allows the identification of the different stages of play and the ways in which those stages are linked together. Full-fledged games that arise in later stages of play are called *subgames* of the full game.

Changing the rules of a game to alter the timing of moves may or may not alter the equilibrium outcome of a game. Simultaneous-move games that are changed to make moves sequential may have the same outcome (if both players have dominant strategies), may have a first-mover or second-mover advantage, or may lead to an outcome in which both players are better off. The sequential version of a simultaneous game will generally have a unique rollback equilibrium even if the simultaneous version has no equilibrium or multiple equilibria. Similarly, a sequential-move game that has a unique rollback equilibrium may have several Nash equilibria when the rules are changed to make the game a simultaneous-move game.

Simultaneous-move games can be illustrated in a game tree by collecting decision nodes in *information sets* when players make decisions without knowing at which specific node they find themselves. Similarly, sequential-move games can be illustrated by using a game table; in this case, each player's full set of strategies must be carefully identified. Solving a sequential-move game from its strategic form may lead to many possible Nash equilibria. The number of potential equilibria can be reduced by using the criteria of *credibility* to eliminate some strategies as possible equilibrium strategies. This process leads to the *subgame-perfect equilibrium (SPE)* of the sequential-move game. These solution processes also work for games with additional players.

KEY TERMS

continuation (175)
credibility (175)
information set (172)
off-equilibrium paths (174)
off-equilibrium subgames (174)
subgame (160)
subgame-perfect equilibrium (SPE) (175)

EXERCISES

1. Consider the two-player sequential-move game illustrated in Exercise 2a in Chapter 3. Reexpress that game in strategic (table) form. Find all of the pure-strategy Nash equilibria in the game. If there are multiple equilibria, indicate which one is subgame-perfect, and, for those equilibria that are not subgame perfect, identify the reason (the source of the lack of credibility).

2. Consider the Airbus–Boeing game in Exercise 5 in Chapter 3. Show that game in strategic form and locate all of the Nash equilibria. Which one of the equilibria is subgame perfect? For those equilibria that are not subgame perfect, identify the source of the lack of credibility.

3. Consider a game in which there are two players, A and B. Player A moves first and chooses either Up or Down. If Up, the game is over, and each player gets a payoff of 2. If A moves Down, then B gets a turn and chooses between Left and Right. If Left, both players get 0; if Right, A gets 3 and B gets 1.
   (a) Draw the tree for this game, and find the subgame-perfect equilibrium.
   (b) Show this sequential-play game in strategic form, and find all the Nash equilibria. Which is or are subgame perfect and which is or are not? If any are not, explain why.
   (c) What method of solution could be used to find the subgame-perfect equilibrium from the strategic form of the game?

4. Consider the cola industry, in which Coke and Pepsi are the two dominant firms. (To keep the analysis simple, just forget about all the others.) The market size is $8 billion. Each firm can choose whether to advertise. Advertising costs $1 billion for each firm that chooses to do so. If one firm advertises and the other doesn't, then the former captures the whole market. If both firms advertise, they split the market 50:50 and pay for the advertising. If neither advertises, they split the market 50:50 but without the expense of advertising.
   (a) Write down the payoff table for this game, and find the equilibrium when the two firms move simultaneously.
   (b) Write down the game tree for this game (assume that it is played sequentially), with Coke moving first and Pepsi following.
   (c) Is either equilibrium in parts a and b best from the joint perspective of Coke and Pepsi? How could the two firms do better?

5. Along a stretch of a beach are 500 children in five clusters of 100 each. (Label the clusters A, B, C, D, and E in that order.) Two ice-cream vendors are deciding simultaneously where to locate. They must choose the exact location of one of the clusters.

   If there is a vendor in a cluster, all 100 children in that cluster will buy an ice cream. For clusters without a vendor, 50 of the 100 children are willing to walk to a vendor who is one cluster away, only 20 are willing to walk to a vendor two clusters away, and none are willing to walk the distance of three or more clusters. The ice cream melts quickly; so the walkers cannot buy for the nonwalkers.

   If the two vendors choose the same cluster, each will get a 50% share of the total demand for ice cream. If they choose different clusters, then those children (locals or walkers) for whom one vendor is closer than the other will go to the closer one, and those for whom the two are equidistant will split 50% each.

Each vendor seeks to maximize her sales.

(a) Construct the five-by-five payoff table for their location game; the entries stated here will give you a start and a check on your calculations:

If both vendors choose to locate at A, each sells 85 units.

If the first vendor chooses B and the second chooses C, the first sells 150 and the second sells 170.

If the first vendor chooses E and the second chooses B, the first sells 150 and the second sells 200.

(b) Eliminate dominated strategies as far as possible.

(c) In the remaining table, locate all pure-strategy Nash equilibria.

(d) If the game is altered to one with sequential moves, where the first vendor chooses her location first and the second vendor follows, what are the locations and the sales that result from the subgame-perfect equilibrium? How does the change in the timing of moves here help players resolve the coordination problem in part c?

6. The street-garden game analyzed in Section 4 of this chapter has a 16 by 4 by 2 game table when the sequential-move version of the game is expressed in strategic form, as seen in Figure 6.13. There are *many* Nash equilibria that can be found in this table.

(a) Use cell-by-cell inspection to find all of the Nash equilibria in the table in Figure 6.13.

(b) Identify the subgame-perfect equilibrium from among your set of all Nash equilibria. Other equilibrium outcomes look identical with the subgame perfect one—they entail the same payoffs for each of the three players—but they arise after different combinations of strategies. Explain how this can happen. Describe the credibility problems that arise in the non-subgame-perfect equilibria.

7. Recall the mall location game, from Exercise 8 in Chapter 3. That three-player sequential game has a game tree that is similar to the one for the street-garden game, shown in Figure 6.13.

(a) Draw the tree for the mall location game, and then illustrate the game in strategic form.

(b) Find all of the pure-strategy Nash equilibria in the game.

(c) Use iterated dominance to find the subgame-perfect equilibrium.

(d) Now assume that the game is played sequentially but with a different order of play: Big Giant, then Titan, then Frieda's. Draw the payoff table and find the rollback equilibrium of the game. How and why is this equilibrium different from the one found in the original version of the game?

8. The rules of the mall location game, analyzed in Exercise 7, specify that, when all three stores request space in Urban Mall, the two bigger (more prestigious) stores get the available spaces. The original version of the game also specifies that the firms move sequentially in requesting mall space.

   (a) Suppose that the three firms make their location requests simultaneously. Draw the payoff table for this version of the game and find all of the Nash equilibria. Which equilibrium is subgame perfect and why?

   Now suppose that when all three stores simultaneously request Urban Mall, the two spaces are allocated by lottery, giving each store an equal chance of getting into Urban Mall. With such a system, each would have a two-thirds probability (or a 66.67% chance) of getting into Urban Mall when all three had requested space there.

   (b) Draw the game table for this new version of the simultaneous-play mall location game. Find all of the Nash equilibria of the game. Identify the subgame-perfect equilibrium.

   (c) Compare and contrast your answers to part b with the equilibria found in part a. Do you get the same Nash equilibria? Why? What equilibrium do you think is focal in this lottery version of the game?

9. Two French aristocrats, Chevalier Chagrin and Marquis de Renard, fight a duel. Each has a pistol loaded with one bullet. They start 10 steps apart and walk toward each other at the same pace, 1 step at a time. After each step, either may fire his gun. When one shoots, the probability of scoring a hit depends on the distance; after k steps it is $k/5$, and so it rises from 0.2 after the first step to 1 (certainty) after 5 steps, at which point they are right up against each other. If one player fires and misses while the other has yet to fire, the walk must continue even though the bulletless one now faces certain death; this rule is dictated by the code of the aristocracy. Each gets a payoff of -1 if he himself is killed and 1 if the other is killed. If neither or both are killed, each gets 0.

   This is a game with five sequential steps and simultaneous moves (shoot or not shoot) at each step. Find the rollback (subgame-perfect) equilibrium of this game.

   Hint: Begin at step 5, when they are right up against each other. Set up the two-by-two table of the simultaneous-move game at this step, and find its Nash equilibrium. Now move back to step 4, where the probability of scoring a hit is 4/5, or 0.8, for each. Set up the two-by-two table of the simultaneous-move game at this step, correctly specifying in the appropriate cell what happens in the future. For example, if one shoots and misses while the other does not shoot, then the other will wait until step 5 and score a sure hit; if neither shoots, then the game will go to the next step, for which you have already found the equilibrium. Using all this information, find the payoffs in the two-by-two table of step 4, and find the Nash equilibrium at this step. Work backward in the same way through the rest of the steps to find the Nash equilibrium strategies of the full game.

10. Think of an example of business competition that is similar in structure to the duel in Exercise 9.