5. In two-person games, when each of the two players has only two possible actions, we can represent the game by a 2×2 payoff matrix. Each player faces four possible combinations. For a one-shot game in pure strategies (i.e., no probabilities or dice rolling), it is sufficient to *rank* the four combinations: best, good, bad, worst. Using payoffs of 4, 3, 2, and 1, respectively, achieves this ranking.

   Consider the well-known Biblical story about Solomon's disposition of a baby for whom two women claim maternity. We can model his judgement in this case as a game devised by Solomon to test the truthfulness of the two women's claims. Although the game as played involved one woman's moving first, Solomon could have set the rules differently—to allow simultaneous moves—and still have achieved the same result. We shall model this as a simultaneous-move game: imagine that the two women are questioned in separate rooms.

   Solomon's game arises from a dispute between two prostitutes who come before him:

   > The first woman said, "Please, my lord! This woman and I live in the same house; and I gave birth to a child while she was in the house. On the third day after I was delivered, this woman also gave birth to a child. We were alone; there was no one else with us in the house, just the two of us in the house. During the night this woman's child died, because she lay on it. She arose in the night and took my son from my side while I was asleep, and laid him in her bosom; and she laid her dead son in my bosom. When I arose in the morning to suckle my son, there he was, dead; but when I looked at him closely in the morning, it was not the son I had borne." [1 Kings 3:17–21]

   The other prostitute disputed this version of their encounter:

   > No; the live one is my son, and the dead one is yours! [1 Kings 3:22]

   The two women continued arguing in Solomon's presence, while he reflected:

   > "One says, 'This is my child, the live one, and the dead one is yours'; and the other says, 'No, the dead boy is yours, mine is the live one.'" So the king gave the order, "Fetch me a sword." [1 Kings 3:23–24]

   Solomon's solution was one of dazling simplicity:

   > Cut the live child in half, and give half to one and half to the

> *other*. [1 Kings 3:25]

The subtlety underlying this solution soon became apparent in the reactions of the two claimants:

> But the woman whose son was the live one pleaded with the king, for she was overcome with compassion for her son. "Please, my lord," she cried, "give her the live child; only don't kill it!" The other insisted, "It shall be neither yours nor mine; cut it in two!" [1 Kings 3:26]

Then Solomon pronounced judgement:

> "Give the live child to her [the first woman]," he said, "and do not put it to death; she is its mother." [1 Kings 3:27]

The story concludes with the following observation:

> When all Israel heard the decision that the king had rendered, they stood in awe of the king; for they saw that he possessed divine wisdom to execute justice. [1 Kings 3:28]

Each woman had two choices: Protest the king's order, or Don't protest the order. Solomon correctly foresaw that their preferences would differ, and that these preferences would be revealed by their strategies: the mother would put the baby's life above all else, even if in protesting the king's order she lost his favour and hence the baby to the imposter, if the imposter accepted the king's order without protest; the imposter would put the king's favour above all else, even if neither woman protested, and the baby was slain. So Solomon was playing a kind of game with the women in which the strategies they chose in the game he devised revealed who was telling the truth, which is in the end what he was interested in uncovering.

a. Draw up a 2×2 *outcome* matrix for this game. Describe the four likely outcomes—as believed by the mother and by the imposter—depending on the two women's strategies. (Make any assumptions necessary, but discuss them and justify them. Consider whether each of them believes the baby will live or die, who each believes will get the boy if he lives, and how each believes the king will regard each woman as a result of her strategies.)

b. Rank these four outcomes for each of the mother and the imposter, from 1 = worst to 4 = best. Draw up your 2×2 *payoff* matrix for this game. Is there a dominant strategy for the mother? Is there a dominant strategy for the imposter?

c. Draw up a 2×2 payoff matrix for this game using your rankings. Is there an equilibrium strategy combination? Explain.

d. Plot the extensive-form decision tree. Show that there is no first-mover advantage in this game. Explain.

e. If the imposter had known what the king would decide after each woman had revealed her preferences, how might she have changed her behaviour? Would the king be able to resolve the dispute as easily under this situation? Do the beliefs of the mother matter: what if she realises the imposter knows what the king would decide (without knowing it herself)? Explain.

f. If the imposter truly believes she is the mother, will Solomon be able to resolve the dispute? Explain.

g. Would Solomon be able to obtain similar results if this problem arose many times again, or even once again? Discuss.