EXPECTATIONS AND THE HOTELLING RULE

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ABSTRACT

Can the socially optimal intertemporal resource allocation of an economy including non-renewable resources be supported as a competitive market allocation? Previous studies have concluded that, in the absence of perfect foresight and a complete set of futures markets, the initial price of the resource, and hence all succeeding prices, may be inconsistent with the shadow prices along the optimal trajectory. This study questions the existence of *any* stable trajectory satisfying the necessary condition that the resource price (net of extraction costs) increases at the interest rate (the Hotelling Rule). Stability analysis of simple disequilibrium models with several formulations of expectation formation and price adjustment suggests that a competitive economy will not support the Hotelling Rule, but rather will in general exhibit stable equilibrium with constant resource price.

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1. Introduction

In the theory of depletable resources, the usual approach is to specify the demand for flow of the resource as a function of the price, and the supply of resource flow as a function of the expected rate of price increase (net of production cost). This latter relationship is usually derived from conditions for intertemporal profit maximization. Market equilibrium is found as a sequence of prices which equate supply and demand at each moment. In all simple cases, the price increases over time. In the case of zero extraction cost, the price increases at the rate of interest.

In the theory of disequilibrium price adjustments, excess supply or excess demand is the driving force for the price changes. In fact the notion of market equilibrium often has two almost completely interchangeable meanings, one describing a situation in which supply equals demand, the other a situation in which prices are unchanging. In the simplest and most frequently used formulations, the rate of price increase (decrease) is proportional to excess demand (supply) in a given market.

These two basic principles are in conflict with each other. In conditions of supply/demand equality for depletable resources, prices cannot change. In conditions of increasing price, supply cannot equal demand.

One simple response to the apparent conflict of the two basic principles has been to assume “rapid” adjustment in response to the disequilibrium, so that prices can increase at the rate of interest when demand is “only slightly” greater than supply. The assumption is that, if adjustment is fast enough, the basic approach of depletable resource economics can remain with but insignificant modifications.

This paper will establish that the simple answer does not provide a solution to the basic dilemma. While in limiting cases the simple solution will provide a price trajectory which satisfies the necessary conditions for an equilibrium, this will not be a stable solution.

For many cases only one stable solution will exist—a constant price at which supply equals demand, at least until physical depletion or increasing demand lead to increasing prices. For a narrower set of cases only the constant price solution will provide any solution at all.

These results imply that, while an efficient allocation of depletable resources corresponds to a competitive equilibrium, this equilibrium will never be found. Not only will the very long run conditions (the transversality conditions) be violated as Stiglitz [1974] has suggested, but so will the short-run conditions.

The theory here may help to explain the striking discrepancy between the theory of depletable resources, which implies increasing prices over time, and the actual behaviour of resource markets, such as petroleum markets in the U.S.A., which have exhibited slowly falling real prices in the past.

To address these issues, this paper drops the usual assumption of trading and production only at market clearing, and allows prices to adjust in response to excess demand (positive or negative) as trading continues. Expectations of future resource price changes are modelled with several different formulations. The paper examines the existence, uniqueness, and stability of short-run equilibria in the resulting model.

To make more transparent the fundamental point, the other relevant markets—those for labour, money, and output—are not modelled. Section 2 discusses the Hotelling Rule. Section 3 introduces expectations and resource supply. The three modes of expectation formation—myopic perfect foresight, simple adaptive expectations, and compound adaptive expectations—are examined in Sections 4, 5, and 6, respectively. Section 7 introduces a model in which there is a direct linkage between the expected and the actual rates of return. Section 8 contains the summary and conclusions.

2. The Hotelling Rule

Stocks of a non-renewable natural resource (which is exhausted in production) are assets, much like reproducible capital. Their ultimate value lies in their potential for being transformed into scarce output, but they can be bought and sold as stocks, and can appreciate or depreciate, as can other assets. In fact, certain resources may be traded on two distinct markets: a stock market, in which the object of buying is to make future anticipated capital gains, and a flow market, in which the object of buying is to obtain quantities of resource as a factor input to production. We implicitly draw this distinction in the use of the phrase "resource flow." Two distinct markets implies two distinct prices, but we assume that arbitrage between the markets (if they exist separately) leads to equality of the prices. We denote the (real) price of resource as v.

If owners of resource stocks regard them as perfect substitutes for other, dividend-earning, assets, then we can speak of a single asset market, on which resource owners can trade stocks of resource and other assets. Asset markets can only be in equilibrium when all assets in a given risk class earn the same rate of return, partly as current dividend, and partly as capital gain.[1] Since the only way that a stock of resource can produce a return for its owner is by appreciating in value, equilibrium in the asset market will only occur when the value of a resource stock is growing at the interest rate r, the assumed common rate of return on assets. Since the value of a stock is also the net present value of future sales from that stock, in asset market equilibrium resource owners must expect the net price of the resource to be growing at a proportional rate equal to the rate of interest.[2] This fundamental principle of the economics of exhaustible resources is known as the Hotelling Rule. Hotelling [1931] thought of it as a condition, not of stock equilibrium on the asset market, but of flow equilibrium on the resource market: if the price of resource (v) is growing at a proportional rate equal to the interest rate, then owners of stocks of resource will be indifferent at the margin between (extracting and) selling, and holding stocks at every instant of time.

Weinstein and Zeckhauser [1975] closely examine market equilibrium in a competitive model. They demonstrate that, if resource producers (or owners) have perfect foresight of future demands for resource flow or if resource consumers can store the resource at zero cost, equilibrium over time can only occur with time paths obeying the Hotelling Rule. Solow [1974] characterizes this as demand for resource flow just equal to its supply at the current price, with clearing in the market for resource flow: "No other time profile for prices can elicit positive production [of resource flow] in every period of time" [1974, p. 3]. If the net price rose too slowly, supply of resource flow would be made earlier, and the resource would be exhausted more quickly, since no-one would want to hold resource stocks and earn less (in capital gains) than the going rate of return on other assets. If the net price rose too fast, holding resource stocks would be an excellent

1. This is the equation of interest (or arbitrage equation), formulated by Samuelson [1937] in the context of capital theory, and recently more generally derived by de La Grandville [1980] in his investigation of optimal growth with exhaustible resources.
2. In a competitive market net price equals market price minus marginal cost; under a monopoly net price equals marginal profit, which equals marginal revenue minus marginal cost.

investment, and owners would postpone selling while they enjoyed super-normal capital gains.

Under conditions of uncertainty, received theory suggests that the Hotelling Rule will still hold. Weinstein and Zeckhauser [1975] show that for risk-neutral resource suppliers market equilibrium requires that the expected resource price grow at a rate equal to the interest rate.

Both Nordhaus [1973] and Stiglitz [1974] note that the condition of equilibrium in the asset market (the Hotelling Rule) results in a family of price trajectories, each having different levels of price at any instant. The unique solution of resource depletion depends on the terminal solution that all resources are used up precisely at the last instant.

Stiglitz [1974] has shown that even if resource producers possess myopic perfect foresight, with the expected rate of change of resource price at any time equal to the actual rate of change of resource price, but without either infinite perfect foresight or a complete set of futures markets extending infinitely far into the future, then in the long run there exists no economic mechanism which will guarantee that the initial price is set so that the economy is on the correct price trajectory. If the initial price is set too low, the resource stock will be used up too soon; if the initial price is too high, there will always be a finite amount of resource stock remaining.[3]

We examine whether the Hotelling Rule can be supported *in the short run,* by considering the micro-foundations of movement along a Hotelling price trajectory, with the asset market in equilibrium. We drop the implicit assumption made by previous authors that arbitrage between resource stock and other asset markets occurs so rapidly that the rates of return on all assets are always equal, and that markets clear instantaneously. We relax the usual assumption of trading only at market clearing, and allow prices to adjust in response to excess demands (positive or negative) as trading continues.[4]

In what follows, we introduce adjusting expectations of future resource price, and examine the existence, uniqueness, and stability of short-run equilibria in the resulting model. We at first assume no autonomous price change: the price on a market changes only in response to an imbalance of supply and demand on that market. We later examine the consequences for the model of allowing expectation of price change to affect actual price change directly.

We find that, despite the present-value-maximizing behaviour assumed of resource suppliers, our economies will not obey the Hotelling Rule under reasonable assumptions.

Three other studies have partially addressed related questions. In papers which examine the consequences for exhaustible resource allocation of the non-existence of a complete set of forward markets, Heal [1975] and [1981] builds three models to examine the resource depletion rates that occur under plausible adjustment mechanisms for price and quantity in disequilibrium situations. In all of these models the behaviour of resource price is strongly dependent on the specific characteristics of the resource demand function. The first model is very simple, with exogenous price expectations. The second

3. This long-run instability of the economy in the absence of future markets is of the same kind that Hahn [1966] and Shell and Stiglitz [1967] have shown can exist in growth models with more than one capital good, which is essentially the same as has often been attributed to speculative markets.

4. The reader should not confuse the prescriptive rules of optimal allocation of resources with perfect arbitrage and perfect foresight of, for example, Samuelson [1937] and de La Grandville [1980] with the outcome of conditions of imperfect foresight, in a decentralised economy.

model, with endogenous expectations of price levels rather than rates of return on holding stocks, does not deal adequately with portfolio adjustment in asset markets. The third model does include expectations of rates of return, but assumes that prices adjust instantaneously to clear markets. Thus, none of these three quite addresses the central issue of price adjustment. Dasgupta and Heal [1979, pp 246–252] build two models of imperfect short-run foresight, but in each study the resource market is modelled to clear instantaneously. None of the papers examines the interaction of the demand for resource stocks and the demand for resource flows in the light of the Hotelling Rule and endogenous price expectations. This is not their purpose. In contrast, we do not assume a specific demand function, nor do we assume that markets always clear, and we model expected rates of return rather than price levels.

3. Expectations and Resource Supply

We want to define a single variable to describe price expectations. This variable will be a link between the supply of resource and the past and present behaviour of the system. It is assumed that suppliers of resource, in deciding whether to hold or sell stocks, compare the return they anticipate receiving from holding stocks with the return they could receive from holding other assets. Since the only possible return from holding natural resource stocks is a capital gain stemming from price increases, resource owners respond to expected capital gains. Not until the actual sale is the capital gain (or loss) realised, and their expectations shown to be correct, or optimistic, or pessimistic. Hence, we adopt the expected rate of return from capital gains as our expectations variable.

We denote the expectation formed at time t of the real resource price at time $(t+h)$ as $v^e(t+h,t)$. Following Burmeister and Turnovsky [1976], we assume that forecasts satisfy the *weak consistency axiom:*

$$v^e(t,t) = v(t). \tag{1}$$

That is, we assume that the expectation formed now for the current price is equal to the actual current price. The expected rate of return on holding stocks of resource over the period $(t, t+h)$ is given by

$$\frac{v^e(t+h,t) - v(t)}{hv(t)}. \tag{2}$$

The instantaneous rate e(t) is obtained by taking the limit as $h \to 0$,

$$e(t) \equiv \lim_{h\to 0} \frac{v^e(t+h,t) - v(t)}{hv(t)} = \frac{v_1^e(t,t)}{v^e(t,t)}, \tag{3}$$

where

$$v_1^e(t,t) \equiv \frac{\partial v^e(s,t)}{\partial s} \Big|_{s=t}. \tag{4}$$

Assuming that $v^e(t,t)$ is twice continuously differentiable, and totally differentiating the weak consistency axiom, equation (1), we obtain

$$\dot{v}^e(t) = \frac{\partial v^e(s,t)}{\partial s}\Big|_{s=t} + \frac{\partial v^e(t,s)}{\partial s}\Big|_{s=t}$$
$$\therefore\ \dot{v}^e(t) = v_1^e(t,t) + v_2^e(t,t) = \dot{v}(t).$$

The right-hand side of equation (5) measures the *actual* rate of price change, while $v_1^e(t,t)$

measures the corresponding *anticipated* rate of change. It follows from equation (5) that $v_2^e(t, t)$ measures the *unanticipated* rate of change, that is, the forecast error (Burmeister and Turnovsky [1978]). With these definitions established, we shall now specify the resource supply function.

3.1 The resource supply function

The supply of resource flow is assumed to be a downwards-sloping function of the expected rate of return from asset value appreciation. This specification has been chosen as an approximation to the Hotelling world: as the expected rate of return increases, ceteris paribus, the supply of resource flow drops. We shall define a quasi-Hotelling supply function, $R^S(e)$, as a functional relationship between the expected rate of price increase (or return) and the resource flow, having the following characteristics:

a. Supply is a continuous, non-increasing function of e.

b. Supply is bounded above by some finite $\bar{R}$ and bounded below by some positive $\underline{R}$.

c. The derivative of supply with respect to e reaches its minimum at $e = r$.

d. When $e = 0$, supply is $\bar{R}$ and completely expectations-inelastic.

Two possible quasi-Hotelling supply functions are shown in Figure 1.

We can think of the finite upper limit $\bar{R}$ on resource flow as implicitly embodying increasing costs of extraction: rather than being a physical limit, it is an economic limit. In the short run it is unprofitable for the resource owners to supply more rapidly than $\bar{R}$ per period because of increasing costs of extraction. The lower limit $\underline{R}$ and the range in which the resource flow supply varies with expectations can be thought of as resulting from incomplete knowledge of investment opportunities or from non-uniform price expectations among resource suppliers.

In what follows we make the following assumption:

> *Assumption (Quasi-Hotelling Supply, or QHS).* The quantity supplied is governed by a quasi-Hotelling supply function, $R^S(e)$.

3.2 Rates of price adjustment

We assume that the rate of price adjustment depends upon the relationship between supply and demand. In most of the following, we assume that prices rise only when demand exceeds supply, and fall only when the converse holds. We shall use the simplest form of this relationship.

> *Assumption (Linear Price Adjustment, or LPA).* The rate of price increase (or decrease), $g(t)$, is a linear function of the difference between the quantity demanded and the quantity supplied:
>
> $$g(t) = \lambda_V . [R^D(v) - R^S(e)], \quad \lambda_V > 0, \tag{6}$$
>
> where $g(t)$, also the rate of return of holding stocks, is defined as
>
> $$g(t) \equiv \lim_{h \to 0} \frac{v(t+h) - v(t)}{hv(t)} = \frac{\dot{v}(t)}{v(t)}, \tag{7}$$

and where $R^D(v)$ is the demand for resource flow, assumed to be a decreasing function of price, v.

In what follows we adopt the two assumptions—quasi-Hotelling supply (QHS) and linear price adjustment (LPA). In addition we shall introduce various assumptions describing expectations formation. It will be shown that, under these two assumptions, virtually all of our postulated modes of expectation formation lead to prices *stable* over time and therefore to violations of the Hotelling Rule.

3.3 Modes of expectation formation

Appropriate modelling of expectations of future prices (and hence expectations of rates of capital gain) is a matter of much professional disagreement. Since the problem is how, in the absence of futures markets, to calculate market-clearing prices for future dates, expectations should depend not only on past prices, but on current and past values of non-price magnitudes. But since, in the short run, quantities do not change greatly, we generally use past prices as driving variables. We examine three modes of expectation formation: *myopic perfect foresight*, *simple adaptive expectations*, and *compound adaptive expectations*. In the third mode, long-run considerations enter through the interest rate.[5]

We define an expectations equilibrium as a state (e^*, v^*) in which there is no tendency for the expected rate of return $e(t)$ and the resource price $v(t)$ to change. In an expectations equilibrium, expectations need not necessarily be fulfilled. If such a state exists it will be self-sustaining, although perhaps unstable. In such an expectations equilibrium the Hotelling Rule will not hold.

The existence of such expectations equilibria will readily be demonstrated graphically. If the expectations equilibrium is locally stable, then the micro-economics of our disequilibrium model do not support the Hotelling Rule in the short run: for e and v initially close to the equilibrium, the resource price v would tend to the constant level characterizing the equilibrium. In what follows, we shall characterize this expectations equilibrium, and establish its local stability for each mode of expectations formation.

4. Myopic Perfect Foresight

Gray and Turnovsky [1979] show that if one assumes that (i) a forecast is for an instantaneously short future period, (ii) forecasters have instantaneous access to relevant information as it becomes available (equation (1)) and a "memory" or some means of storing the information, and (iii) the variable being forecast is differentiable, then expectations must satisfy myopic perfect foresight, with $v_2^e(t,t) = 0$ in equation (5). Assumptions (i) and (ii) will hold throughout,[6] but we show below that the differentiability assumption is too restrictive.

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5. Dasgupta and Heal [1979] consider an expectation formulation in which the resource-utilisation ratio of resource flow to resource stock is used as a depletion index on which expectations of the rate of capital gains are based. Imposing equilibrium in the asset market, they find that the economy is hopelessly inefficient, with expectations never fulfilled.
6. Gray and Turnovsky [1979] examine the implications of relaxing assumptions (i) and (ii).

Assumption (Myopic Perfect Foresight, or MPF). The instantaneous expected rate of return equals the actual rate at every moment:

$$e(t) = g(t), \qquad \text{for all } t. \tag{8}$$

Myopic perfect foresight, the deterministic equivalent of rational expectations, is the formulation used in Hahn [1966] and Shell and Stiglitz [1967].

Under the assumption of MPF, plus assumptions QHS and LPA, equations (6) and (8) must hold simultaneously. Combining these two equations gives:

$$R^S(e) = R^D(v) - \frac{e}{\lambda_V}. \tag{9}$$

The components of this equation are plotted in Figure 2 to obtain three values of e, (e_1, e_2, e_3), which are consistent with the initial price. Figure 3 demonstrates that for a higher price (v_2) expected rates of return of e_4, e_5, and e_6 are obtained.

Using equations (8) and (9), one can plot the expected rates of return consistent with myopic perfect expectations for each possible price. The result is a locus of points as plotted in Figure 4.

In Figure 4, we have indicated by arrows the direction of price changes for the various points on the locus. If we start at a point to the right of point A, prices are rising faster than the interest rate, with the rate of price rise constantly falling. If the system starts between points A and B, prices rise at an increasing rate.[7] To the left of point D, prices fall.

From point A, prices must still rise. A tiny rise forces a jump to point C and an instant change in the expected rate of return. From then, prices must fall. Thus, there may be a discontinuity in expectations and in the rate of return. Turnovsky and Gray's assumption of differentiable g is inconsistent with MPF. This dilemma can be resolved by the observation that MPF cannot describe *unfulfilled* expectations, with $e \neq g$ and non-zero unanticipated rate of return $v_2^e(t, t)$ in equation (5).

One and only one equilibrium exists under MPF: that indicated by point D. At point D, $e = 0$ and v is constant. Hence, if the system converges to the equilibrium, then prices will remain constant, and the expectations of no price change will be fulfilled.

This equilibrium always exists so long as $R^D(v^*) = R^S(0)$, for some price, v^*. These conditions occur if the rate of production of the resource is bounded above by some finite level and the demand is unbounded as price approaches zero.

If demand at zero price is bounded below supply $\bar{R}$ at zero expected return, then Figure 4 must be modified, as indicated in Figure 5. In this situation, prices fall to zero and excess supply remains. Prices can be expected to remain at zero from then on.

The equilibrium in either case is globally stable if we add the restriction that e and g change discontinuously only when necessary to maintain the assumption MPF. Without that assumption, expectations could randomly jump among the three branches of the locus of Figure 4, and the system might not converge.

We can summarize these results in Proposition 1.

7. The points A and B satisfy the condition that $\partial R^S | \partial e = -1 | \lambda_V$.

Proposition 1. Under assumptions QHS, LPA, and the assumption MPF, a unique equilibrium exists. The equilibrium is characterized by constant prices. Furthermore, if expectations change discontinuously only when necessary to preserve the assumption MPF, then this equilibrium will be globally stable.

Proposition 1 thus shows that with MPF the Hotelling Rule will not characterize the economy. Prices and expectations will adjust to a point at which markets are cleared with an expectation of constant price, an expectation which is fulfilled. Prices will not adjust at the rate of interest.

Assumption MPF suffers from the possible discontinuity in the expected rate of return. In addition, by assuming that expectations are always fulfilled, it ignores the process by which expectations may adjust in response to unanticipated price change. The following formulation addresses these concerns. The basic conclusion, as represented by Proposition 1, will remain intact.

5. Simple Adaptive Expectations

In this formulation (Cagan [1956]) resource owners revise upwards their expectations of the rate of return if the actual rate of return exceeds what has been expected, and vice versa.

Assumption (Simple Adaptive Expectations, or SAE). The rate of change of the expected rate of return is a linear function of the difference between the actual rate and the expected rate. This can be written:[8]

$$\dot{e}(t) = \mu . [g(t) - e(t)], \qquad \mu > 0. \tag{10}$$

If the expected rate of return at any moment is not fulfilled, then the expectation is revised so as to approach the observed behaviour. Note that the formulation MPF is identical to the formulation SAE with the constant μ unbounded above, so that $e = g$.

In order to develop a phase diagram under the assumptions of SAE and LPA, equations (6) and (10) can be combined to give:

$$\dot{e} = \mu \lambda_V [R^D(v) - \frac{e}{\lambda_V} - R^S(e)]. \tag{11}$$

Equation (11) should be compared to equation (9). Note that $\dot{e} = 0$ for all combinations of e and v at which equation (9) holds. Therefore, the locus of points for which $\dot{e} = 0$ is identical to the locus of points plotted in Figure 4. Increasing v with e held constant reduces the value of $\dot{e}$. Thus, below this locus $\dot{e} > 0$, and above $\dot{e} < 0$.

The locus of points at which $g = 0$ can be derived by noting that g must exceed e whenever $\dot{e} > 0$, and must be smaller whenever $\dot{e} < 0$ [see equation (10)]. Thus, if $e > 0$, then $g = 0$ for some value of v at which $\dot{e} < 0$. Similarly, the $g = 0$ locus must pass through the region of $\dot{e} > 0$ for $e < 0$. The locus of $g = 0$ is plotted in the phase diagram of Figure 6.

An equilibrium of the system occurs when both $\dot{v} = 0$ and $\dot{e} = 0$, as plotted in Figure 6. This equilibrium is unique, and is characterized by constant price. Notice that

8. Burmeister and Turnovsky [1976] show that this formulation is well-defined if μ is finite and if the weak consistency axiom, equation (1), holds.

the equilibrium occurs at precisely the same point as in Figure 4.

From the two-dimensional phase diagram we can see that adjustment will move in a counter-clockwise direction. For example, starting from a low price and from the expectations that price will increase faster than r leads to a state of excess demand. Price increases, as does e. Once the $\dot{e} = 0$ locus is crossed, prices rise, but expectations of the rate of return fall. When the $g = 0$ locus is reached, prices are high and demand has been reduced so that the market clears. Therefore, prices temporarily remain constant, and the expectation of the rate of return itself falls. The falling expectations increase supply, leading to excess supply and to downwards pressure on prices. At a later point e becomes negative (owners expect the price to fall) and ultimately the $\dot{e}$ locus is crossed. Prices drop and drops are expected, but expectations increase towards the actual rate of return. The market clears as the $g = 0$ locus is crossed. Prices begin to rise again and expectations follow.

Analysis in Appendix A.1 shows that the system is locally stable. Thus, if the system begins close enough to the equilibrium, it will converge to that point.

If the system is globally stable, then it will ultimately converge to the equilibrium at $e = 0$; the Hotelling Rule will not hold. Global stability, however, has not been established.

If the system is not globally stable, then it will oscillate, with prices cycling around the equilibrium level. The Hotelling Rule will not hold in this case either.

These conclusions can be summarized in Proposition 2.

> *Proposition 2.* Under assumptions QHS and LPA and the assumption of SAE, an equilibrium exists, characterized by constant prices. If the equilibrium is stable, then prices will adjust to the constant level. If it is unstable, prices will oscillate around the equilibrium level. The Hotelling Rule will not hold.

The formulation SAE cannot capture the phenomenon known as *regressive expectations.* With SAE, if the price rises slightly from its "normal" level, then this price increase is extrapolated and the expected return increases (*extrapolative expectations*). But it is equally plausible that some owners expect the price (and hence the return) to regress to its previous level.[9] We have noted that under the Hotelling Rule equilibrium in the market for stocks of resource is assumed to occur when $g = r$, when the actual rate of return equals the interest rate. These observations lead to the following formulation.

6. Compound Adaptive Expectations

In this formulation the expected instantaneous rate of return is a function of the actual rate of return and the equilibrium rate of return.

> *Assumption (Compound Adaptive Expectations, or CAE).* The rate of change of the expected rate of return is a function of the difference between the interest rate and the actual rate of return, and of the difference between the actual rate

9. Stiglitz [1974] postulates a formulation equivalent to

$$v_1^e(t,t) = \alpha.(v(t) - \bar{v}(t)),$$

where $\bar{v}(t)$ is an exponentially weighted average of past prices and where $\alpha < 0$ corresponds to regressive expectations, and $\alpha > 0$ corresponds to extrapolative expectations, but it is readily shown that this is equivalent to myopic perfect foresight (Marks [1979]).

and the expected rate of return. A particular formulation of this is

$$\dot{e}(t) = \gamma.[r - g(t)] + \mu.[g(t) - e(t)], \qquad \mu > 0. \tag{12}$$

Here $\gamma > 0$ corresponds to regressive expectations in the sense that if the expected rate of return equals the actual at any moment, and if the actual rate of return is less (more) that the interest rate, then the expected rate of return increases (decreases). This models the expectation that the actual rate of return will tend toward the interest rate. $\gamma < 0$ corresponds to the extrapolative expectation that the stock market equilibrium ($g = r$) is a knife-edge, and that any deviation from it will be amplified.[10]

With explicit formulation of extrapolative or regressive expectations, we can examine whether the stability (or not) is dependent on the expectations of the traders in the market.[11] The formulation CAE results from the resource owners' being aware of the Hotelling Rule: they consider the capital gains that will accrue as the price of resource v rises, and as they compare the actual rate of return g with the rate of return r available from alternative investments.[12]

The formulation CAE of equation (12) is very similar to that of

$$\dot{e}(t) = \sigma.[r - e(t)] + \nu.[g(t) - e(t)], \qquad \nu > 0. \tag{13}$$

This formulation differs from CAE in that the interest rate is compared, not with the actual rate of return from holding resource stocks, but with the expected rate of return. Since the formulation is attempting to model the process whereby the resource owners change their expectations, the formulation of equation (13) is more direct, even if the comparison may not take place. The parameter ν is positive, since the expected approaches the actual rate of return as in the simple adaptive formulation. The parameter σ can take either sign: σ positive corresponds to regressive expectations, since if the expected rate of return e is less (more) than the interest rate r, e increases (decreases); σ negative corresponds to extrapolative expectations, a knife-edge.

The two versions of the compound adaptive expectations formulation turn out to be very similar mathematically: each can be obtained as a linear transformation of the other. By analyzing the formulation CAE, we are also analyzing that of equation (13). Comparison shows that the two are identical if $\nu = \mu - \gamma$, and $\sigma = \gamma$.

The formulation CAE with its tendency for expectations to approach the interest rate may seen rather strange: in normal markets there is equilibrium with constant price level. But, as we have argued in Section 2.1 above, rational resource owners are concerned with the current return from and opportunity cost of holding stocks of non-

10. Writers on the topic have disagreed on whether in fact the stock market equilibrium is stable: Stiglitz [1974] emphasized its instability, and Kay and Mirrlees [1975] its stability.

11. The formulation CAE is similar to the two-stage formulation of expectations of price rises of Frenkel [1975], in the context of inflation. Frenkel includes a long-run expected average inflation rate instead of the (fixed) rate of return r, with an adaptive adjustment equation for this long-run expectation similar to equation (12). In order to ensure that the model produces behaviour consistent with the empirical evidence of the short-run effects of changes in the rate of monetary expansion, Frenkel (in our terms) requires that $\gamma > \mu > 0$, that is, he assumes regressive expectations.

12. The formulation CAE is also similar to that of the third model of Heal [1975] and [1981], in which the rate of change of the expected return is a function both of the difference between actual and expected returns, and of the resource-utilisation ratio, which in turn is a function of the difference between the interest rate and the actual return.

renewable resource. The formulation CAE attempts to model this behaviour, a case perhaps of economic theory (that of exhaustible resources) affecting the behaviour of economic actors, who forecast using Hotelling-type models. Note expectations are always stationary when $e = g = r$: the formulation CAE does not exclude fulfilled expectations of the Hotelling Rule.

Phase diagrams of the system with assumptions LPA and CAE, equations (6) and (12), can be developed for each combination of μ and γ. The $\dot{v} = 0$ locus is identical to that obtained for the formulation MPF; the $\dot{e} = 0$ locus is different. We show that, unlike the formulation SAE of Section 4, the formulation CAE admits of the possibility of equilibrium with $g = 0$ and $e^* \neq 0$. Indeed, $e^* = 0$ occurs only when $\gamma = 0$, which is the formulation SAE. The equilibrium expected rate of change of price, e^*, by equation (12) with $g = 0$, is equal to:

$$e^* = \gamma \frac{r}{\mu}. \tag{14}$$

With regressive expectations ($\gamma > 0$), e^* is positive, but with extrapolative expectations ($\gamma < 0$), the expected rate of return at equilibrium e^* is negative: owners expect the price to change despite the fact that it is constant at equilibrium.

Figure 7 shows the system plotted in the (e, v)-plane for various values of the parameters μ and γ. Case (a), with $\mu = 0$ and regressive expectations ($\gamma > 0$), is unstable, with the system moving to the right, tending asymptotically to the constant-v locus, with ever-increasing expected rate of price growth, e. The resource market will tend to a situation where clearing occurs with $R^D(v) = \underline{R}$ and constant price. Case (f) is similarly unstable, with negative and ever-decreasing expected rate of return and the economy tending asymptotically to the constant price locus. In this case, the resource market will tend to a situation where clearing occurs with $R^D(v) = \bar{R}$ and constant price. For $\mu \neq 0$, we see that $e^* = \gamma r/\mu$ is finite: with regressive expectations e^* is positive; with extrapolative expectations, negative.

Cases (b) through (e) all show unique equilibria characterized by constant price but with expected rates of price change which may vary from zero. Although owners may expect price changes, no such changes occur.

Analysis in Appendix A.2 shows that in cases (b), (c), and (e) the system is locally stable, in cases (a) and (f) locally unstable, and in case (d) locally stable iff the slope of the quasi-Hotelling supply function is bounded towards zero (from inequality (A6))

$$-\frac{(\mu|\lambda_V - v^* R_v^D)}{\mu - \gamma} < R_e^S(e^*) < 0. \tag{15}$$

In none of these cases does the economy tend towards a trajectory in which the Hotelling Rule is obeyed. The set of results so far can be summarized in Proposition 3.

> *Proposition 3.* Under assumptions QHS and LPA and any of the three expectations formulations (MPF, SAE, or CAE), prices will converge to a constant level or will oscillate. The Hotelling Rule will not hold.

The next task is to relax the assumption of linear price adjustment. This is done in the following section.

7. An Expectations-Augmented Model

In this section we introduce a formulation which has often been postulated in models of optimal growth with non-renewable natural resources, in which expectations of the rate of price increase directly affect the actual rate of increase.

> *Assumption (Expectations-Augmented Price Adjustment, or EAPA).* The rate of price increase (or decrease), $g(t)$, is a function of the difference between the quantity demanded and the quantity supplied and of the expected rate of price increase (or decrease). A specific formulation of this is:
>
> $$g(t) = \lambda_V (R^D - R^S) + e, \tag{16}$$

That is, even if the market for resource flow is clearing, the price will change at a rate equal to the expected rate. Then, if resource owners expect the Hotelling Rule to hold, it can, even if all markets clear.

We can argue some justification for it, other than existence of the Hotelling Rule, from the micro-conditions of the economy. The formulation of equation (16) would result from a situation where

$$\frac{\dot{V}}{V} = \lambda_V . (R^D - R^S) + \frac{\dot{V}^e}{V},$$
$$\frac{\dot{P}}{P} = \lambda_P . (Y^D - Y^S) + \frac{\dot{P}^e}{P},$$
$$e = \frac{\dot{V}^e}{V} - \frac{\dot{P}^e}{P}.$$

$\dot{V}^e | V$ is the expected rate of change of the nominal price V of resource flow, $\dot{P}^e | P$ is the expected rate of change of the nominal price P of output (a proxy for the price level). The second equation of (17) can be thought of as the outcome of the following process: in the market for output (Y) there are two direct forces on the price level—imbalance of the market for output, and participants' expectations of future price inflation, which through their behaviour will affect the price of output, even if the output market clears. Similarly, in the first equation of (17) we can think of the direct effects of imbalance of the market for resource flow and participants' expectations of the future nominal price increases on the market. Subtracting the second from the first leads to a formulation similar to that of equation (16), but with changing price level P.

Although the motivation for the formulation EAPA is not as strong as for the formulation LPA used previously, it has been used in the literature to lead to economies which obey the Hotelling Rule. We see that, in our model, with the formulation CAE of Section 5, it still cannot save the Hotelling Rule.

Phase diagrams of the system with assumptions EAPA and CAE, equations (16) and (12), are plotted in Figure 8 for various values of the parameters μ and γ. The constant-v locus (and hence, from equation (12), the constant-e locus) is steeper in Figure 8 than in Figure 7.

Case (a), with $\mu = 0$ and regressive expectations ($\gamma > 0$), is still unstable, with the economy moving to the northeast between the constant-v and constant-e loci, with $\dot{e}$ positive and g positive and less than r. It is readily shown that if

$$\lim_{v \to \infty} (v R_v^D) \equiv -c < 0, \tag{18}$$

then

$$\lim_{t \to \infty} g(t) = \frac{\gamma r}{\gamma + \lambda_V c} < r, \text{ and}$$
$$\lim_{t \to \infty} \dot{e}(t) < \gamma r.$$

Case (*f*) is similarly unstable, except that expectations are extrapolative and $\gamma < 0$.

For $\mu \neq 0$ we see that $e^* = \gamma r / \mu$ is finite: with regressive expectations e^* is positive, with extrapolative expectations, negative.

In each case an equilibrium exists with price remaining constant. Analysis in Appendix A.3 shows that in cases (*b*) and (*c*) the system is locally stable, in cases (*a*) and (*f*) locally unstable, in case (*d*) locally stable iff the slope of the quasi-Hotelling supply function is bounded towards zero (from inequality (A7))

$$-\frac{(\gamma| \lambda_V - v^* R_v^D)}{\mu - \gamma} < R_e^S(e^*) < 0, \tag{20}$$

and in case (*e*) locally stable iff the price elasticity of resource demand is bounded away from zero (from inequality (A8))

$$\frac{v^* R_v^D}{R^D(v^*)} < \frac{\gamma}{\lambda_V R^D(v^*)} < 0. \tag{21}$$

In none of these cases is the Hotelling Rule obeyed, even asymptotically.

8. Summary and Conclusions

In this paper, we have begun with the heuristic, Walrasian formulation of price adjustment (LPA), in which the rate of change of price is proportional to the excess demand in that market. In Section 6 we have augmented this by including expectations of price movement as a factor affecting price adjustment (EAPA). We have considered three modes of expectation formation, each successively more general than the previous case, in an attempt to obtain more robust results. The most general of these is the compound adaptive formulation (CAE) of Section 5, which exhibits eight cases corresponding to various values of its two parameters: one which models the degree to which the expectations of rate of return from holding resource stocks are regressive or extrapolative, and the other which models the degree of responsiveness of changes in the expected return to the difference between the expected and the actual rates of return.

Using these formulations, we have argued that in an economy without perfect foresight there appears to exist a failure of competitive markets to satisfy the Hotelling Rule. It is not merely that the resource stocks are depleted at the wrong time, but that in many cases there exists a stable equilibrium with a constant-price trajectory. This apparent failure occurs in general because the changing price, which might satisfy the Hotelling Rule and which is the result of excess demand in the resource market, will itself result in eventual market clearing after adjustment, with constant price. Nor can the inclusion of a direct effect on the price movement from the expectation of price movements (EAPA), which allows the rate of return to equal the interest rate even with market clearing, overcome this tendency towards constant-price equilibrium.

If a locally unstable equilibrium exists, then the price trajectories away from the equilibrium point may coincide with the Hotelling Rule trajectories. But as seen in Figures 7 and 8, in no cases do the price trajectories obey the Hotelling Rule, even asymptotically.

The only cases in which no constant-price equilibria exist are cases (a) and (f), with regressive and extrapolative expectations, respectively. They model cases in which the level of the expected return has no direct effect on its rate of change: $\dot{e} = \gamma.(r - g)$. In general, however, one would expect resource stock owners to compare their expectations with their experience, and adjust their expectations accordingly. Such comparisons preclude cases (a) and (f).

We have shown that even with expectation augmentation the actual rate of return in case (a) is strictly less than the interest rate, and in case (f), strictly greater. Expectations will not be stationary, although they tend to a bounded, constant rate of change, which is a function of the interest rate and degree of regression/extrapolation of the expectations.

Thus, we conclude that in cases where equilibria exist, such equilibria may be locally stable or unstable. In a few cases equilibria do not exist. But in no case does the economy obey the Hotelling Rule, even asymptotically. Thus, the analysis of this paper shows that under reasonable conditions the long-run growth paths derived for most depletable resource models cannot be supported in the short run.

9. Appendix

Consider the system

$$\dot{e} = (\mu - \gamma)g - \mu e + \gamma r,$$
$$g \equiv \frac{\dot{v}}{v} = \lambda_V[R^D(v) - R^S(e)] + \theta e, \quad \theta = 0, 1.$$

With $\gamma = \theta = 0$, this system includes the assumptions of linear price adjustment (LPA) and simple adaptive expectations (SAE) of equations (6) and (10). With $\theta = 0$ the system includes the LPA assumption and the assumption of compound adaptive expectations (CAE) of equations (6) and (12), γ positive corresponding to regressive expectations, and γ negative to extrapolative expectations. With $\theta = 1$ the system includes the CAE assumption and the assumption of expectations-augmented price adjustment (EAPA) of equations (12) and (16).

The system is locally dynamically stable iff both roots of the characteristic equation,

$$Z^2 + AZ + B = 0, \tag{A2}$$

have negative real parts, which will occur iff the functions A and B are positive when evaluated at the equilibrium. These functions at equilibrium are given by

$$A \equiv -v^* g_v - (\mu - \gamma)g_e + \mu, \quad \text{and}$$
$$B \equiv -\mu v^* g v.$$

Differentiating the second of equations (A1), we obtain

$$g_v = \lambda_V R_v^D < 0, \quad \text{and}$$
$$g_e = -\lambda_V R_e^S + \theta \geq \theta.$$

9.1 Simple Adaptive Expectations

We now set $\gamma = \theta = 0$ to convert equations (A1) into the system of SAE with LPA, equations (6) and (10). From equation (A3), functions A and B are positive at equilibrium. Since equilibrium occurs with constant v^*, g must be zero, and so e^* too from Figure 6. From Figure 1 and the quasi-Hotelling supply assumption (QHS),

$$R^S(0) = \bar{R}, \text{ and } R_e^S(0) = 0. \tag{A5}$$

Thus, $g_e = 0$, and the system is stable.

9.2 Compound Adaptive Expectations

We now allow γ to be non-zero; positive γ corresponds to regressive expectations and negative γ to extrapolative expectations; we maintain a zero θ. This converts equations (A1) into the system of CAE with LPA, equations (6) and (12).

Phase diagrams of this system are plotted in Figure 7. We note that e^* is not necessarily zero, and that under the QHS assumption the resource supply at equilibrium may be a function of the expected rate of price increase. This can occur in cases (*b*), (*c*), and (*d*) of Figure 7. Otherwise, the resource supply is completely expectations inelastic.

It is necessary and sufficient for local dynamic stability of the system of equations (6) and (12)) that the two functions A and B be positive, equations (A3). B is non-negative iff μ is non-negative. No equilibria exist in cases (*a*) and (*f*); the equilibrium in case (c) is stable, since $\mu = \gamma > 0$.

From equation (A3) and (A4), A is positive iff

$$-(\mu - \gamma) R_e^S(e^*) < \frac{\mu}{\lambda_V - v^* R_v^D}, \tag{A6}$$

which is true if resource supply is independent of price expectations, as in case (e) and perhaps in cases (b) and (d). Function A is definitely positive in case (b) since $(\mu - \gamma) R_e^S(e^*)$ is non-negative in this case.

9.3 Expectations-Augmented Price Adjustment

We now set $\theta = 1$, while allowing γ to be non-zero. This converts equations (A1) into the system of CAE and EAPA, equations (12) and (16).

Phase diagrams of this system are plotted in Figure 8. As in Appendix A.2, e^* is not necessarily zero, and the resource supply can be a function of the expected rate of price increase. This can occur in cases (b), (c), and (d) of Figure 8. Otherwise, the resource supply is completely expectations inelastic.

It is necessary and sufficient for local dynamic stability of the system of equations (12) and (16) that the two functions A and B of equation (A3) be positive. B is non-negative iff μ is non-negative. No equilibria exist in cases (a) and (f); the equilibrium in case (c) is stable, since $\mu = \gamma > 0$.

From equations (A3) and (A4), function A is positive iff

$$-(\mu - \gamma) R_e^S(e^*) < \frac{\gamma}{\lambda_V - v^* R_v^D}, \tag{A7}$$

which is true if supply is independent of price expectations and if γ is positive, as perhaps in case (d). In case (b) function A is positive, since γ is positive and $(\mu - \gamma) R_e^S(e^*)$ is non-negative. If supply is independent of price expectations and if γ is negative, as in case (e), then A is positive iff

$$v^* \lambda_V R_v^D < \gamma < 0. \tag{A8}$$

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