Simulation in the Social Sciences

Lectures:

1. **Introduction: Modelling.**

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1. **Introduction: Modelling.**
   and Simulation.

2.

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2. **Micro Analysis and Cellular Automata**

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6. **Learning and Simulation.**

1. Modelling — from March & Lave (1975)

1.1 Overview

A. What is a model?

B.

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A. A model:

- a simplified picture of a part of the real world.
- has some of the real world's attributes, but not all.
- a picture simpler than reality.

We construct models in order to explain and understand.

Why model? (from Josh Epstein, *JASSS*, 2008))

To:

1. Explain (very distinct from predict)
2. Guide data collection
3. Illuminate core dynamics
4. Suggest dynamical analogies
5. Discover new questions
6. Promote a scientific habit of mind
7. Bound (bracket) outcomes to plausible ranges
8. Illuminate core uncertainties.
9. Offer crisis options in near-real time
10. Demonstrate tradeoffs / suggest efficiencies
11. Challenge the robustness of prevailing theory through perturbations
12. Expose prevailing wisdom as incompatible with available data
13. Train practitioners
14. Discipline the policy dialogue
15. Educate the general public
16. Reveal the apparently simple (complex) to be complex (simple)

Three Rules of Thumb for Model Building:

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That is, an interplay between the real world (truth), world of æsthetics (beauty), world of ethics (justice), and the model world.

(See the March & Lave extract at the SimSS web page.)
http://www.agsm.edu.au/bobm/teaching/Taiwan/marchlave.pdf

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Prices, Costs, and Values → Profits

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We use verbal, graphical, and algebraic models of how consumers, firms, and markets work.

We assume *rationality*: that economic actors (consumers and firms) will not consistently behave in their own worst interests.

Not a predictive model of how individuals (always) act, but nonetheless robust in aggregate.

1.2 Modelling

Speculations about human behaviour/social and organisation interactions.

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- **contemplating**
- **testing**
- **revising**

— models of behaviour.

What is a model?

- We can have several models of the same thing, depending on which aspects we want to emphasise, how we will use the model.

- Models are constructs to explain and appreciate aspects of the real world.

So ...

Need *skills* of:

- —

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Q: If we cannot understand individual behaviour, then how are we to understand systemic/social/bureaucratic behaviour?

Six familiar models in the social sciences:

1. individual choice under uncertainty
2. exchange/trade
3. adaptation of ideas/technology
4. diffusion of ideas/technology
5. transition
6. demography

Each is treated by March & Lave (1975).

1.3 Model of the Model-Building Process

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4. Are these *true*? If not, speculate on other models/processes.

Case 1: Contact and Friendship.

Why are some people friends and not others?

e.g. **In a hall of residence,**
obtain lists of friends and their room numbers

Observe: friends live close together.

A process to generate this?

What is a possible process that might produce the observed result?

Two Speculations about Process:

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   What changes in these friendship clusters over time?

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We want to include earlier predictions but find a more general model that predicts new behaviours as well, more widely.

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   Ordinarily, the more situations a model applies to, the better it is and the greater the variety of its possible implications.

1.5 Evaluation of Speculative Models

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II. Beauty
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- **Surprise!**

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And suppose (we still have a couple more) that no one divorces (an Irish folk tale) or sleeps around (a Scottish folk tale) without precautions (a Swedish folk tale).

And suppose that the expected sex (technical term) of a birth if all couples are producing equally is half male ♂, half female ♀ (though mostly they are one or the other)."

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"Question: (Are you ready?) What will be the ratio of boys (with) to girls (without) in such a society?"

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for *society*: more girls than boys,

Let's simulate this using NetLogo.

http://www.agsm.edu.au/bobm/teaching/SimSS/NetLogo41-models/boysngirls2.html

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Need *Critical Experiments:*

compare alternative models
with the same question $\rightarrow$ different answers:
this is critical.

Beware Circular Models:

a. "when the rain-dance ceremony is properly performed, and all the participants have pure hearts, then it will rain" — testable?

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c. Monty Python's "the man who claims he can send bricks to sleep"

e.g. Case 3): The Case of the Stupid Question

e.g. "a surfer asked a stupid question in class"

Speculations:

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Speculations:

A. **not enough time to study**

B. **success on the board is sufficient for her**

C. **jealous of her prowess at surfing, the rest of us look down on her classroom performance and interpret her questions as "stupid"**

D. **Others?**

How do the Implications Differ?

	Speculation		
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Q1: will surfers ask stupid questions out of season?	no	yes	yes

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	Speculation A	Speculation B	Speculation C
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Q3: will surfers who don't look like surfers ask stupid questions?	yes	yes	no

(Note: we have already generalised: *surfers* plural.)

The Importance Of Being Wrong

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- evaluate rather then defend (avoid "falling in love" with your model)
- delight in finding fault — be skeptical and playful
- always think of alternative models and explanations the academic ability of the surfer

2. Simulation

The anecdote about the economist looking for his lost car keys:

"An accurate answer to the wrong question"? (using closed-form methods)

or: simulation (numerical methods)

"Approximate answers to the right questions"

Helped by the developments in computer hardware and software.

Meanwhile: Computer Science has borrowed simulation tools from the natural world:

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Want: dynamics, out-of-equilibrium characterisations in our models.

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But at the micro level, the agents in social science models are people, with self-conscious motivations and actions.

Beware: Aggregate behaviour may be well described by differential equations, with little difference from models of inanimate agents at the micro level.

The Five Functions of Simulations:

(from Hartmann 1996)

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4. As a **Tool for Experimentalists** — to support experiments.
5. As a **Pedagogic Tool** — to gain understanding of a process.

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  Q: does more realistic assumption → more accurate prediction?

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- Solution of a set of equations describing a complex (e.g. bottom-up) interaction.
- *Discrete* (Cellular Automata): if the model behaviour ≠ empirical, it must be because of the transition rules.
- *Continuous*: not so clear-cut: background theory v. model assumptions

  Q: does more realistic assumption → more accurate prediction?

"A simulation is no better than the assumptions built into it" — Herbert Simon

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Steve Durlauf: Is there an underlying optimisation by agents? (his "Complexity and Empirical Economics," *EJ*, 2005)

3. As a Substitute for Experiment

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- ***pragmatically* impossible: scale, time; or**
- ***theoretically* impossible: counterfactuals; or**
- ***ethically* impossible: e.g. taxation, no minimum wage;**

***or* to complement lab experiments**
(See the link to Monte Carlo Probablistic Sampling.)

e.g. Agent-Based Models v. Economic Experiments

Hailu & Schilizzi (2004, p.155) compare and contrast ABMs with experiments using human subjects, under the headings:

- **Approach to inference, or micro-macro relationship**
- **Specification of behavioural rules**
- **Informational problems**
- **Degree of control**
- **Explanation of agents' choices**
- **Temporal length of analysis**
- **Representativeness / realism**
- **Data**
- **Cost**

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- to inspire experiments
- to preselect possible systems & set-ups
- to analyse experiments
  (statistical adjustment of data)

5. For Learning

A pedagogic device through play ...

See Mitchell Resnick. *Turtles, termites, and traffic jams: Explorations in massively parallel microworlds*. MIT Press, 1994.

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Play with NetLogo models, and experience emergence: Life is famous, and others too.

See the Models Library that comes with the NetLogo download. e.g. Segregation

Summary

A simulation imitates one process by another process

With Social Sciences: few good descriptions of static aspects, and even fewer of dynamic aspects

(Remember the economists' focus on: existence, uniqueness, stability)

Robust Predictions from Simple Theory

(from Latané, 1996)

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3. **As an "easy" alternative to thinking: robust coding**
4. **As a machine for discovering consequences of theory: if this, then that. (i.e. sufficiency).**

A Third Way of Doing Science DIS

(from Axelrod & Tesfatsion 2006)

Deduction + Induction + Simulation.

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- Deduction: deriving theorems from assumptions
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- Simulation: assumptions $\rightarrow$ data for inductive analysis

S differs from D & I in its implementation & goals.

S permits increased understanding of systems through controlled computer experiments

Emergence of self-organisation (Miller & Page, 2007, Ch. 4)

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Adam Smith's Invisible Hand $\rightarrow$ prices

Schelling's residential tipping (segregation) model:
People move because of a weak preference for a neighbourhood that has at least 33% of those adjoining the same (colour, race, whatever) $\rightarrow$ segregation.

Need models with more than one level to explore emergent phenomena.

Families of Simulation Models

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4. Learning Models LM
   (from Simulated Evolution and from Psychology)

Comparison of Simulation Techniques

Gilbert & Troitzsch compare these (and others):

Technique	Number of Levels	Communication between agents	Complexity of agents	Number of agents

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This is necessary for investigating emergent phenomena.

So "agent-based models" excludes simple Systems Dynamics (SD) models, but can include the others.

Simulation: The Big Questions

from: www.csse.monash.edu.au/~korb/subjects/cse467/questions.html

- What is a simulation?
- What is a model?
- What is a theory?
- How do we test the validity of any of the above?
- When do we trust them, what sort of understanding do they afford us?
- What is an experiment? What does it mean to experiment with a simulation?
- What is the role of the computer in simulation?
- How does general systems dynamics influence simulations?
- How do we handle sensitivity to initial conditions?
- How precisely can a simulation approximate real life / a model?
- How do we decide whether to use a theory / model / simulation / lab experiment / intuition for a given problem?
- Does a simulation have to tell us something?
- How complex is too complex, how simple is too simple?
- How much information do we need to (a) build and (b) test a simulation?
- How/when can the transition from a quantitative to a qualitative claim be made?

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To Verify: use a suite of tests, and run them every time you change the simulation code — to verify the changes have not introduced extra bugs.

Perhaps code using a different platform, or dock.

Validation

Ideally: compare the simulation output with the real world.

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Use Sensitivity Analysis, to ask:

- **robustness of the model to assumptions made**
- **which are the crucial initial conditions/parameters?**

use: randomised Monte Carlo, with many runs.

Judd's ideas (2006)

"Far better an approximate answer to the right question ... than an exact answer to the wrong question."

— John Tukey, 1962.

That is, economists face a tradeoff between:

the numerical errors of computational work
and
the specification errors of analytically tractable models.

Judd on Validation

Several suggestions:

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   If found, then insights into when the proposition fails to hold.
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5. Synergies between Simulation and Conventional Theory.

Axelrod on Model Replication and "Docking"

***Docking*: a simulation model written for one purpose is aligned or "docked" with a general purpose simulation system written for a different purpose.**

Four lessons:

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4. **Minor procedural differences (e.g. sampling with or without replacement) can block replication, even at (b).**

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4. Software and/or hardware subtleties.
   e.g. different floating-point number representation.

(See Axelrod 2006.)

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- **what should be? (i.e. prescription, normative)**

Consider historical market data:

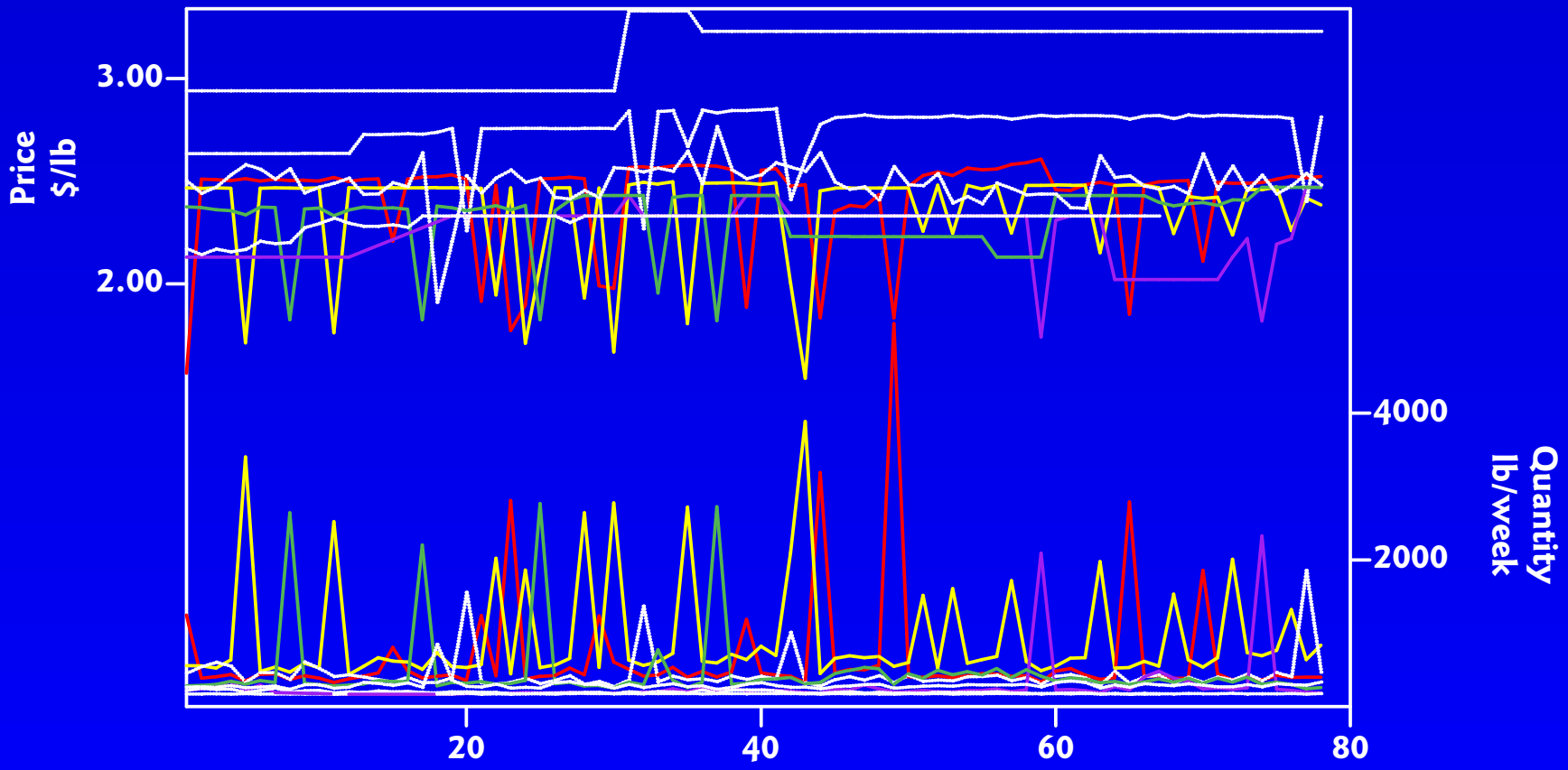

Figure 1: Weekly Prices and Sales (Source: Midgley et al. 1997)
(Coloured lines: Folgers, Maxwell House, Hills Bros, CFON)

Stylised Facts of the Market Behaviour

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- **Much movement in prices and quantities of four brands — a rivalrous dance.**
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— reactions by brands?
— actions by the supermarket chain?
— unobserved marketing actions?

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Similar (qualitatively or quantitatively) to the brands' behaviours of pricing and sales.

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Note: assuming profit-maximising (or purposeful) agents means that we are not simply curve-fitting or description using D.E.s. Going beyond the rivalrous dance.

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Examine:

- **limits of behaviour**
  (Miller's Automated Non-linear Testing System)
- **regime-switching**
- **range of behaviour generated**
- **sensitivity of the aggregate (or energent behaviour) to a single agent's behaviour.**

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