoff, 1960). The ZIP rule at time t is

$$m(t + 1) = (p(t) + F(t))/(\lambda - 1)$$

where $p(t)$ is the trader's valuation as defined above and $F(t)$ is

$$F(t + 1) = \gamma F(t) + (1 - \gamma)\delta(t)$$

and

$$\delta(t) = \beta(\tau(t) - p(t))$$

γ is the momentum term and $\tau(t)$ is the target output price and is calculated as

$$\tau(t) = R(t)q(t) + A(t)$$

where $R(t)$ is a small relative perturbation, $A(t)$ is a small absolute perturbation and $q(t)$ is the shouted price.

Cliff and Bruten demonstrate that markets populated by Zero Intelligence Plus traders behave in line with experimental evidence under a wider range of supply and demand schedules than the standard Zero Intelligence traders. In particular they perform significantly better in some of those in which Zero Intelligence traders fail to converge.

This enhancement expands the range over which the model captures reality at the cost of lost simplicity. The original Zero Intelligence model gained its explanatory power from the agents' lack of strategy. By comparing the cases in which traders had no strategy with experimental results the effect of strategy could be deduced. By adding intelligence, even as small an amount as is added in this case, the same deductions are no longer possible. The difference between experimental and model results can no longer be put down to the effect of strategy and the difference between random behaviour and the model behaviour can no longer be said to be due to the effect of the market mechanism, instead the conclusions are more blurred. Adding increased cognitive ability increased the range over which conclusions from the model can be said to be applicable, however, the conclusions that can be drawn are weaker.

The addition of simple strategies does allow this model to be used in answering alternative questions which could not be approached with the Zero Intelligence model. Cliff and Bruten (1999) examine an alternative market mechanism demonstrating its inferiority to the continuous double auction and ZIP traders ability to reproduce experimental findings. Whilst Cliff (2003) use a genetic algorithm to evolve strategies with which he demonstrated that supply and demand conditions in addition to the presence and nature of market shocks effect the optimal strategies and the ideal market design. The parameter space of this model has more recently been extended from 8 to 60 parameters Cliff (2006) allowing finer control of behaviour in different market conditions. Zero Intelligence Plus traders have bee used to investigate sealed bid auctions Bagnall and Toft (2004, 2005) and to examine the behaviour of markets with incomplete flows of information and trade Ladley and Bullock (2006).

3 Finance

In addition to economic theory Zero Intelligence models have made a considerable contribution to the understanding of financial markets. Financial markets are frequently based on the order book market mechanism (as investigated by Bollerslev and Domowitz, 1993). The exact rules differ from market to market, however, several such as the Paris Bourse use a pure order book mechanism[5].

Financial markets present an attractive application for these models due to the large amount of data available. Analysis has exposed the existence of certain regularities or "stylised facts" within price time series, for instance clustered volatility or correlations in price movements at different lags have both been frequently observed (see Cont, 2001, for a review). More recently large data sets at the level of individual orders have become available. These

[5]In contrast markets such as the New York Stock Exchange simultaneously employ an order book and a specialist who makes strategic decisions in order to make profit whilst simultaneously supplying a service to traders such as liquidity and continuity of price.

data sets allow for reconstruction of the order book and a like for like comparison to be carried out between the dynamics of the Zero Intelligence models and real markets. Explanation of these elements has been a central goal of many of the Zero Intelligence financial models as the causes of these facts are rarely obvious. Many can be explained through the complex strategic interactions of traders, however, in order to do this assumptions regarding the nature and quantities of traders within the market are required. In comparison a Zero Intelligence approach has allowed researchers to explain many of these stylised facts as being a result of the market mechanism without strategic trader behaviour.

The papers presented within this section employ a variety of techniques. Early work tended to use mathematical and statistical techniques to examine the arrival of random limit orders at an order book. Physicists in addition to economists have conducted a large body of the more recent work. As a consequence as well as being agent based many of the models have strong links with physical systems in terms of the rules or equations governing the dynamics. In particular these models frequently use analogies to particle systems, mean fields or more simply statistical techniques that are commonly employed within the physics community. As this work was performed separately from the economics work discussed above (and in some cases predates it), much of the work does not describe itself as Zero Intelligence or justify it methodology in the same manner.

There were several early mathematical approaches. The first being that of Mendelson (1982) who employed analytical techniques to investigate the accumulation of limits orders within an order book subject to periodic clearing. The analysis demonstrated the effect of market clearing on the probability of trade, the spread and volatility. In addition a critical trading volume was also shown to exist. If the expected number of transactions between clearing events is less than this value the market may lack liquidity, if more are present then the market behaves in a relatively smooth manner. Success was also gained by modelling the order book as queues, subject to random order flow, allowing the use of standard results form this field. Notably Cohen et al. (1985), employed a model which allowed only two prices and later Domowitz and Wang (1994) developed a model removing this restriction. These models both employ Poisson processes to simulate the arrival of limit and market orders in addition to order cancellation which allowed results to be derived regarding the waiting time to trade and the distribution of buy and sell limit orders. Additionally Domowitz and Wang used their model to compare the same dynamics in continuous double auctions and single price periodic auctions. Bollerslev et al. (1997) went on to test this model using Deutschemark/US Dollar data.

In a series of analytical papers Delong, Shleifer, Summers and Waldman DeLong et al. (1989, 1990a,b, 1991) introduce and develop a model of noise traders; traders who make random trading decisions in a similar manner to Zero Intelligence traders. Through their random behaviour they increase the risk of investing in the risky asset. As a consequence even in the presence of completely rational traders, noise traders create their own niche allowing them to survive.

Bak et al. (1997) take a particle model approach involving noise and fundamental traders. Noise traders employ a range of strategies, the most simple is to increase or decrease their price by one tick randomly irrespective of other traders behaviours, in effect adopting Zero Intelligence behaviour. Several formulations of the model are investigated, gradually strategy is added to the noise trader behaviour: allowing drift towards the current price and copying of other trader behaviours. In the simplest form of this model only the most random instantiation of the noise traders are considered. This allows the model to be analysed as a particle system of the form $A + B \rightarrow 0$ (in this case buy and sell orders react and are both removed). Fundamental traders, when they are investigated, are risk averse and attempt to maximise a short run utility function dependant to the dividend of the share. These traders then buy or sell the stock if certain floors or ceilings are crossed in the price level. The model reproduces results seen in empirical data including larger than Gaussian fluctuations at short time scales and some of the power law form of price variations. Whilst the Hurst exponent[6] of $\frac{1}{4}$ produced by the model disagrees with that measured in financial data, typically $0.6 - 0.7$, it does demonstrate that non trivial behaviours can be generated by simple systems with many interacting agents. The model is powerful in that it is amenable to analytical solution, however, its limited ability to capture the dynamics of limit order markets does limit its applicability. The addition of fundamentalist traders and more realistic behaviour for the noise traders results in "statistics of price variation which are consistent with empirical results" (Bak et al., 1997). The addition of rational traders also leads to results that show when the number of noise traders relative to rational traders is high bubbles are common.

Eliezer and Kogan (1998) extended the above model to investigate the scaling laws of several measures of liquidity. They showed that the spread scales as $\sqrt{\frac{1}{DealRate}}$ before going on to investigate extensions to the basic

[6] The Hurst exponent characterises the tendency of a time series to regress towards the mean. Low Hurst exponents, below 0.5, indicate a tendency to revert to the mean, high Hurst exponents, above 0.5, indicate that extreme events are more common. A Hurst exponent of 0.5 characterises a random walk.

reaction diffusion setup. Tang and Tian (1999) investigated the diffusive dynamics within the Bak et al. (1997) model. They showed that by biasing diffusion towards the market price the Hurst exponent converges to that of a random walk at long time periods in line with empirical evidence.

Maslov (2000) takes a different approach. Within this model traders randomly choose between submitting a limit order or market order. All orders are for a single unit of the stock, therefore market orders are matched against and eliminate the best order on the opposite side of the book. Limit orders were placed at a price based on the last transaction price. If the last price was a buy the limit order was placed at a price a small random number (uniformly distributed $[0, 4]$) bellow the last transaction price and vice versa for sells. From this model Maslov is able to approximate several stylised facts observed in financial markets including similarities in the autocorrelation function of the absolute value of price changes and short range correlations in the signs of price movements along with evidence of fat tails in the histogram of price increments. In line with previous work their model produces a non trivial Hurst exponent of the price graph of $\frac{1}{4}$, again different from that found in real markets. Slanina (2001) furthers this work by using a mean field approximation to solve the model. From this work additional conclusions may be drawn regarding the behaviour of the system, in particular negative short-time autocorrelation of price changes and differences in the power law exponent of price changes due to difference in assumption regarding the order density.

Challet and Stinchcombe (2001) conduct static and dynamic analysis's of limit order market behaviour based on empirical data (the Island ECN order book). Form this analysis they criticise the price behaviour of both the models of Bak et al. (1997) and Maslov (2000). In particular the Hurst exponent of $\frac{1}{4}$ produced by both models at all time scales rather than the empirically observed $\geq \frac{1}{2}$ at short time scales and $\frac{1}{2}$ at longer scales. They suggest a particle model which partially resolves this problem. Buy and sell offers are generated at random and submitted to the order book. The quantity submitted is always one unit. The price is drawn from a Gaussian distribution centred on the best bid or best ask as appropriate, consequently the submission of orders changes the probability of orders being submitted at each price subsequently. Orders are removed (evaporate) with uniform probability. By assuming that the number of orders deposited is drawn from an exponential distribution Challet and Stinchcombe (2001) are able to reduce the area in which the price is under diffusive. In addition they observe power law distributions and under certain parameters volatility clustering. Bouchaud et al. (2002) develops a model along similar lines to that of Challet and Stinchcombe (2001). They use data obtained from the Paris Bourse to inform the creation of a simple Zero Intelligence numerical model with which they are able to replicate features of the order book shape and statistics of limit order prices observed in the empirical data.

In a series of papers Daniels et al. (2003), Smith et al. (2003) and Farmer et al. (2005) develop a Zero Intelligence model based on a random order submission processes. They model two types of Zero Intelligence traders: patient and impatient. If $a(t)$ is the best ask price and $b(t)$ is the best bid price, patient buyers place orders uniformly in the range $[-\infty, a(t)]$. This is defined analogously for sellers. Limit orders arrive with Poisson Rate parameter α which corresponds to shares per unit price per unit time. Orders are of size σ shares and are subject to Poisson cancellation process with rate δ. Impatient traders only place market orders, these market orders have a total quantity of μ per unit time.

Daniels et al. (2003) introduces the model and makes some initial predictions. Smith et al. (2003) goes on to develop the model analysis using both analytical techniques based on mean field theory and simulation. They are able to demonstrate properties of the bid ask spread, the probability of orders being filled and price volatility amongst others. They also emphasise the importance of order size rather than tick size in governing market dynamics. In their later paper Farmer et al. (2005) go on to estimate the values of the parameters, α,σ,δ,μ, from data form the London Stock Exchange and the predictions of the model are compared with empirical findings. This simple Zero Intelligence model does an excellent job of explaining certain aspects of the market behaviour. With regard to the spread the Zero Intelligence model explains 96% of the variance of the spread within the data set. Simultaneously the model explains 76% of the variance in the price diffusion whilst also providing insights into the market impact of orders.

Ladley and Schenk-Hoppé (2007) extended the model of Bollerslev and Domowitz (1993) by allowing traders to make orders for multiple units of the asset and introducing a process by which traders enter and leave the market at random. As a consequence the supply of intra-marginal units was never exhausted and so trade continued indefinitely resulting in a better fit to the dynamics of a financial market. Ladley and Schenk-Hoppé were particularly interested in the distribution of orders within the market. In order to study this orders were separated into 12 different classes based on their aggressiveness[7]. This model demonstrated that the positive correlation between order types observed in financial data (e.g. Biais et al., 1995) and elements of the order book shape are a result of the market mechanism.

[7] Order aggressiveness was defined by Biais et al. (1995) based on the quantity of the order and the price relative to the best bid and offer quotes.

Where as the relative frequencies of different order types highlight strategic trade offs. The model also demonstrates that some of the price predictability observed in financial data originates from the structure of the market.

4 Evaluation: Strengths and Weaknesses of Zero Intelligence

The preceding sections have described the application of Zero Intelligence within economics and finance. This section will examine the strengths and weaknesses of this approach in general.

Zero Intelligence models have allowed researchers to gain insight into market dynamics without having to make diverse behavioural assumptions regarding the strategies of traders. From the earliest analytical (Becker, 1962) and simulation (Gode and Sunder, 1993a) models the Zero Intelligence framework has demonstrated to researchers that the market mechanism is capable of imposing rationality at the market level even though individual participants may behave completely at random. As a consequence of this rationality a Zero Intelligence model of a market may display characteristics closely matching those of a real market even though the traders within it differ in their sophistication.

The complete lack of strategy of Zero Intelligence traders eases the understanding of the behaviour of markets they occupy. The process of order submission to a market, in particular an order book market, naturally results in complicated dynamics. Traders submit orders that depending on the state of the order book or best quotes may or may not result in a trade and may or may not result in a change in the best prices. The next order to be submitted to the market may then arrive in the same or a different market situation to the previous order. The state of the market changes over time, the same order submitted at two different points in time may have a completely different effect. The addition of strategy makes this situation considerably more complex. The submission of an order is then dependant on the state of the market at the time of the submittal. A feedback loop is formed: a trader submits orders which effect the state of the market which effect the decisions of the trader on what order to submit. Finally if traders have heterogeneous strategies the complexity is increased. The previous feedback loop exists, however, the order in which traders submit orders becomes significant, different traders perform different actions in the same situation. Zero Intelligence traders eliminate the second and third forms of complexity[8], their decisions ignore the state of the market and the motivations of the other individuals. As a consequence individual traders behave in a very simple predictable manner.

This simplicity is useful to the researcher in understanding the behaviour of their model. For instance Gode and Sunder (1997, 2004); Ladley and Schenk-Hoppé (2007) all use the simplicity and regularity of Zero Intelligence traders to explain complex market behaviours. In contrast models that attempt to capture strategic behaviour can encounter difficulties in pinpointing the cause of a particular effect. For instance the Sante Fe Artificial Stock Market Arthur et al. (1997), contains an evolutionary process, a classifier system, learning and multiple traders in addition to a market mechanism. This model is able to demonstrate and reproduce several features observed in financial markets, however, the interaction of the many process make explaining the appearance of these elements very difficult.

The simplicity of these model often provides an additional benefit, in some cases they are analytically tractable. The use of both standard mathematical and physics based analyses has allowed several general rules to be drawn regarding the behaviour of continuous double auctions and order book markets (e.g. Farmer et al. (2005). These rules may then be tested against empirical data in order to confirm the validity of the rule and to make predictions. Verification in this manner allows further development and refinement of the agent based model and consequently further understanding of the system to proceed with confidence. As such Zero Intelligence models provide a link between analytics, empiric's, simulation and experiments.

Zero Intelligence may provide a baseline against which other experiments can be performed. Studies such as that of Chiarella and Iori (2002) and Bottazzi et al. (2005) investigate the behaviour of order book markets populated by strategic traders within an order book market setting. Models of this type are important in understanding the behaviour of markets, in particular when coupled with empirical and experimental work informing the design of the strategies of the market participants. Zero Intelligence models can play an important role in this area by highlighting which aspects of the market behaviour are explained by the market mechanism allowing the researcher to focus on those which require trader strategy. As Cason and Friedman (1996) state Zero Intelligence is the natural source for a null hypothesis in assessing a more complicated model.

Although Zero Intelligence models do have notable advantages for in-

[8] In many models, in particular those which are agent based, the individuals do differ e.g. Gode and Sunder (1993a) in which each trader has their own supply of demand function. This does not cause a large problem in general for zero intelligence models as the simplicity of strategy aids in reducing complexity.

vestigating markets there are also several notable problems. As Cliff and Bruten (1997) show, Zero Intelligence markets only accurately reproduce the behaviour of experimental markets under conditions in which the supply and demand schedules are near symmetric. When there is a heavy imbalance the market may converge to the wrong price or fail to converge at all. As a consequence a Zero Intelligence model would not be appropriate for examining markets in these types of situations as results obtained would not accurately reflect reality.

Zero Intelligence traders do not learn from their experiences, as a consequence they poorly replicate experimental results when a market situation are repeated or expert traders are employed (Cason and Friedman, 1996). As Gode and Sunder (2004) point out the convergence of real traders towards the market equilibrium price is well captured by a Zero Intelligence model. However, if repeated trading days are conducted with the same traders and supply and demand functions human traders learn from their previous experience and so start trading very close to the equilibrium price, they do not have to repeat the convergence process. In contrast Zero Intelligence traders return to the start of the convergence process. In modelling markets in which the supply and demand of the commodity remain constant for many trading periods, the Zero Intelligence traders inability to learn may lead to spurious results. If only the convergence process is of interest or if the market supply and demand regularly change Zero Intelligence models may be appropriate. This is the case in financial markets where the asset regularly changes value and is evidenced by the large number of successful studies in this area. For markets in which this is not the case it may be necessary to explicitly model the traders learning process e.g. Cliff and Bruten (1997). By adding complexity the model is able to perform well under a wider range of conditions, at the cost of increased difficulty in understanding the market dynamics and separating the effect of trader behaviour from the market mechanism.

In order to capture the most benefit form these models and insight into market behaviour it is necessary to accurately capture the dynamics of the market mechanism, including the process of order submission, being modelled. Since traders have no intelligence, the market mechanism is the sole governor of individual behaviour. As a consequence if the rules of the mechanism, how orders are submitted, traded and cancelled, are not accurately captured the dynamics of the model may be misleading. For instance Bak et al. (1997) base their model on the reactions of two gases injected into a tube from either end. The advantage of this mechanism is that the model is sufficiently close to certain physics models that it can be analysed using the same tools. However, limit orders do not diffuse randomly in the same manner as gas particles which can be seen in the under-diffusive behaviour in the price process. As a consequence if the model of the market mechanism and order submission are not good reflections of reality the resulting conclusions may be misleading. Extra care must also be taken in the case of markets which do not easily fit this paradigm such as hybrid markets like the NYSE. These market cannot be expected to behave in the same manner as a pure limit order or continuous double auction market modelled in the majority of Zero Intelligence experiments as the market mechanism includes a specialist, an individual who has a significant effect on the market mechanism through his strategic decisions.

5 Conclusion

This paper has discussed the Zero Intelligence model in the investigation of economic and financial systems. The use of the Zero Intelligence model together with agent-based computational economics techniques has allowed economists, physicists and computer scientists to investigate issues that had proved to be analytically very difficult to penetrate and to generate results of interest to the economics and science communities at large. Within this paper we have examined the key models within this area, starting from the analytical models which inspired the later work and then considering in detail those that have been key to it's development. We have also examined the strengths and weaknesses of this approach, highlighting the successes it can bring when combined with other methodologies whilst pointing out the potential dangers of its application.

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