6

Pricing Bonds in the Australian Market

by
Christopher M. Bilson †
Timothy J. Brailsford §
Luke J. Sullivan ‡
Sirimon Treepongkaruna †

Abstract:
This paper provides an examination of term structure models in the Australian bond market. Specifically, we examine the comparative ability of various models to forecast at the short, medium and long ends of the yield curve. Overall, we find that model performance varies along the yield curve. Out-of-sample pricing tests show that most of the term structure models underprice a bond at the short and medium ends of the term structure and generally overprice bonds at the long end. Further, the level of mispricing is related to time-to-maturity, coupon payments and interest rate volatility. The results have implications for bond pricing in relatively illiquid markets like Australia's.

Keywords:
INTEREST RATES; TERM STRUCTURE; BOND PRICING; YIELD CURVE.

† School of Finance and Applied Statistics, Australian National University, Canberra ACT 0200. Email: Chris.bilson@anu.edu.au
§ UQ Business School, The University of Queensland, St Lucia QLD, 4072
‡ Citigroup Investment Research, New York USA.

Australian Journal of Management, Vol. 33, No. 1 June 2008, *© The University of New South Wales*

1. Introduction

The term structure of interest rates is a fundamental concept used in the pricing of almost all fixed interest securities and is essential in any valuation of interest rate contingency claims. For instance, a primary use of the term structure is in the valuation of a coupon bond, whereby a coupon bond can be decomposed into individual cash flows, which are valued as zero-coupon bonds. The zero-coupon bonds are then priced using spot rates derived from the term structure. In a similar way, interest rate swaps can be notionally deconstructed into two sets of interest cash flows. Using a similar approach as that used for coupon bonds, these cash flows can be valued to determine the value of the swap. The term structure can also be used in a similar fashion to price interest rate derivatives. Yet a further application of the term structure is in the predictive ability of interest rates.

In theory, an efficient market leads to an expectation that discontinuities, kinks and other irregularities in the term structure will be quickly priced out by arbitrageurs, and thus, a degree of smoothness and functional predictability within the term structure should be evident. However, in reality, the nature of financial markets restricts the identification of an observable continuous term structure. Combined with the inability of coupon bonds to properly substitute for the term structure, various techniques and models to estimate the term structure have developed.

Models of the term structure can be broadly grouped into four categories.[1] The first category comprises single-factor models using the short rate as the only explanatory variable. Such approaches include the well-known models of Merton (1973), Vasicek (1977), and Cox, Ingersoll and Ross (1985). The second category consists of multi-factor models, which typically incorporate the short-rate in addition to another factor. Examples of such factors include the long rate, as in Brennan and Schwartz (1979), the spread from Fong and Vasicek (1991), and the volatility factor from Longstaff and Schwartz (1993). In the case of Chen (1996), three factors are introduced, namely the short rate, in addition to its mean and volatility. More recently, Ang and Bekaert (2004) have proposed a model that incorporates inflation and time-varying prices of risk. The third category of models seeks to specify the dynamics of the term structure as a whole using one state variable of infinite dimension. This approach typically involves forward curve-based models such as Ho and Lee (1986) and Heath, Jarrow and Morton (1992). The final category of models includes the so-called ‘affine’ term structure models of Duffie and Kan (1996). These models propose a no-arbitrage relationship over the entire yield curve using the zero-coupon yields as factors. As such, these models are analogous to a multivariate form of the Cox, Ingersoll and Ross (1985) model.

Empirical tests of the term structure generally take one of three approaches. First, models are fitted to cross-sectional data across bonds of different maturities—sometimes referred to as a no-arbitrage fit. Second, a time-series approach is used wherein parameters are inferred from observed interest rates, typically at the short end—sometimes referred to as an equilibrium fit. Third, indirect tests are conducted by pricing interest-rate contingent claims which possess

1. For extensive reviews of the literature, see Langetieg and Smoot (1989) and Gibson, Habitant and Talay (2001).

an embedded term structure model. Generally, the literature has tended to adopt the second approach, and focus on an analysis of specific models, rather than a comparative examination across models, with some recent exceptions.

The evidence on term structure estimation is quite mixed and appears to depend on the sample data and method. For instance, Brown and Dybvig (1986) cast doubt over the Cox, Ingersoll and Ross model, which Pearson and Sun (1994) subsequently confirm, yet Gibbons and Ramaswamy (1993) reject this conclusion. Chan, Karolyi, Longstaff and Sanders (1992) adopt the time-series approach and propose a general framework to compare nested models. They find that volatility is a key parameter in the US market. In contrast, Nowman (1997) finds that mean reversion is critical in the UK market, while Dahlquist (1996) finds evidence of both volatility and mean reversion influences in the Danish market. Ait Sahalia (1996) rejects all linear drift short-term models. Carverhill (1995) finds support for the specific one-factor Markov model using Heath-Jarrow-Morton, while Amin and Morton (1994) prefer the two-factor model. Diebold and Li (2006) argue that neither the no-arbitrage nor the equilibrium approaches provide much use in forecasting and instead advocate the use of a time-varying approach.

In the case of Australia, the comparative analysis of different term structure estimation models is relatively untested. Australian research has tended to focus on the time-series of the short end of the curve. Examples include Brailsford and Maheswaran (1998), Treepongkaruna and Gray (2003, 2006), Treepongkaruna (2005), Chan (2005) and Faff and Gray (2006), who examine the fit of various one-factor and conditional models. These studies follow the earlier tests of the Expectations Theory, mainly tested over the short end of the term structure, which document limited support (Bloch 1974; Tease 1988; Alles 1995; Moschos 1995).[2] Treepongkaruna (2005) and Treepongkaruna and Gray (2006), for example, find that the forecasting ability of short-rate models in Australia is limited. Despite this fact, there is very little evidence of Australian studies using alternatives to the short-rate model. To the extent that one-factor models are unable to fully account for the term structure, the focus on the short rate is a major limitation in understanding the Australian yield curve.

The Australian market is of interest because it is generally regarded as a sophisticated market in terms of economic development and institutional structures; however, the fixed interest market is generally small and relatively illiquid. Given that prior research has failed to document consistent results, the more evidence we can gather from alternate markets possessing different characteristics, the better our likely understanding.

From the previous discussion, it is clear that there is no consensus in the extant literature as to the appropriate functional form that term structure models should take. This is particularly so in the Australian market, where the empirical evidence has been primarily limited to short-rate models. To this end, this study conducts an empirical investigation of a number of term structure models not previously examined in an Australian context. Our approach involves fitting alternative models across the yield curve and then repeating the analysis on a rolling time-series. As a consequence, we are able to undertake residual analysis to detect misspecification of the underlying pricing equation as related to the term

2. For contrasting evidence, see Juttner, Madden and Tuckwell (1975). Note that Heaney (1994) argues for caution in that previous papers may suffer from spurious regression.

structure models. The study follows in the spirit of Maes (2003, p. 1) who notes 'given the lack of uniform data samples and the widely differing estimation methods, much robustness work remains to be done.' Our method also allows for a comparison of each model's forecasting ability at varying points along the yield curve. In contrast to prior studies in the Australian market which focus predominantly on the short-end of the term structure, we also consider the predictive ability at the medium and long ends.

The remainder of this paper is organised as follows. Section 2 discusses the alternate term structure models examined in this study. Section 3 describes the data used, and methodologies employed, in the construction and comparison of the term structure models. Section 4 contains a discussion of the results, while section 5 concludes the paper.

2. Models of the Yield Curve

The yield curve describes the relationship between the yields of risk-free zero-coupon bonds and their time-to-maturity. Coupon bonds can be decomposed into a series of zero-coupon bonds, which can then be priced using continuously compounding interest. The price of a bond can be expressed as:

$$P_0 = \sum_{t=1}^{T} C_t e^{-r_t t} + FV_T e^{-r_T T} \quad (1)$$

where: C_t = the coupon;
r_t = the periodic spot rate;
FV_T = the face value of the bond; and,
t = time, ranging from 1...T, where T is the time to maturity of the bond.

Applying the intuition of McCulloch (1971, 1975), estimation of the term structure can be accomplished by using a discount function, D_t. Similar to a discount factor, the discount function describes the present value of \$1, repayable at time t. Expressing equation (1) in terms of the discount function $D_t = e^{-r_t t}$ gives;

$$P_0 = \sum_{t=1}^{T} CF_t D_t \quad (2)$$

where: CF_t = cash flow, either coupon payment or face value, at time t; and
D_t = discount function in continuous time.

Different term structure models assume different functional forms of the discount function. Given an assigned functional form, we can estimate the discount function, which allows us to estimate the term structure and any number of related curves. Following McCulloch (1971, 1975), a curvature is assumed to exist in the continuous discount function, as follows:

$$D_t = a_0 + a_1 t + a_2 t^2 + ... + a_T t^T \quad (3)$$

Substituting equation (3) into equation (2) yields:

$$P_0 = \sum_{t=1}^{T} CF_t (a_0 + a_1 t + a_2 t^2 + ... + a_T t^T) \quad (4)$$

Rearranging:

$$P_0 = a_0 \left[\sum_{t=1}^{T} CF_t \right] + a_1 \left[\sum_{t=1}^{T} tCF_t \right] + a_2 \left[\sum_{t=1}^{T} t^2 CF_t \right] + ... + a_T \left[\sum_{t=1}^{T} t^T CF_T \right] \quad (5)$$

Given that P_0 and CF_t are known, the form in (5) is advantageous as it allows $a_0, a_1, a_2, ..., a_T$ to be estimated using regression. Inserting these estimates into equation (3) produces an estimate of the discount function, which can then be placed into (2) to yield a prediction of the bond price.

In the absence of a universally true functional form for the discount function, a number of smoothing models have been developed to address this problem. These models range from linear interpolation to polynomial splines. Significantly, no consensus concerning the best functional form and the degree of smoothing has yet been reached. In this paper, we focus five models, of which four are non-linear.

2.1 Polynomial Models

Under the polynomial models, a yield curve can be approximated using a single polynomial discount function. This study investigates two forms of polynomials: a quadratic and a cubic function. The quadratic polynomial function is given by:

$$D_t = a_0 + a_1 t + a_2 t^2 \quad (6)$$

Similarly, the cubic polynomial function can be expressed as:

$$D_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad (7)$$

Substituting (6) and (7) into (4) produces the following two model specifications, the parameters of which can be estimated using regression:

Quadratic Polynomial:

$$P_0 = a_0 \left[\sum_{t=1}^{T} CF_t \right] + a_1 \left[\sum_{t=1}^{T} tCF_t \right] + a_2 \left[\sum_{t=1}^{T} t^2 CF_t \right] \quad (8)$$

Cubic Polynomial:

$$P_0 = a_0 \left[\sum_{t=1}^{T} CF_t \right] + a_1 \left[\sum_{t=1}^{T} tCF_t \right] + a_2 \left[\sum_{t=1}^{T} t^2 CF_t \right] + a_3 \left[\sum_{t=1}^{T} t^3 CF_t \right] \quad (9)$$

In order to find the estimated price of a zero-coupon bond, the following procedure is implemented. The estimated coefficients a_0, a_1 and a_2 (and a_3 in the case of the cubic model) are estimated by fitting models (8) and (9) using known price and cash flow data. The estimated coefficients are then substituted into models (6) and (7), to generate the respective discount function of the bond at time t. The price of the bond is then found by inserting the estimated discount function into (2).

2.2 Duration-Based Models

Like polynomial models, duration-based models assume that a yield curve can be approximated using a single polynomial discount function. However, instead of defining the discount function as a function of time, duration-based models define the discount function as a function of duration. There are questions about which form of duration should be adopted, however the simple Macaulay duration measure appears to capture most of the interest rate risk (Nelson & Schaefer 1983). Moreover, Hunt (1995) finds that the higher convexity term has only limited explanatory power in this context. Hence, we follow the path of parsimony and focus on the duration model as follows:

$$y_t = a_0 + a_1 M + a_2 M^2 + ... + a_t M^t \tag{10}$$

where y_t = the yield to maturity of a bond with t periods to maturity; and,
M = Macaulay duration measure of the bond.

Two variations of the duration-based models considered include a quadratic and cubic duration model. The quadratic duration-based model is defined as:

$$y_t = a_0 + a_1 M + a_2 M^2 \tag{11}$$

While the cubic duration-based model takes the following form:

$$y_t = a_0 + a_1 M + a_2 M^2 + a_3 M^3 \tag{12}$$

To form an estimate of a bond price, the duration-based models (as in (11) and (12)) are fitted using observed yields to maturity and duration measures. The fitted values from these models provide estimates of the yield at particular values of duration. These yields proxy for the spot rate r_t, which can be substituted into $D_t = e^{-r_t t}$ to provide an estimate of the discount function. The price of the bond can then be obtained by inserting the estimated discount function into (2).

2.3 Piece-Wise Linear Model

Under the piece-wise linear model, a straight line between two consecutive yield observations is plotted. Using linear interpolation, the yield for any zero-coupon bond with time to maturity j, is estimated as follows:

$$y_j = y_t + \left(\frac{y_{t+1} - y_t}{t_{t+1} - t_t} \right)(t_j - t_t) \tag{13}$$

The piece-wise linear model in (13) can be used to obtain estimates of yields at various maturities. Using a similar approach to that used for the duration-based models, these yield estimates proxy for the spot rate r_t, which provides an estimate of the discount function, via $D_t = e^{-r_t t}$. As above, the price of the bond can again be obtained by inserting the estimated discount function into (2).

3. Data and Method

3.1 Data

One of the major problems with estimating the term structure is the issue of liquidity. There is substantial evidence throughout the literature that liquidity has an impact on bond prices, and as a consequence, on yield to maturity and term structure estimation (Amihud & Mendelson 1991; Sarig & Warga 1989; Warga 1992). In contrast, a more recent study Elton and Green (1998) find that such liquidity effects are quite small. It must be noted, however, that Elton and Green's examination is based on the US Treasury market, which is far more liquid than Australia's market. As such, greater liquidity effects are expected to exist in the Australian market. To mitigate the impact of this potential bias, we conduct our analysis using a sample constructed from high liquidity periods and also filter illiquid bond issues from the sample.

The sample period commences shortly after the new financial year in 1990 owing to limited data availability prior to this date. As we are conscious of the potential impact of illiquidity, the end date is guided by the overall liquidity of the market. The budget surpluses of the Commonwealth Government since the turn of the millennium has created a downturn in the issuance of Commonwealth Government Bonds (CGBs) and caused much conjecture in Australia over the future of the long-term bond market. Hence, we avoid this issue and any speculative impact by ending the sample shortly after the financial year of 2000. Thus, the sample spans from 22 August 1990 to 23 August 2000. We collect data on 93 CGBs or 25,928 (weekly) observations.

As an additional filter, we also exclude potentially illiquid bond issues using the following procedure. First, the Datastream Research Extranet Dead Bond search facility is used to obtain information on bond issues which have previously expired or been redeemed early. To avoid the impact of illiquidity, bonds are not used if they are within six months of maturity.[3] Similarly, we exclude issues with volume less than AUD$50 million.[4] Bonds are also discarded if the

3. See Langetieg and Smoot (1989) who also note that this filter avoids any roll-over or hedging effects which exist with bonds close to maturity.
4. This approach follows Garbade and Silber (1976), Sarig and Warga (1989) and Warga (1992). This filter removes 17 full CGB issues and 8 partial CGB issues.

time-to-maturity is within six months of issuance.[5] Further, bonds with special tax features are removed.[6] The final sample comprises 75 CGBs or 18,465 (weekly) observations.

Table 1 provides descriptive statistics for the yield to maturity across the sample of bonds and the interbank rates. The means and medians differ between the interbank and CGB observations. The mean of the CGBs is 8.08%, while the mean of the interbank rate is 6.64%, a difference of 1.44%.[7,8] This positive difference conforms to expectations of where the yield curve was positioned over the decade. The short-end and long-end yields have similar volatility. The interbank rates have a fixed time-to-maturity due to the nature of zero coupon bonds. However, coupon bonds by their nature have a constantly changing time-to-maturity. We analyse the cross-sectional description of the time-to-maturity characteristics of the bond sample and note that the significant reduction in the mean values over the first three years of the data set coupled with the 'levelling off' of the mean values can be attributed to the initial low level of bond issues with time-to-maturity of greater than 10 years.

The maturity of the very long-end of the term structure, declined gradually over the period from a maximum value of 29.76 semi-annual periods in 1990 to 21.59 semi-annual periods in 2000. This reduction in the length of the term structure corresponds with a reduction in the number of issued CGBs in the market due to liquidity considerations.[9] The number of issued bonds over the sample period is depicted in figure 1 which shows the peak in 1996–97.

3.2 Term Structure Models

We analyse the ability of five models to accurate predict bond prices along the yield curve. To assess the performance of each model, we conduct out-of-sample tests of each model's predictive ability at the short-end (2-year), medium-end (5-year) and long-end (10-year). The database for the out-of-sample tests is constructed following Adams and van Deventer (1994), who suggest omitting known data, then smoothing the remaining data and comparing the estimated yields with the true market data. To gauge the model performance along the term structure, we select three bonds from each week over the sample period, and omit

5. Price behaviour of new issues differs from that of existing issues, for example, underpricing in initial public offering due to a lack of seasoning (Modigliani & Sutch 1966; Weinstein 1978; Wasserfallen & Wydler 1988), market imperfections due to transaction costs and issuer price-setting policy (Wasserfallen & Wydler 1988). Note that the 1998-99 Commonwealth Debt Management Annual Report indicates that the introduction of a new benchmark line (June 2011) produced lower prices due to a liquidity premium demanded by the market. The illiquidity in the new benchmark line meant that the weighted average yield achieved at tender and in the prevailing secondary market mid-rate was higher than in the previous fiscal year.
6. This filter removes three bonds, which are subject to rebate under section 160A of the Income Tax Assessment Act 1936.
7. The maximum value in the data set originates from the interbank sample. This occurs in the first few observations and is a product of the inverse nature of the yield curve at the beginning of the sample.
8. The interbank rates are combined for the purposes of Table 1. However, separate analysis of the individual series reveals very similar means and medians for the one-month, two-month and three-month interbank rates. The mean of the six-month rate was slightly higher consistent with the term premium.
9. The Commonwealth Debt Management (1998-99, p. 7) cites that the reduced debt issue program over the 1990s was a consequence of broader debt management objectives concerning the liquidity and efficiency of the CGB market rather than fiscal requirements.

these data. The bonds are selected as those with a time-to-maturity numerically closest to two, five and ten years. [10]

Table 1

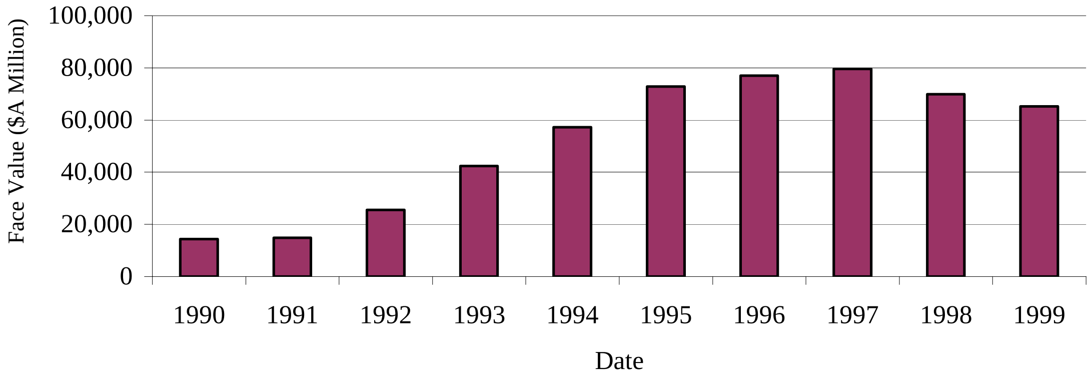
Descriptive Statistics of Yield-To-Maturity Observations for the Interbank Interest Rates and Commonwealth Government Bonds

Data are drawn over the period 22 August 1990 to 23 August 2000. The interbank rates collected include the call, one, two, three, and six-month rates—the descriptive statistics presented are based on the combined sample of these rates. The Commonwealth Government Bond data represent the population of 93 CGBs of domestic issuance in Australian dollars with available historical data.

	Interbank Rates		Commonwealth Government Bonds	
	Yield (% pa)	Maturity (Years)	Yield (% pa)	Maturity (Years)
Mean	6.64	0.20	8.08	5.12
Median	5.92	0.17	7.83	4.50
Maximum	14.15	0.50	13.92	14.89
Minimum	4.60	0.01	4.15	0.50
Std Devn	2.10	0.17	2.15	3.32
Skewness	155.22	0.70	53.60	0.61
Kurtosis	489.88	2.29	259.75	2.51
Observations	2,615	2,615	15,850	15,850

Figure 1

Commonwealth Government Bonds on Issue: 30 June 1990 to 30 June 1999



Note: Sourced Various Commonwealth Debt Management Annual Reports, Australian Government.

10. Given that we do not get exact matches with these maturities every week, we select those that are closest. In the event that the ten-year observation has the longest maturity, we select the next longest bond to avoid extrapolating the term structure (see Bliss 1996). Further, where two or more observations are selected with identical time-to-maturity values, the bond with a coupon rate closest to their respective yield-to-maturity is selected as evidence suggests that the coupon bias is greater, the further a coupon is from a security's yield-to-maturity.

The term structure is then estimated, with the exclusion of the omitted bond data, following the models described above. Once the parameters of the term structure are estimated, they can be used to estimate the price of a bond with identical characteristics to those observations removed from the database. These estimated prices are constructed by decomposing the coupon bond into strips of zero coupon bonds. Note that an adjustment is required for the accrued interest around ex-coupon dates.[11] The Reserve Bank of Australia (RBA) recommends the following formula in the calculation of Australian Government securities which takes into account these adjustments:[12]

$$\mathrm{P} = v^{f/d}\left(c + g a_{n\neg} + 100 v^{n} \right) \tag{14}$$

where
P = price per \$100 Face Value;
i = yield % p.a./200;
v = 1/(1+i);
f = the number of days to the next interest payment date;
d = the number of days in the half-year ending on the next interest payment date;
c = the amount of the interest payment (if any) per \$100 face value at the next interest payment date;
g = the fixed half-yearly interest payable (equal to the annual fixed rate divided by 2);
$a_{n\neg} = v + v^2 + \ldots + v^n = (1-v^n)/i$; and,
n = the number of full half-years between the next interest payment date and the date of maturity.

The result of this approach is an out-of-sample testing procedure that compares the theoretical yield derived from each term structure estimation model with the observed yield of the three excluded bonds. We apply these yields to a bond with a face value of \$1 to generate a pricing error. This approach leads to a 'fitted-price error' for each of the two, five and ten-year bonds. These errors, which are available each week over the sample period, can be analysed to determine the relative accuracy of each model. The fitted-price error series are assessed using the standard performance metrics of the Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE).

3.3 Pricing Influences

Past research has paid minimal attention to the influence of external factors on term structure estimation errors. However, these errors are potentially a rich source of information. Two types of influences are relevant. First, systematic distortions in

11. The coupon payments on CGBs transfer from cum-coupon to ex-coupon seven days prior to the coupon date. Since interest is paid on the 15th of the month, accrued interest will be negative for any settlement falling on or after the 8th of the month and before the coupon date. This rule is incorporated into the price calculations.
12. Prospectus—Treasury Bonds (1996, p. 3).

pricing errors may be induced by estimation model misspecification. One way to detect this bias is to analyse the explanatory relationships related to the characteristics of the estimated bond. A second type of influence may be external factors related to economic forces, such as those sometimes proposed in multi-factor models. Such external forces may be identified through the literature and include those factors that may be relevant to the pricing process.

Recall from the previous discussion that, for each of the term structure models, we have an estimated price (predicted from the term structure model) and an observed price for each of the 523 weeks in the sample. In order to analyse the relationship between systematic distortions and/or external factors, the weekly fitted pricing error is used as the dependent variable in a cross-sectional regression. The following regression is then estimated for each term structure model, excluding prices for bonds with maturities of two, five and ten years, viz:

$$FPE = \beta_0 + \beta_1 COUPON + \beta_2 TTM + \beta_3 NUMBER + \beta_4 PREM + \beta_5 VOL + \beta_6 REL\_RTN + E \quad (15)$$

where *FPE* = fitted-price error, defined as the observed price minus the estimated price obtained from the term structure model;

COUPON = coupon size;

TTM = time-to-maturity;

NUMBER = number of bonds in the term structure;

PREM = term premium—the difference between the short rate (six-month interbank rate) and the long rate (ten-year bond yield);

VOL = interest rate volatility, $\sqrt{\frac{\sum_{t=-13}^{n=-1}(y_t - \bar{y})^2}{n-1}}$, where y_t is the 90-day bank accepted bill rate, with t measured in months;

REL_RTN = relative market return premium, the difference between the dividend yield on the equity market and the interest yield on the debt market (in this case, ten year bond); and,

E = an error term.

COUPON, *TTM* and *NUMBER* are included in the regression to account for potential systematic influences associated with bond characteristics. *COUPON* and *TTM* are integral components to the valuation. *COUPON* is directly related to the coupon effect, while *TTM* determines the size of the discount factor.[13] *NUMBER* is included as we believe that the number of data estimation points used will impact on the accuracy of the estimation process.

13. The coupon effect results from the assumption that all coupon payments are immediately reinvested at the current yield-to-maturity until the bond's expiration. Clearly, where the yield curve is sloping upwards (downwards), the larger the coupon, the lower (higher) the yield-to-maturity when compared to the spot rate at the bond's maturity. Nelson (1972) argues that the coupon bias is minimal and that coupon bond yields should be treated as essentially zero-coupon yields. In contrast, Echols and Elliot (1976, p. 102) find that the impact of the coupon level upon the 'yield is statistically significant, consistently positive in direction, substantial in magnitude, and variable over time'.

PREM, *VOL* and *REL_RTN* are included in this regression in order to examine the influence of other factors on the accuracy (or otherwise) of the term structure model predictions. The term premium reflects the multi-factor approach and stems from Brennan and Schwartz (1979). *VOL* is similarly included as per Longstaff and Schwartz (1993) and Fong and Vasicek (1991). *REL_RTN* attempts to reflect market sentiment and flows between different markets, specifically, between the equity and debt markets. The variable loosely proxies for the relative demand for interest rate securities, as demand is likely to impact on the pricing of debt at least in the short-term.

4. Results

4.1 Pricing Accuracy

Table 2 reports the fitted price errors for each term structure model at specific points on the yield curve. Specifically, panel A contains fitted price errors for the short-end (2-year maturity), panel B the medium-end (5-year maturity) and panel C the long-end (10-year maturity).

Several observations can be made from table 2. First, the value of the error is less than \$0.01 (on the \$1 face value) in all cases at the short-end and medium-end. However, the error increases dramatically for some models at the long-end as demonstrated by the duration-based cubic model where the pricing error is almost \$0.05. More generally, all models tend to forecast progressively worse as the bond maturity lengthens. We believe that this is caused by the fewer number of nearby observations with longer maturities. Thus, the models have to stretch over successively longer periods along the term structure. This evidence is consistent with the findings of Adams and van Deventer (1994) who use mean absolute errors for US bonds, and report that errors in the seven-year out-of-sample bonds increase by two to four times when compared to the results of the three-year bonds.

Second, the conclusions regarding model superiority depend upon the end of the term structure under examination. For instance, the duration-based models and the piece-wise linear model perform well for two-year bonds but progressively worse for five-year bonds and ten-year bonds. While the polynomial models also perform worse over the longer maturities, they are clearly the preferred model for five-year and ten-year bonds.

Third, at the short-end, the cubic forms of the models are superior to the quadratic forms, consistent with a better fit provided by the extra parameters. However, this result is reversed at both the medium and long ends of the curve, where the quadratic forms dominate. Fourth, there is an overall tendency to underprice the two- and five-year bonds and overprice the ten-year bonds. Finally, the fitted-price errors depart significantly from normality. This raises the possibility of other systematic factors having an influence on the performance of these models.[14]

14. This issue is considered below in section 4.2.

Table 2
Descriptive Statistics of Fitted-Price Errors

The figures in the table are fitted price errors of a bond with a nominal face value of $1 calculated from the difference between observed yields and estimated yields from the following term structure models:

Quadratic polynomial $D_t = a_0 + a_1 t + a_2 t^2$ ...(6)

Cubic polynomial $D_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ ...(7)

Quadratic duration $y_t = a_0 + a_1 M + a_2 M^2$ ...(11)

Cubic duration $y_t = a_0 + a_1 t + a_2 M^2 + a_3 M^3$ ...(12)

Piece-wise linear $y_j = y_t + (y_{t+1} - y_t) \times (t_j - t_t) / (t_{t+1} - t_t)$ ...(13)

Data are drawn over the period 22 August 1990 to 23 August 2000. The data comprise interbank rates for call, one, two, three, and six-month rates and Commonwealth Government Bond data comprising 93 CGBs of domestic issuance in Australian dollars. Fitted price errors are obtained for each of the 523 weekly observations excluding yields for bonds with maturities of two, five and ten years, for each of the short-end (panel A), medium-end (panel B) and long-end (panel C) regressions, respectively. * denotes significant at the 5% level.

	Mean	Median	Std Dev	Maximum	Minimum	Jarque-Bera
Panel A: Short-end (2 years)						
Polynomial Quadratic	0.0039	0.0034	0.0036	0.0156	–0.0050	8.95*
Polynomial Cubic	0.0019	0.0016	0.0026	0.0130	–0.0062	61.49*
Duration Quadratic	0.0004	0.0006	0.0037	0.0119	–0.0128	4.06
Duration Cubic	–0.0001	–0.0002	0.0032	0.0106	–0.0092	2.91
Piece-wise Linear	–0.0005	–0.0004	0.0023	0.0090	–0.0074	140.09*
Panel B: Medium-end (5 years)						
Polynomial Quadratic	0.0009	0.0008	0.0034	0.0200	–0.0062	492.79*
Polynomial Cubic	0.0097	0.0079	0.0126	0.0485	–0.0209	47.09*
Duration Quadratic	0.0038	0.0037	0.0046	0.0187	–0.0088	12.44*
Duration Cubic	0.0051	0.0055	0.0047	0.0261	–0.0063	137.34*
Piece-wise Linear	–0.0025	–0.0023	0.0032	0.0067	–0.0135	19.89*
Panel C: Long-end (10 years)						
Polynomial Quadratic	–0.0015	0.0004	0.0112	0.0305	–0.0364	48.63*
Polynomial Cubic	0.0019	0.0015	0.0088	0.0296	–0.0198	123.25*
Duration Quadratic	–0.0213	–0.0142	0.0371	0.0655	–0.1036	26.00*
Duration Cubic	–0.0473	–0.0446	0.0697	0.1349	–0.2437	1.25
Piece-wise Linear	–0.0102	–0.0076	0.0143	0.0329	–0.0473	44.28*

The mean values of the fitted-price errors, presented in table 2 can be misleading because positive and negative values offset each other. This is a well-known issue with this metric, thus other error metrics are presented in table 3 including the root mean squared error (RMSE) in panel A and mean absolute percentage error (MAPE) in panel B.

Table 3
Error Metrics of Fitted-Price Errors

The figures in the table are error metrics of fitted price errors (FPEs) of a bond with a nominal face value of $1 calculated from the difference between observed yields and estimated yields from the following five term structure models:

Quadratic polynomial $D_t = a_0 + a_1 t + a_2 t^2$ ...(6)
Cubic polynomial $D_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ ...(7)
Quadratic duration $y_t = a_0 + a_1 M + a_2 M^2$ ...(11)
Cubic duration $y_t = a_0 + a_1 t + a_2 M^2 + a_3 M^3$ ...(12)
Piece-wise linear $y_j = y_t + (y_{t+1} - y_t) \times (t_j - t_t) / (t_{t+1} - t_t)$ ...(13)

Data are drawn over the period 22 August 1990 to 23 August 2000. The data comprise interbank rates for call, one, two, three, and six-month rates and Commonwealth Government Bond data comprising 93 CGBs of domestic issuance in Australian dollars. Fitted price errors are obtained for each of the 523 weekly observations excluding yields for bonds with maturities of two, five and ten years, for each of the short-end (panel A), medium-end (panel B) and long-end (panel C) regressions, respectively. The RMSE is calculated as $\sqrt{\frac{1}{n}\sum_{t=1}^{T}\left(\Phi_t - \hat{\Phi}_t\right)^2}$ The MAPE is calculated as $\frac{1}{n}\sum_{t=1}^{T}\left|\left(\frac{\Phi_t - \hat{\Phi}_t}{\Phi_t}\right)\times\left(\frac{100}{1}\right)\right|$,
where Φ_t is the bond price based on observed yields $\hat{\Phi}_t$ is the bond price based on estimated yields.

	RMSE (Rank)	MAPE % (Rank)
Panel A: Short-end (2 years)		
Polynomial Quadratic	0.0043 (5)	0.3899 (5)
Polynomial Cubic	0.0025 (2)	0.2252 (2)
Duration Quadratic	0.0030 (4)	0.2750 (4)
Duration Cubic	0.0026 (3)	0.2305 (3)
Piece-wise Linear	0.0016 (1)	0.1466 (1)
Panel B: Medium-end (5 years)		
Polynomial Quadratic	0.0025 (1)	0.2285 (1)
Polynomial Cubic	0.0119 (5)	1.0405 (5)
Duration Quadratic	0.0047 (3)	0.4183 (3)
Duration Cubic	0.0057 (4)	0.4969 (4)
Piece-wise Linear	0.0032 (2)	0.2843 (2)
Panel C: Long-end (10 years)		
Polynomial Quadratic	0.0070 (2)	0.6874 (2)
Polynomial Cubic	0.0059 (1)	0.6075 (1)
Duration Quadratic	0.0334 (4)	3.2173 (4)
Duration Cubic	0.0654 (5)	6.2134 (5)
Piece-wise Linear	0.0136 (3)	1.2587 (3)

Under the analysis of the RMSE metric, no model consistently outperforms the other models over all bonds and the general conclusions are similar to those drawn from table 2. That is, the piece-wise linear model performs well at the short-end ranking first, while the polynomial models perform the best for other maturities with the polynomial quadratic ranking first for the five-year bond and the polynomial cubic ranking first for the ten-year bond. Again we suspect that the extrapolation involved in the piece-wise linear approach works well when there are nearby observations and the length of the extrapolation is small, however the relatively larger gaps between bond observations at the longer term means that this model struggles at the long-end of the term structure.

Both duration-based models perform poorly relative to the other three models across all bonds. This is especially highlighted by the price deviations in the ten-year bonds where the cubic duration model has a RMSE of 0.0654, which is five to ten times larger than the RMSE of the polynomial and piece-wise linear models. The relative model rankings also provide an indication of the consistently poor performance of the duration-based approach, with neither duration model receiving a ranking of higher than three for any of the bonds. This result provides some support for the criticisms levelled by Garbade (1996), as to the inappropriate use of the yield-duration relationship.

Analysis of the MAPE metric provides a guide as to the economic significance of the pricing errors. For the two-year and five-year bonds, the MAPE figures are generally less then 0.5%. However for the ten-year bonds, the MAPE figures jump considerably with large and clearly economically significant errors observed, especially for the duration-based models.

4.2 Analysis of Pricing Errors

We now turn to the final test which seeks to examine whether there are factors that are systematically related to the pricing errors.[15] Table 4 presents the results of the fitted price error regression as in (15). Panel A contains results for the short-end (2-year maturity), panel B the medium-end (5-year maturity), and panel C the long-end (10-year maturity) of the term structure. The regressions provide a strong explanation of pricing errors, with R^2 values ranging from 10 to 64% with the higher values obtained for the long-term bonds.[16]

The regression results indicate that each of the variables plays a role in explaining pricing error variation. The specific features associated with the estimation procedure, namely coupon size, term to maturity and number of observations used in the cross-sectional estimation are generally significant and of changeable sign. That is, these factors appear to contain residual information in addition to their influence over the estimated term structure. The sporadic significant relationships with the more macro-based factors provide some support for a multifactor term structure. However, the general lack of consistency across the three bonds suggests that no single model is likely to be universally superior. The inconsistent results across the three bonds also suggest that segmented sections of the term structure may be subject to different market forces which could influence pricing behaviour. Note that the long-rate premium potentially proxies for the slope of the term structure. The term structure was positively sloped for the majority of the sample period and the models generally over-predict the yield at the short- and medium-ends and under-predict at the long-end (as demonstrated by the

15. It is important to note that we only consider linear relations between the variables, and as such any non-linearities would not be identified in our analysis.
16. Bliss (1996) runs similar regressions with R^2 values of between 15 to 30%.

Table 4

Regression of Fitted Price Errors Against Pricing Influences

The table reports estimates of the regression:

$$FPE = \beta_0 + \beta_1 COUPON + \beta_2 TTM + \beta_3 NUMBER + \beta_4 PREM + \beta_5 VOL + \beta_6 REL\_RTN \ ,$$

where FPE represents the fitted price errors of a bond with a nominal face value of $1 calculated from the difference between observed yields and estimated yields from the following five term structure models:

Quadratic polynomial $D_t = a_0 + a_1 t + a_2 t^2$ ...(6)
Cubic polynomial $D_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ ...(7)
Quadratic duration $y_t = a_0 + a_1 M + a_2 M^2$ ...(11)
Cubic duration $y_t = a_0 + a_1 t + a_2 M^2 + a_3 M^3$ ...(12)
Piece-wise linear $y_j = y_t + (y_{t+1} - y_t) \ x \ (t_j - t_t) / (t_{t+1} - t_t)$ ...(13)

COUPON is the size of the coupon payment; *TTM* is the time-to-maturity of the bond; *NUMBER* is the number of bonds in the estimation; *PREM* is the long-rate premium; *VOL* is the historical interest rate volatility; and *REL_RTN* is the difference between the equity and debt market return. Fitted price errors (FPEs) are obtained for each of the 523 weekly observations excluding yields for bonds with maturities of two, five and ten years, for each of the short-end (panel A), medium-end (panel B) and long-end (panel C) regressions, respectively. * denotes significance at the 5% level.

	intercept	COUPON	TTM	NUMBER	PREM	VOL	REL_RTN	R-square	F-statistic
				Panel A: Short-end (2 years)					
Polynomial Quadratic	0.0076*	0.0033	–0.0009*	–0.0001*	0.0030*	–0.0014*	0.0001	0.45	71.17*
	(4.53)	(0.36)	(–2.15)	(–2.86)	(13.16)	(–3.81)	(1.09)		
Polynomial Cubic	0.0099*	–0.0241*	–0.0015*	0.0000	0.0006*	–0.0010*	0.0000	0.12	11.57*
	(5.57)	(–2.50)	(–3.43)	(–1.10)	(3.33)	(–3.52)	(0.09)		
Duration Quadratic	–0.0015	–0.0498*	0.0015*	–0.0001*	0.0004	–0.0008	0.0006*	0.10	9.87*
	(–0.81)	(–4.19)	(3.18)	(–2.07)	(1.16)	(–1.48)	(3.79)		
Duration Cubic	–0.0019	–0.0563*	0.0016*	–0.0001	0.0000	–0.0008*	0.0002	0.12	11.46*
	(–1.09)	(–5.01)	(3.75)	(–1.49)	(0.13)	(–2.02)	(1.92)		
Piece-Wise Linear	0.0025	–0.0293*	–0.0007*	0.0001*	–0.0013*	–0.0007*	–0.0002*	0.16	16.09*
	(1.86)	(–3.58)	(–2.03)	(4.54)	(–6.75)	(–2.47)	(–2.34)		

Table 4 Cont.

	intercept	COUPON	TTM	NUMBER	PREM	VOL	REL_RTN	R-square	F-statistic
Panel B: Medium-end (5 years)									
Polynomial Quadratic	0.0151*	–0.0106	–0.0014*	–0.0001*	0.0020*	0.0019*	0.0003*	0.20	21.29*
	(4.62)	(–0.83)	(–4.23)	(–4.67)	(8.14)	(4.15)	(2.83)		
Polynomial Cubic	–0.0099	–0.1577*	0.0015	–0.0001	0.0119*	–0.0016	0.0005	0.55	105.80*
	(–0.95)	(–3.63)	(1.52)	(–0.85)	(17.76)	(–1.11)	(1.53)		
Duration Quadratic	0.0052	0.0644*	–0.0006	–0.0001	0.0029*	0.0007	–0.0004*	0.42	61.48*
	(1.36)	(4.88)	(–1.48)	(–1.56)	(10.45)	(1.35)	(–2.81)		
Duration Cubic	0.0053	0.0011	–0.0002	–0.0002*	0.0046*	0.0006	0.0008*	0.46	72.05*
	(1.43)	(0.07)	(–0.67)	(–6.51)	(15.90)	(1.21)	(6.79)		
Piece-Wise Linear	0.2017*	–0.9287*	–0.0080*	0.0013*	–0.0283*	0.0196*	–0.0203*	0.48	77.84*
	(2.88)	(–4.03)	(–2.21)	(2.30)	(–6.69)	(2.05)	(–8.96)		
Panel C: Long-end (10 years)									
Polynomial Quadratic	0.0397*	–0.7686*	–0.0005	0.0002	–0.0057*	–0.0015	0.0013*	0.56	109.38*
	(3.64)	(–20.78)	(–0.78)	(1.74)	(–8.52)	(–1.38)	(5.03)		
Polynomial Cubic	0.0104*	–0.2515*	0.0000	0.0001	0.0019*	–0.0010*	–0.0001	0.57	55.47*
	(2.42)	(–15.89)	(–0.03)	(1.93)	(6.64)	(–2.17)	(–0.34)		
Duration Quadratic	–0.0083	–0.5020*	0.0047*	–0.0025*	–0.0062*	0.0002	0.0036*	0.43	64.29*
	(–0.20)	(–3.54)	(2.19)	(–9.53)	(–2.60)	(0.04)	(3.04)		
Duration Cubic	0.0450*	–1.0829*	–0.0005	0.0003*	–0.0082*	0.0001	0.0010*	0.64	151.23*
	(3.29)	(–23.21)	(–0.72)	(2.75)	(–11.96)	(0.04)	(3.42)		
Piece-Wise Linear	–0.0018	0.0476*	0.0001	0.0000	–0.0021*	–0.0015*	0.0002*	0.39	54.30*
	(–0.61)	(3.61)	(0.31)	(–1.78)	(–10.70)	(–3.09)	(2.65)		

positive mean fitted price errors in table 2 for two-year and five-year bonds, and the negative fitted price errors for ten-year bonds), thereby implying a flatter line than observed. The generally positive (significant) relationship with the relative return variable is consistent with shifts in investor sentiment and resultant funds moving from the debt market to the equity market when return differentials between the markets change.

5. Conclusion

Using an Australian data set, this paper has provided an examination of bond pricing methodologies, focusing on out-of-sample pricing performance. Overall, the results show the piece-wise linear model exhibits superior performance over the short end of the term structure, however all the models are reasonably accurate, with MAPE measures less than 0.5%. However, at the long end, the polynomial model, and specifically its cubic form, provides superior performance. The duration-based models perform poorly in all tests, especially in the long end of the term structure. Of note, the performance of all models worsens as the maturity of the bond increases. We suspect this is due to the relative lack of nearby observations at the long end and hence the implied interpolation in the procedure becomes important. Generally the models imply a flatter term structure than observed. We document that the pricing error is significantly related to features of the bonds (coupon size and term to maturity), and sensitive to the number of bonds used in the estimation. Further, we find that interest rate volatility, the term premium and the differential return between equity and bond markets all have some explanatory power over the bond pricing errors.

These empirical findings have implications for the estimation and simulation of yield curves and bond prices. First, the evidence herein does not support the use of duration-based models. Second, at the short end, a simple piece-wise linear model appears to work as well as more complex models. Third, at the long end, and we suspect in illiquid markets, the cubic polynomial model is preferred. Irrespective of which model is adopted, no single model is likely to be consistently superior across the yield curve at all points in time. Finally, other factors appear relevant and it remains to be seen whether multifactor models are capable of capturing this variation.

(Date of receipt of final transcript: October 25, 2007.
Accepted by David Gallagher & Garry Twite, Area Editors.)

References

Adams, K. & van Deventer D. 1994, 'Fitting yield curves and forward rate curves with maximum smoothness', *Journal of Fixed Income*, vol. 4, pp. 52–62.

Amihud, Y. & Mendelson, H. 1991, 'Liquidity, maturity, and the yields on US Treasury securities', *Journal of Finance*, vol. 46, pp. 1411–25.

Ait Sahalia, Y. 1996. 'Testing continuous-time models of the spot interest rate', *Review of Finance*, vol. 9, pp. 385–426.

Alles, L.A. 1995, 'Time-varying risk premium and the predictive power of the Australian term structure of interest rates', *Accounting and Finance*, vol. 35, pp. 77–96.

Amin, K.I. & Morton, A.J. 1994, 'Implied volatility functions n arbitrage-free structure models,' *Journal of Financial Economics*, vol. 35, no. 2, pp. 141–80.

Ang, A. & Bekaert, G. 2004, 'The term structure of real rates and expected inflation', European Finance Association 2004 Maastricht Meetings Paper No. 1220.

Bliss, R. 1996, 'Testing term structure estimation methods', working paper, Federal Reserve Bank of Atlanta.

Bloch, F.A. 1974, 'The term structure of interest rates: A test using the error-learning model', *The Economic Record*, vol. 50, pp. 77–93.

Brailsford, T.J. & Maheswaran, K. 1998, 'The dynamics of the Australian short-term interest rate', *Australian Journal of Management*, vol. 23, pp. 213–33.

Brennan, M. & Schwartz, E.S. 1979, 'A continuous time approach to the pricing of bonds', *Journal of Banking and Finance*, vol. 3, pp. 133–55.

Brown, S. & Dybvig, P.H. 1986, 'The empirical implications of the Cox, Ingersoll, Ross theory of the term structure of interest rates', *Journal of Finance*, vol. 41, no. 3, pp. 616–28.

Carverhill, A. 1995,'A note on the models of Hull and White for pricing options on the term structure', *Journal of Fixed Income*, vol. 5, no. 3, pp. 89–96.

Chan, K.F. 2005, 'Modelling conditional heteroscedasticity and jumps in Australian short-term interest rates', *Accounting and Finance*, vol. 45, pp. 537–51.

Chan, K.C., Karolyi, G.A., Longstaff, F.A. & Sanders, A.B. 1992, 'An empirical comparison of alternative models of the short-term interest rate', *Journal of Finance*, vol. 47, pp. 1209–27.

Chen, L. 1996, 'A bond pricing formula under a non-trivial, three-factor model of interest rates', *Economics Letters*, vol. 51, no. 1, pp. 95–9.

Cox, J.C., Ingersoll, Jr. J.E. & Ross, S.A. 1985, 'A theory of the term structure of interest rates', *Econometrica*, vol. 53, pp. 385–467.

Dahlquist, M. 1996, 'On alternative interest rate processes', *Journal of Banking and Finance*, vol. 20, pp. 1093–119.

Diebold, F.X. & Li, C. 2006, 'Forecasting the term structure of government bond yields', *Journal of Econometrics*, vol. 130, no. 2, pp. 337–64.

Duffie, D. & Kan, R. 1996, 'A yield factor model of interest rates', *Mathematical Finance*, vol. 6, no. 4, pp. 379–406.

Echols, M.E. & Elliott, J.W. 1976, 'A quantitative yield curve model for estimating the term structure of interest rates', *Journal of Financial and Quantitative Analysis*, vol. 11, pp. 87–113.

Elton E.J. & Green, T.C. 1998, 'Tax and liquidity effects in pricing government bonds', *Journal of Finance*, vol. 53, no. 5, pp. 1533–62.

Faff, R. & Gray, P. 2006, 'On the estimation and comparison of short-rate models using the generalised method of moments', *Journal of Banking and Finance*, vol. 30, pp. 3131–46.

Fong, H.G. & Vasicek, O.A. 1991, 'Fixed-income volatility management', *Journal of Portfolio Management*, vol. 17, no. 4, pp. 41–6.

Garbade, K.D. (ed.), 1996, *Fixed Income Analytics*, MIT Press, New York.

Garbade, K.D. & Silber, W.L 1976, 'Price dispersion in the government securities market', *Journal of Political Economy*, vol. 84, pp. 721–20.

Gibbons, M. & Ramaswamy, K. 1993, 'A test of the Cox, Ingersoll and Ross model of the term structure', *Review of Financial Studies*, vol. 6, pp. 619–58.

Gibson, R., Habitant, F. & Talay, D. 2001, 'Modeling the term structure of interest rates: A review of the literature', working paper, HEC.

Heath, D., Jarrow, R. & Morton, A. 1992, 'Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation', *Econometrica*, vol. 60, no. 1, pp. 77–106.

Heaney, R.A. 1994, 'Predictive power of the term structure in Australia in the late 1980s: A note', *Accounting and Finance*, vol. 34, pp. 68–72.

Ho, T.S.Y. & Lee, S.B. 1986, 'Term structure movements and pricing interest rate contingent claims', *Journal of Finance*, vol. 41, pp. 1011–29.

Hunt, B.F. 1995, 'Modelling the yields on Australian coupon paying bonds', working paper, University of Technology, Sydney.

Juttner, D.J., Madden, G.M. & Tuckwell, R.H. 1975, 'Time series analysis of the term structure of Australian interest rates', *Economic Record*, vol. 51, pp. 19–30.

Langetieg, T.C. & Smoot, S.J. 1989, 'Estimation of the term structure of interest rates', *Research in Financial Services*, vol. 1, pp. 181–222.

Longstaff, F.A. & Schwartz, E.S. 1993, 'Interest rate volatility and bond prices', *Financial Analysts Journal*, vol. 49, no. 4, pp. 70–4.

Maes, K. 2003, 'Modeling the term structure of interest rates: Where do we stand?', National Bank of Belgium Working Paper Series No. 42.

McCulloch, J.H. 1971, 'Measuring the term structure of interest rates', *Journal of Business*, vol. 44, pp. 19–31.

McCulloch, J.H. 1975, 'An estimate of the liquidity premium', *Journal of Political Economy*, vol. 83, pp. 95–120.

Merton, R.C. 1973, 'The theory of rational option pricing', *Bell Journal of Economics*, vol. 4, no. 1, pp. 141–83.

Modigliani, F. & Sutch, R. 1966, 'Innovations in interest rate policy', *American Economic Review (Papers and Proceedings)*, vol. 56, pp. 178–97.

Moschos, D.M. 1995, 'The information content of the yield curve in Australia', *Journal of Macroeconomics*, vol. 17, pp. 93–109.

Nelson, C.R. 1972, 'Estimation of term premiums from average yield differentials in the term structure of interest rates', *Econometrica*, vol. 40, pp. 277–87.

Nelson, J. & Schaefer, S. 1983, 'The dynamics of the term structure and alternative portfolio immunization strategies', in *Innovations in Bond Portfolio Management*, G.G. Kaufman, G.O. Bierwag & A. Toevs, eds., JAI Press, Greenwich, Connecticut.

Nowman, K.B. 1997, 'Gaussian estimation of single-factor continuous time models of the term structure of interest rates', *Journal of Finance*, vol. 52, no. 4, pp. 1695–1706.

Pearson, N.D. & Sun, T-S. 1994, 'Exploiting the conditional density in estimating the term structure: An application of the Cox, Ingersoll and Ross model,' *Journal of Finance*, vol. 49, no. 4, pp. 1279–304.

Sarig, O. & Warga, A. 1989, 'Bond price data and bond market liquidity', *Journal of Financial and Quantitative Analysis*, vol. 24, pp. 367–78.

Tease, W.J. 1988, 'The expectations theory's of the term structure of interest rates in Australia', *The Economic Record*, vol. 64, pp. 120–7.

Treepongkaruna, S. 2005, 'Forecasting and finite sample performance of short rate models: International evidence', *International Review of Finance*, vol. 5, pp. 175–98.

Treepongkaruna, S. & Gray, S.F. 2003, 'On the robustness of short-term interest rate models', *Accounting and Finance*, vol. 43, pp. 87–121.

Treepongkaruna, S. & Gray, S.F. 2006, 'Are there non-linearities in short-term interest rates?', *Accounting and Finance*, vol. 46, pp. 149–67.

Vasicek, O. 1977, 'An equilibrium characterization of the term structure', *Journal of Financial Economics*, vol. 5, pp. 177–88.

Warga, A. 1992, 'Bond returns, liquidity, and missing data', *Journal of Financial and Quantitative Analysis*, vol. 27, pp. 605–17.

Wasserfallen, W. & Wydler, D. 1988, 'Underpricing of newly issued bonds: Evidence from the Swiss capital market', *Journal of Finance*, vol. 43, pp. 1177–91.

Weinstein, M.I. 1978, 'The seasoning process of new corporate bond issues', *Journal of Finance*, vol. 33, pp. 1343–54.