COMPLEX SYSTEMS: BEYOND THE METAPHOR: Your Mathematical Toolkit

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## OUTLINE

1. Modelling.

Simulation.

- 2. Agent-Based Modelling.
- 3. Learning and Simulation.

1. Modelling — from March & Lave

**1.1 Overview** 

- A. What is a model?
- B. What is a good model?

Α.

1. Modelling — from March & Lave

**1.1 Overview** 

- A. What is a model?
- B. What is a good model?
- A. A model:
  - a simplified picture of a part of the real world.
  - has some of the real world's attributes, but not all.
  - a picture simpler than reality.

We construct models in order to explain and understand.

Three Rules of Thumb for Model Building:

- Think "process".
- Develop interesting implications.
- Look for generality.

Judge models using: truth, beauty, justice.

# Interplay between the real world (truth), world of æsthetics (beauty), world of ethics (justice), and the model world.

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Example: The firm — *Prices, Costs, and Values* → *Profits* 

We use verbal, graphical, and algebraic models of how consumers, firms, and markets work.

We assume rationality: that economic actors (consumers and firms) will not consistently behave in their worst interests.

Not a predictive model of how individuals act, but robust in aggregate.

## **1.2 Modelling**

Speculations about human behaviour/social and organisation interactions.

## **Explore the arts of**

- developing
- elaborating
- contemplating
- testing
- revising

models of behaviour.

#### What is a model?

- We can have several models of the same thing, depending on which aspects we want to emphasise, how we will use the model.
- Models are constructs to explain and appreciate the real world.

## So ...

## Need skills of:

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We can produce more complex behaviour than we are capable of understanding:

the behaviour of a baby baffles a psychologist (and vice versa)

If we cannot understand individual behaviour, then how are we to understand systemic/social/bureaucratic behaviour?

## Six familiar models in the social sciences:

- individual choice under uncertainty
- exchange
- adaptation
- diffusion
- transition
- demography

Each is treated by March & Lave (1975).

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4. Are these *true*? If not, speculate on other models/processes.

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Case: Contact and Friendship.

Why are some people friends and not others?

e.g. In a hall of residence, lists of friends Observe: friends live close together.

#### **Process?**

What is a possible process that might produce the observed result?

 $\Rightarrow$ 

**Two Speculations about Process:** 

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observe this? yes.

#### Generalisation

We want to include earlier predictions but find a more general model that predicts new behaviours as well, more widely.

**Can we generalise this?** 

- beyond the university?
- communication  $\rightarrow$  friendship?
- enemies as well as friends?

- **1.4 Three Rules of Thumb** 
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- 2. Develop interesting implications Much of the *fun* in model building comes in finding interesting implications in your models. A good strategy for producing interesting predictions: look for natural experiments.
- 3. Look for generality Ordinarily, the more situations a model applies to, the better it is and the greater the variety of possible implications.

**1.5 Evaluation of Speculative Models** 

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**Beauty:** 

- Simplicity, or parsimony
- Fertility (many predictions/assumptions)
- Surprise!

#### e.g. Parental preference for sons.

"Suppose that each couple agreed (knowing the relative value of things) to produce children (in the usual way) until each couple had more boys (the ones with penises) than girls (the ones without).

And further suppose that the probability of such coupling (technical term) resulting in a boy (the ones with) varies from couple to couple, but not from coupling to coupling for any one couple.

And (we still have a couple more) that no one divorces (an Irish folk tale) or sleeps around (a Scottish folk tale) without precautions (a Swedish folk tale).

And that the expected sex (technical term) of a birth if all couples are producing equally is half male, half female (though mostly they are one or the other)."

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"Question: (Are you ready?) What will be the ratio of boys (with) to girls (without) in such a society?"

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A Surprise —

 $\rightarrow$  for *society*: more girls than boys,

but —

for most couples: more sons than daughters.

Let's simulate this using NetLogo.

http://www.agsm.edu.au/~bobm/teaching/SimSS/NetLogomodels/boysngirls.html

### Truth:

- correct (or more correct) models
- requires clever, responsible detective work to find the truth (aim for objectivity, but face subjectivity if it exists)
- test derivatives, not assumptions
- predicting is not equivalent to understanding, necessarily

### **Need Critical Experiments:**

compare alternative models with the same question  $\rightarrow$  different answers: critical.

### **Beware Circular Models:**

- a. "when the rain-dance ceremony is properly performed, and all the participants have pure hearts, then it will rain" testable?
- b. "people pursue their own self-interest" — don't predict values from behaviour and then predict the same behaviour from the values just derived.
- c. Monty Python's "the man who claims he can send bricks to sleep"

The Importance Of Being Wrong

- evaluate rather then defend (avoid "falling in love" with your model)
- delight in finding fault be skeptical and playful
- always think of alternative models

2. Simulation Social Science, not Physical Science

At the aggregate level, similar.

But at the micro level, the agents in social science models are people, with self-conscious motivations and actions.

Aggregate behaviour may be well described by differential equations, with little difference from models of inanimate agents at the micro level.





- 1. As a Technique to investigate the detailed dynamics of a system.
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### (from Hartmann 1996)

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- 4. As a Tool for Experimentalists to support experiments.
- 5. As a Pedagogic Tool to gain understanding of a process.

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- Solution of a set of equations describing a complex (e.g. bottom-up) interaction.
- Discrete (CA): if the model behaviour ≠ empirical, it must be because of the transition rules.
- Continuous: not so clear-cut: background theory v. model assumptions

**Q:** does more realistic assumption  $\rightarrow$  more accurate prediction?

"A simulation is no better than the assumptions built into it" — Herbert Simon

## **2. Heuristic Tool**

Where the theory is not well developed, and the causal relationships are not well understood:

- theory development = guessing suitable assumptions that may imitate the change process itself
- but how to assess assumptions independently?

Durlauf: Is there an underlying optimisation by agents? (Complexity and Empirical Economics, *EJ*, 2005)

### **3. Substitute for Experiment**

When actual experiments are perhaps:

- pragmatically impossible: scale, time
- theoretically impossible: counterfactuals
- ethically impossible: e.g. taxation, no minimum wage

or to complement lab experiments

## **Agent-Based Models v. Economic Experiments**

Hailu & Schilizzi (2004, p.155) compare and contrast ABMs with experiments using human subjects, under the headings:

- Approach to inference, or micro-macro relationship
- Specification of behavioural rules
- Informational problems
- Degree of control
- Explanation of agents' choices
- Temporal length of analysis
- Representativeness / realism
- Data
- Cost

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### 4. Tool for Experimentalists

- to inspire experiments
- to preselect possible systems & set-ups
- to analyse experiments (statistical adjustment of data)

## **5. For Learning**

A pedagogic device through play ...

See Mitchell Resnick. *Turtles, termites, and traffic jams: Explorations in massively parallel microworlds*. MIT Press, 1997.

Play with NetLogo models, and experience emergence: Life is famous, and others too.

### Summary

A simulation imitates one process by another process

With Social Sciences: few good descriptions of static aspects, and even fewer of dynamic aspects (Remember: existence, uniqueness, stability)

# **Robust Predictions from Simple Theory** (from Latané, 1996)

Four conceptions of simulation as a tool for doing social science:

- 1. As a scientific tool: theory + simulation + experimentation
- 2. As a language for expressing theory:
  - natural language,
  - mathematical equations (i.e., closed form), and
  - computer programs, such as C++, Java, etc.
- 3. As an "easy" alternative to thinking: robust coding
- 4. As a machine for discovering consequences of theory: if this, then that.

### A Third Way of Doing Science

(from Axelrod & Tesfatsion 2006)

**Deduction + Induction + Simulation.** 

- Deduction: deriving theorems from assumptions
- Induction: finding patters in empirical data
- Simulation: assumptions  $\rightarrow$  data for inductive analaysis
- S differs from D & I in its implementation & goals.

S permits increased understanding of systems through controlled computer experiments

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#### Adam Smith's Invisible Hand $\rightarrow$ prices

Schelling's residential tipping (segregation) model: People move because of a weak preference for a neighbourhood that has at least 33% of those adjoining the same (colour, race, whatever)  $\rightarrow$  segregation.

Need models with more than one level to explore emergent phenomena.

### **Families of Simulation Models**

- 1. System Dynamics SD (from differential equations)
- 2. Cellular Automata CA (from von Neumann & Ulam, related to Game Theory)
- 3. Multi-agent Models MAM (from Artificial Intelligence)
- 4. Learning Models LM (from Simulated Evolution and from Psychology)

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So "agent-based models" excludes Systems Dynamics models, but can include the others.

### **Simulation: The Big Questions**

from: www.csse.monash.edu.au/~korb/subjects/cse467/questions.html

- What is a simulation?
- What is a model?
- What is a theory?
- How do we test the validity of any of the above?
- When do we trust them, what sort of understanding do they afford us?
- What is an experiment? What does it mean to experiment with a simulation?
- What is the role of the computer in simulation?
- How does general systems dynamics influence simulations?
- How do we handle sensitivity to initial conditions?
- How precisely can a simulation approximate real life / a model?
- How do we decide whether to use a theory / model / simulation / lab experiment / intuition for a given problem?
- Does a simulation have to tell us something?
- How complex is too complex, how simple is too simple?
- How much information do we need to (a) build and (b) test a simulation?
- How/when can the transition from a quantitative to a qualitative claim be made?

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To Verify: use a suite of tests, and run them every time you change the simulation code — to verify the changes have not introduced extra bugs.

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- **Use Sensitivity Analysis, to ask:** 
  - robustness of the model to assumptions made
  - which are the crucial initial conditions/parameters?
- use: randomised Monte Carlo, with many runs.

Judd's ideas (2006)

"Far better an approximate answer to the right question ... than an exact answer to the wrong question." — John Tukey, 1962.

That is, economists face a tradeoff between: the numerical errors of computational work and the specification errors of analytically tractable models.



**Several suggestions:** 

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- 5. Synergies between Simulation and Conventional Theory.

*Docking*: a simulation model written for one purpose is aligned or "docked" with a general purpose simulation system written for a different purpose.

Four lessons:

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  - a. numerical identity
  - b. distributional equivalence
  - c. relational equivalence
- 3. Which null hypothesis? And sample size.
- Minor procedural differences (e.g. sampling with or without replacement) can block replication, even at (b).

#### **Reasons for Errors in Docking**

- 1. Ambiguity in published model descriptions.
- 2. Gaps in published model descriptions.
- 3. Errors in published model descriptions.
- 4. Software and/or hardware subtleties.

e.g. different floating-point number representation.

## (See Axelrod 2006.)

For whom?

For whom?

With regard to what?

For whom?

# With regard to what?

A good simulation is one that achives its goals:

- to explore
- to predict
- to explore

Or

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For whom?

# With regard to what?

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- to explore
- Or
- what is?
- what could be?
- what should be?

# **Consider historical market data:**

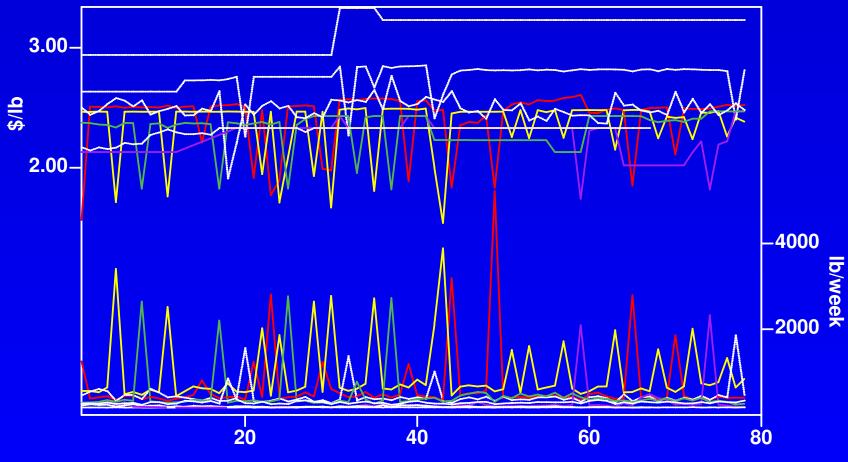


Figure 1: Weekly Prices and Sales (Source: Midgley et al. 1997)

#### **Stylised Facts of the Market Behaviour**

- Much movement in prices and quantities of four brands — a rivalrous dance.
- Pattern: high price (and low quantity) punctuated by low price (and high quantity).
- Another four brands: stable prices and quantities

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#### **Questions:**

What is the cause of these patterns?

- shifts in brand demand?
- reactions by brands?
- actions by the supermarket chain?
- unobserved marketing actions?

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Similar (qualitatively or quantitatively) to the brands' behaviours of pricing and sales.

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Note: assuming profit-maximising (or purposeful) agents means that we are not simply curve-fitting or description using D.E.s. Going beyond the rivalrous dance.

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#### **Examine:**

- limits of behaviour (Miller's Automated Non-linear Testing System)
- regime-switching
- range of behaviour generated
- sensitivity of the aggregate (or energent behaviour) to a single agent's behaviour.

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