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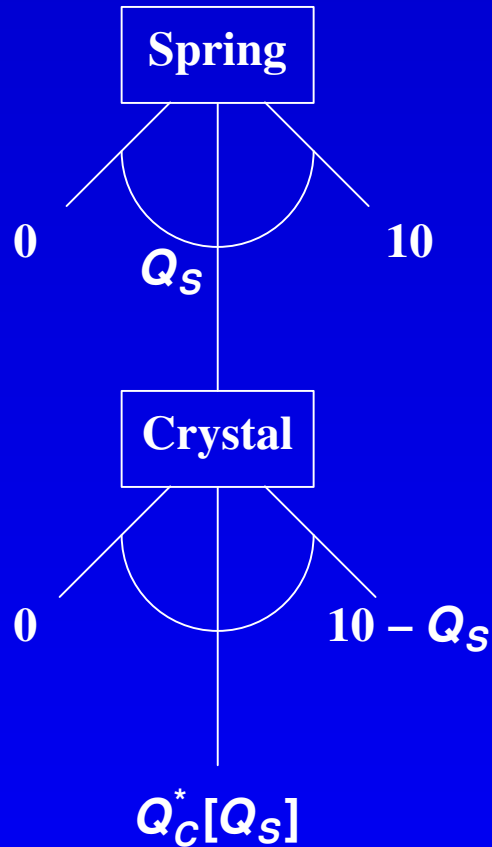
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What is the Follower's *reaction function*?



### *Cournot Rivalry Game Tree*

**(Quant.  $\Rightarrow$  Cournot,  
Price  $\Rightarrow$  Bertrand.)**

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- This function is known as the *reaction function*, since it tells us how the Follower will react to the Leader's choice (of output in this case, but it could be price).

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- leadership — first mover
- leadership — innovator, monopolist, faced with threat of entry
- incumbent erects barriers to entry by new-comer
- long-term contracts reduce incumbent's flexibility and increase the credibility of defence

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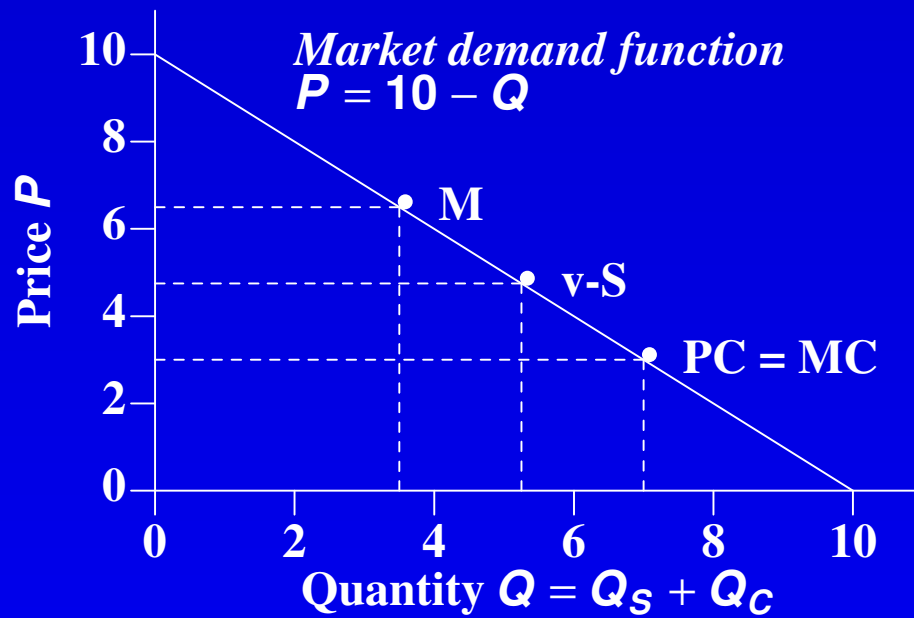
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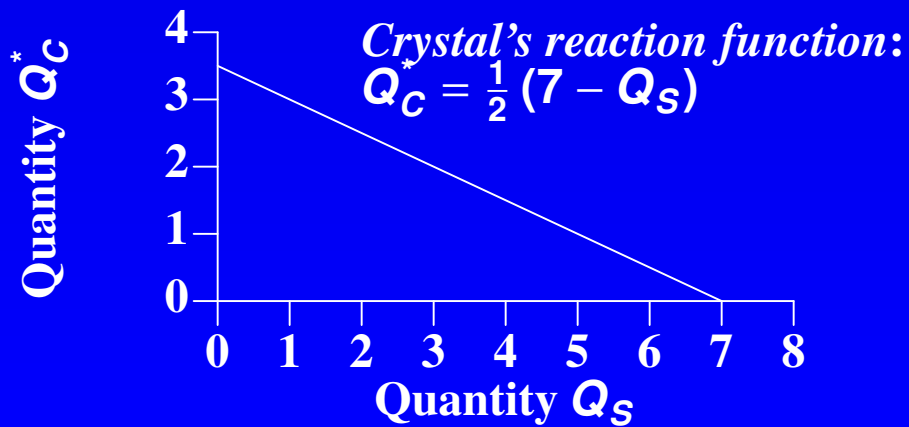
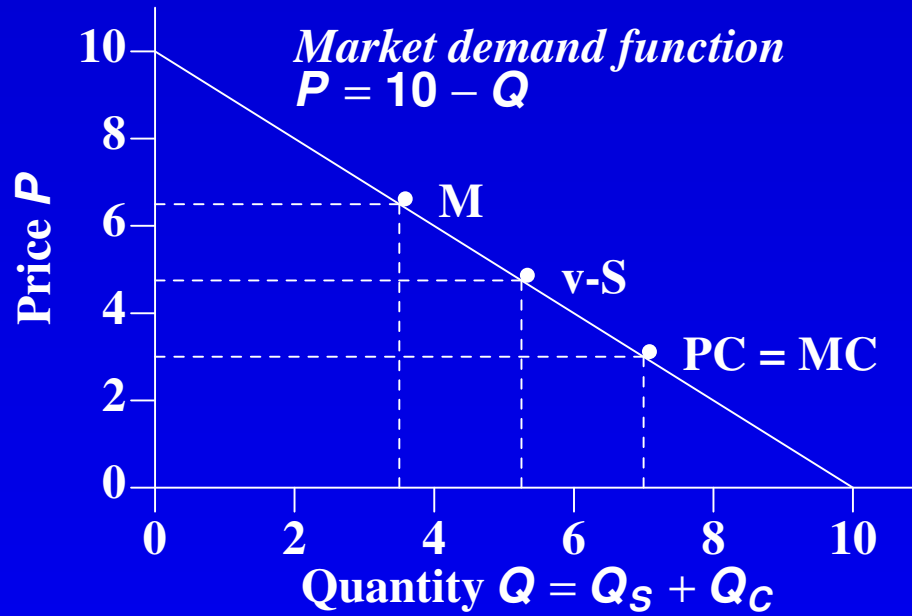
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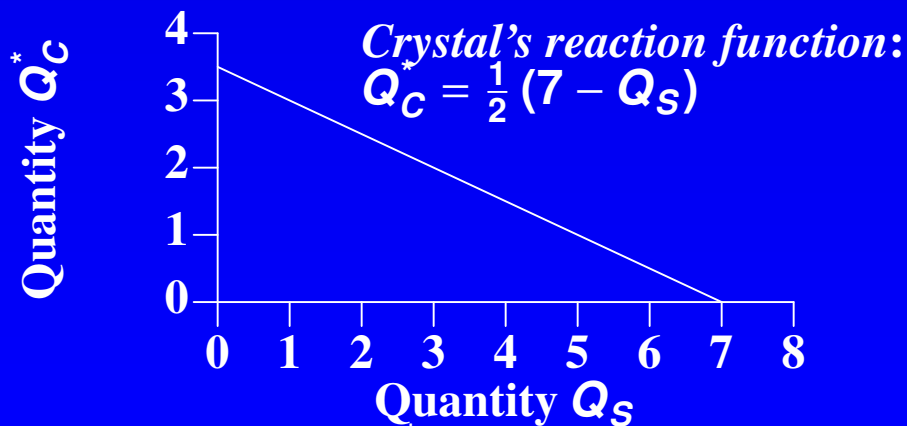
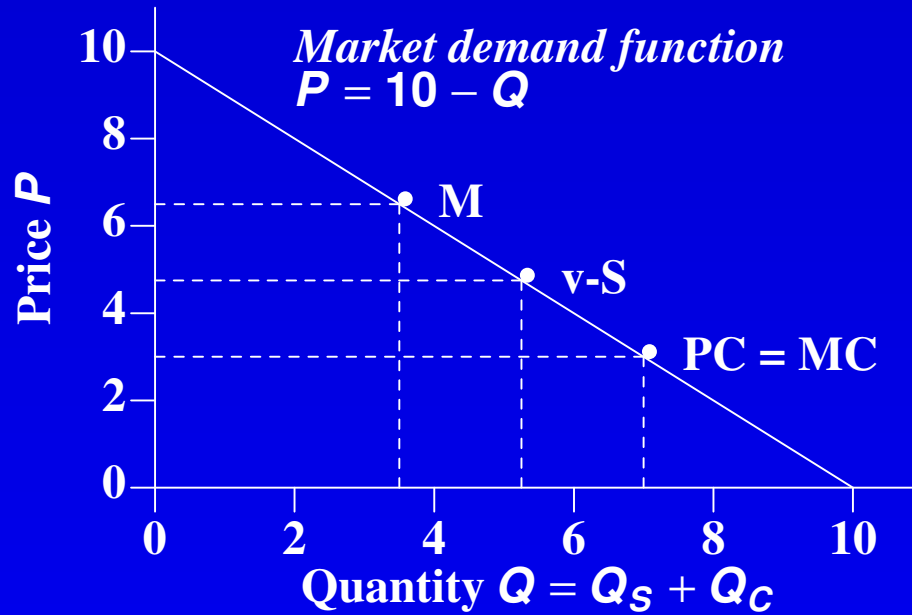
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This means that the Leader could become the Monopolist by paying the Follower not to enter the market, and offering him his (Follower's) profit of \$3.06 ( $= \pi_C$ ) not to, and still be ahead by:  
 $\$12.25 - 3.06 - 6.125 = \$3.06$ .

$$\text{i.e., } \pi_M - \pi_C - \pi_S = \$3.06$$







i.e. as  $Q_S$  rises,  $Q_C^*$  falls, & vice versa: *strategic substitutes*.

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There will be an industry-wide equilibrium when both firms resolve this balance.

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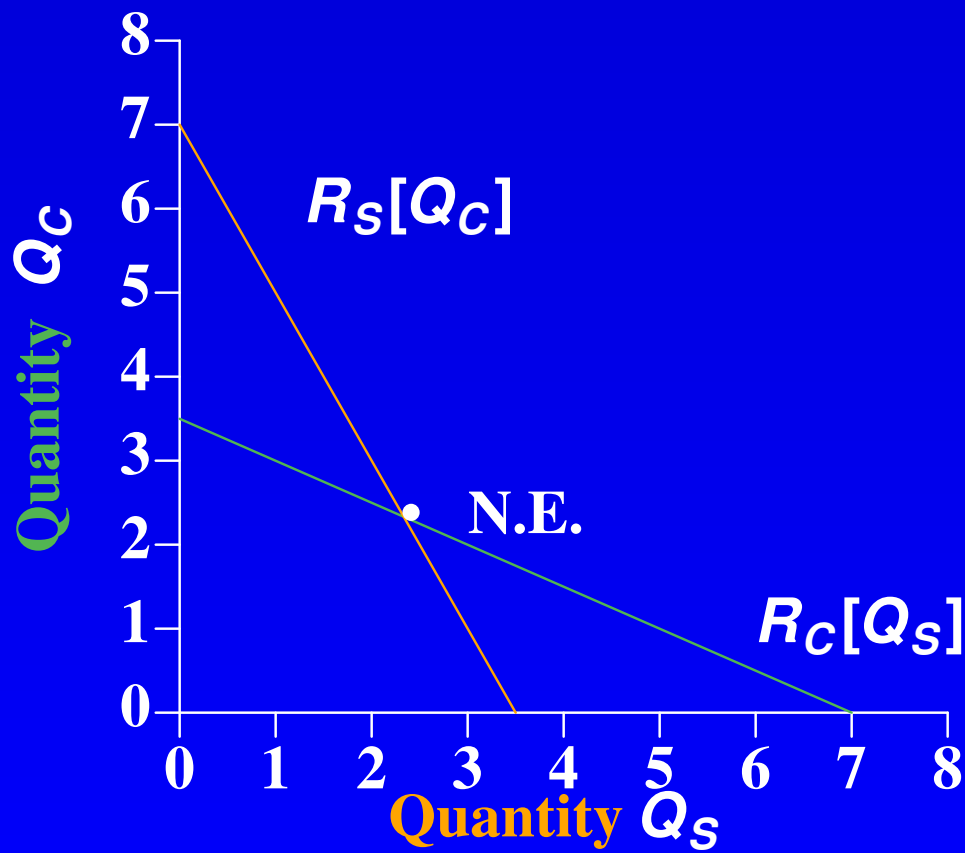
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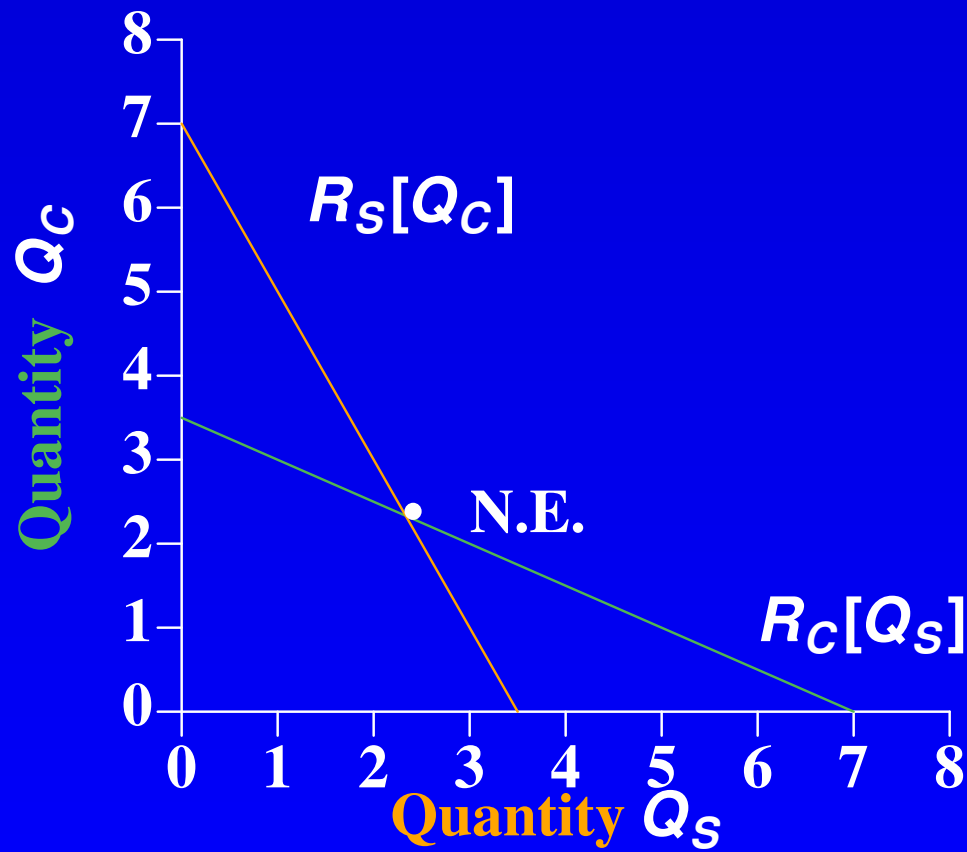
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**Price/unit =  $\$5\frac{1}{3}$ , profit of each =  $\$5.44$**

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It costs \$6 to make each pizza.

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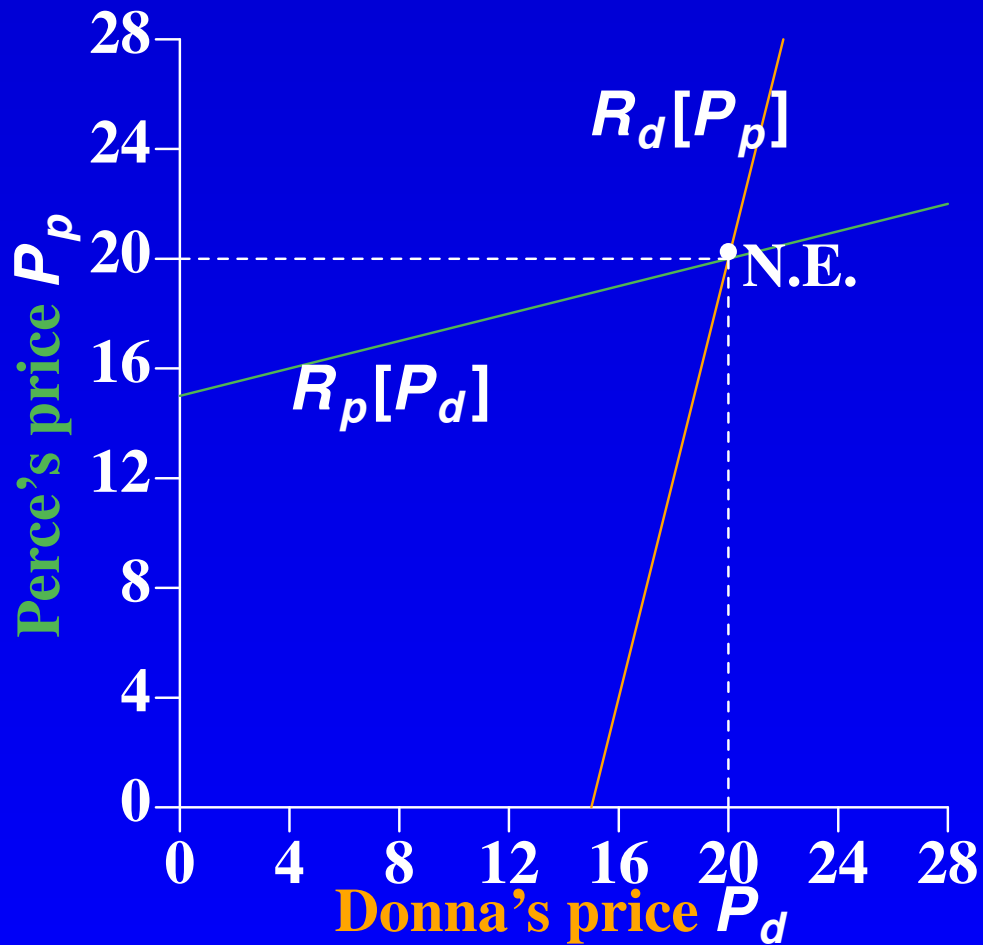
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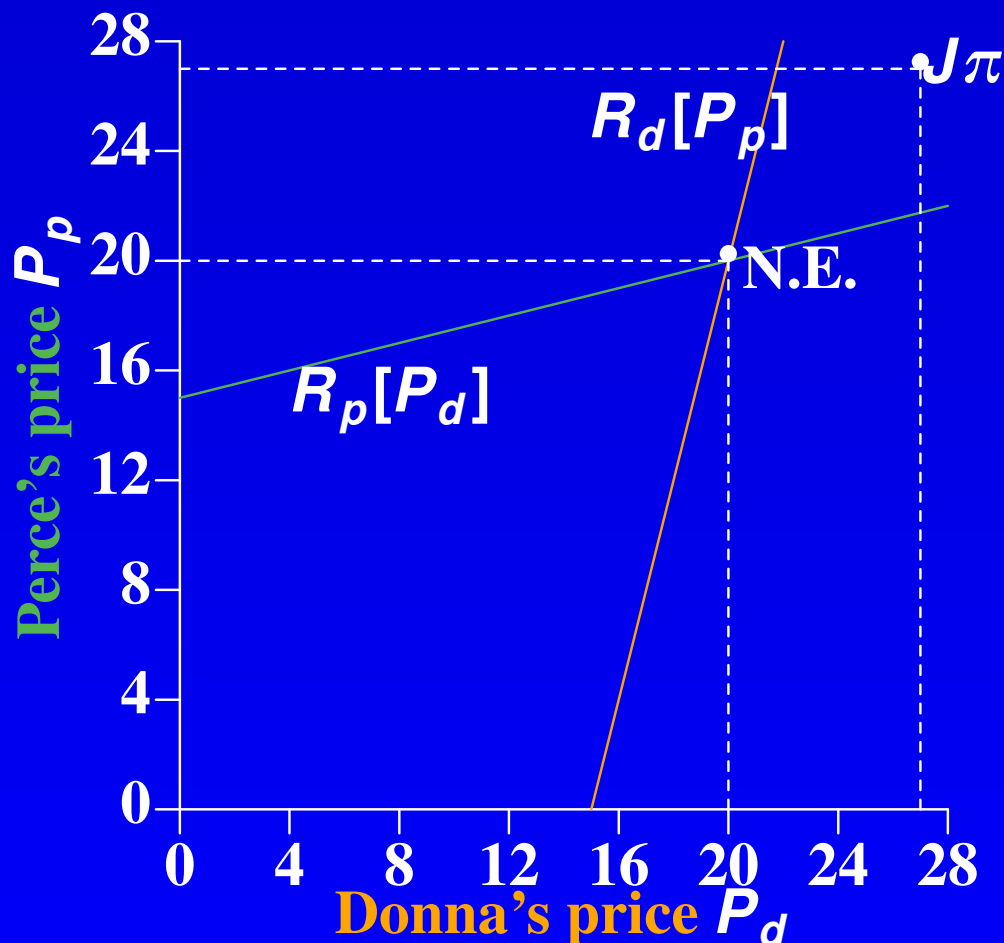
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## Plotting the Response Curves:



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*The Two Best-Response Curves (Reaction Functions): Bertrand*

*(Strategic complements: as  $P_p$  rises, so does  $P_d$ .)*

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**∴ a shop that increases its price is helping increase the profits of its rival, but this side-effect is uncaptured (and so ignored) by each shop independently.**



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Note: if the market demands were *not* symmetric, then it would be wrong to charge the same price  $P$  for both pizzas. Need to choose the two prices to  $\max \pi = \pi_d + \pi_p$ .

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## **Meaning?**

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**Compare the Cournot and Bertrand duopoly profits below.**

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Two companies produce homogeneous output.

Linear industry demand curve of  $P = 10 - Q$ ,  
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Since  $P_{PC} = AC$ , their profits are zero:  $\pi_1 = \pi_2 = 0$ .

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Each produces output  $y_1 = y_2 = 2.25$  units, and earns  $\pi_1 = \pi_2 = \$10.125$  profit.



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So  $Q_{Co} = 6$  units, price  $P_{Co}$  is then \$4/unit, and the profit of each firm is \$9.

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Profits are  $\pi_1 = \$10.125$  (the same as in the cartel case 2. above) and  $\pi_2 = \$5.063$  (half the cartel profit).

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**Identical to the price-taking case above.**

**Note: Were  $MC_1$  greater than  $MC_2$ , then Firm 2 would capture the whole market at a price just below  $MC_1$ , and would make a positive profit; and  $y_1 = 0$ .**



## 6 Comparing the Five Market Types

Market	Output $y_1$	Profit $\pi_1$	Output $y_2$	Profit $\pi_2$	Price $P$	Quantity $Q = y_1 + y_2$
1 Price-taking	4.5	0	4.5	0	1	9
2 Cartel	2.25	10.125	2.25	10.125	5.5	4.5
3 Cournot	3	9	3	9	4	6
4 von Stackelberg	4.5	10.125	2.25	5.063	3.25	6.75
5 Bertrand	4.5	0	4.5	0	1	9

*Summary of Outcomes.*

