# STRATEGIC GAME THEORY FOR MANAGERS

**Outline of subject:**

<table>
<thead>
<tr>
<th>Lecture</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td><strong>1. Strategic Decision Making</strong></td>
</tr>
<tr>
<td>2.</td>
<td>Firms Behaving Badly.</td>
</tr>
<tr>
<td>3.</td>
<td>Further Equilibria.</td>
</tr>
<tr>
<td>5.</td>
<td><strong>2. Decision Analysis:</strong></td>
</tr>
<tr>
<td></td>
<td>Games Against Nature</td>
</tr>
<tr>
<td>7.</td>
<td>Gaining Insight.</td>
</tr>
<tr>
<td>8.</td>
<td>Utility.</td>
</tr>
<tr>
<td>9.</td>
<td><strong>6. Unpredictability</strong></td>
</tr>
</tbody>
</table>
11. **8. Using Information Strategically**
12. Screening, Signalling.
15. **10. Contracting, or the Rules of the Game**
17. **13. Choosing the Right Game: Coopetition**
18. Concluded.
19. **Student presentations**
20. Concluded.
Books

Recommended (but not required) text:


As well, the following books will be found useful:


Package

Assessment
Quotable Quotes

Game theory:

“the greatest auction in history”

“When government auctioneers need worldly advice, where can they turn? To mathematical economists, of course ... As for the firms that want to get their hands on a sliver of the airwaves, their best bet is to go out first and hire themselves a good game theorist.”
The Economist, July 23, 1994, p.70.

the “most dramatic example of game theory’s new power ... It was a triumph, not only for the FCC and the taxpayers, but also for game theory (and game theorists).”
Fortune, February 6, 1995, p.36.

“Game theory, long an intellectual pastime, came into its own as a business tool.”

“Game theory is hot.”
On-line References

... but game theory is not new!

For a history of game theory, see
www.econ.canterbury.ac.nz/hist.htm

For more game theory on the Web, see
www.economics.harvard.edu/~aroth/alroth.html

For a glossary of terms you should be familiar with after Chongwoo’s SET introduction, see
Game Theory

“Conventional economics takes the structure of markets as fixed. People are thought of as simple stimulus-response machines. Sellers and buyers assume that products and prices are fixed, and they optimize production and consumption accordingly. Conventional economics has its place in describing the operation of established, mature markets, but it doesn’t capture people’s creativity in finding new ways of interacting with one another.

Game theory is a different way of looking at the world. In game theory, nothing is fixed. The economy is dynamic and evolving. The players create new markets and take on multiple roles. They innovate. No one takes products or prices as given. If this sounds like the free-form and rapidly transforming marketplace, that’s why game theory may be the kernel of a new economics for the new economy.”

— Brandenburger & Nalebuff
Foreword to Co-opetition
1. **Strategic Decision Making**

**Business is war and peace.**

- Cooperation in creating value.
- Competition in dividing it up.
- No cycles of War, Peace, War, .... but simultaneously war and peace.

“You have to compete and cooperate at the same time.”
— Ray Noorda of Novell.

☞ **Co-opetition**

(See Lectures 17 and 18 later and Brandenburger & Nalebuff in the Package.)
Manual for “Co-opetition”

How to:
— cooperate without being a saint
— compete without killing the opposition.

➡ Game Theory
1.1 Business is a Game, of Sorts

Business is a game, but different from structured board games or arcade games or computer games:

➢ it is not win-lose (not zero-sum): possible for all players to win

➢ apart from the law, there is no rule book

➢ others will change the game to their advantage

➢ the game is made up of five PARTS (see below)

➢ success comes from playing the right game

So game theory provides a framework for an ever-rapidly changing world.
Wider issues.

In Lectures 17 and 18 we go beyond the more micro issues → wider issues:
Which game should your firm/organisation be in?

It’s no good sticking to your knitting if there’s no demand for jumpers.
**Question: High or low?**

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>High</th>
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<tbody>
<tr>
<td>You</td>
<td>$40 m</td>
<td>$80 m</td>
</tr>
<tr>
<td>Rival</td>
<td>$20 m</td>
<td>$160 m</td>
</tr>
</tbody>
</table>
The PARTS of the Game

Players: customers, suppliers, substitutors, complementors; change any, including yourself.

Added Values: what each player adds to the game (taking the player out would subtract their added value). Ways to raise yours, or lower theirs.

Rules: give structure to the game; in business — no universal set of rules from law custom, practicality, or contracts can revise exiting rules, or devise new ones

Tactics: moves to shape the way:
— players perceive the game, and hence
— how they play

Tactics to reduce misperception, or to create or maintain misperception.

Scope: the bounds of the game: expand or shrink.

PARTS gives more than a framework, it provides a complete set of levers.

PARTS provides a method to promote non-routine thinking, see later.
## A Case: The New York Post v. the New York News

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<tr>
<th></th>
<th>N.Y. Post</th>
<th>N.Y. News</th>
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<tbody>
<tr>
<td>January 1994</td>
<td>40¢</td>
<td>40¢</td>
</tr>
<tr>
<td>February 1994</td>
<td>50¢</td>
<td>40¢</td>
</tr>
<tr>
<td>March 1994</td>
<td>25¢</td>
<td>40¢</td>
</tr>
<tr>
<td>(in Staten Island)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 1994</td>
<td>50¢</td>
<td>50¢</td>
</tr>
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</table>

What had changed?
How?
Who ended up as price leader?
1.2 A gentle introduction

Piemax Inc. bakes and sells dessert pies. Its decision:

— price high or low for today’s pies?

Things to be considered:
— prices of rivals’ pies?
— prices of non-pie substitutes?

A naïve option:

simply optimise its pricing policy given its beliefs about rivals’ prices, or
Think strategically...

Alternative:
try to predict those prices,
using Piemax’ knowledge of the industry,
in particular: its knowledge that its rivals will choose their prices based on their own predictions of the market environment, including Piemax’ own prices.

Game Theory →
• Piemax should build a model of the behaviour of each individual competitor,
• Which behaviour would be most reasonable to expect?

Later: what is an equilibrium?
Later: ought Piemax to believe that the market outcome → equilibrium?

Now: what kind of model?
The simplest kind of model.

— All bakers operate for one day only (a so-called one-shot model)
— All bakers know the production technologies and objectives of the others
— Study with the tools of:
  ➢ **payoff matrix** games and
  ➢ **Nash equilibrium**

Nash Equilibrium: no player has any incentive to change his or her action, assuming that the other player(s) have chosen their best actions for themselves.

Nash equilibria are self-reinforcing.

In two-player games, a Nash equilibrium prescribes strategies that are mutually best response (not universally best responses, as with dominant strategies).
Repeated interactions.

If more than one day (a repeated game or interaction):

— then Piemax’s objectives?

(more than maximising today’s profits)

e.g. low price today may:

→ customers switch from a rival brand

→ increase Piemax’ market share in the future

e.g. baking a large batch of pies may

→ allow learning by doing by the staff

& lower production costs in the future
But there are dangers!

Its rivals may be influenced by Piemax’s price today
→ a low Piemax price may trigger
→ a price war.

Such dynamic games can be dealt with using
  — extensive-form game trees and
  — the solution concept of subgame perfection

Subgame Perfect Equilibrium:
a Nash equilibrium that does not rely on non-credible threats (that satisfies backwards induction).
How about information?

- What if Piemax is uncertain of the cost functions or the long-term objectives of its rivals?
  - Has Cupcake Pty Ltd just made a breakthrough in large-batch production?
  - Does Sweetstuff plc care more about market share than about current profits?
  - And how much do these rivals know about Piemax?

Incomplete information games.

Acting in a fog: perceptions rule!
And learning?

➢ If the industry continues for several periods, then Piemax ought to learn about Cupcake’s and Sweetstuff’s private information from their current pricing behaviour and use this information to improve its future strategy.

➢ In anticipation, Cupcake and Sweetstuff may be loath to let their prices reveal information that enhances Piemax’s competitive position:

➢ They may attempt to manipulate Piemax’s information.
In a nutshell ...

Game theory is the study of rational behaviour in situations involving interdependence:

- May involve common interests: coordination
- May involve competing interests: rivalry
- Rational behaviour: players do the best they can, in their eyes;
- Because of the players’ interdependence, a rational decision in a game must be based on a prediction of others’ responses.

Put yourself in the other’s shoes and predicting what action the other person will choose, you can decide your own best action.
1.3 Strategic Interaction

- Game theory → a game plan, a specification of actions covering all possible eventualities in strategic interactions.

- Strategic situations:
  involving two or more participants, each trying to influence, to outguess, or to adapt to the decisions or lines of behaviour that others have just adopted or are expected to adopt (Tom Schelling).

Look forward and reason backwards!
The flat tyre and myopia ...

Two college students, very confident about their mid-term exam performance in a subject, decided to attend a party the weekend before the final exam. The party was so good that they overslept the whole Sunday.

Instead of taking the exam unprepared on Monday, they pleaded to the professor to give them a make-up exam. Their excuse was a flat tyre without a spare and any help. The professor agreed.

On Tuesday morning, the professor placed them in separate rooms and handed them the test. The test had just one question:

Which tyre?
And the applications ...

— a procurement manager trying to induce a subcontractor to search for cost-reducing innovations
— an entrepreneur negotiating a royalty arrangement with a manufacturing firm to license the use of a new technology
— a sales manager devising a commission-payments scheme to motivate salespeople
— a production manager deciding between piece-rate and wage payments to workers
— designing a managerial incentive system
— how low to bid for a government procurement contract
— how high to bid in an auction
— a takeover raider’s decision on what price to offer for a firm
— a negotiation between a multinational and a foreign government over the setting up of a manufacturing plant
— the haggling between a buyer and seller of a used car
— collective bargaining between a trade union/employees and an employer
1.4 Some Interactions

1.4.1 Auctioning a Five-Dollar Note

Rules:
➢ First bid: 20¢
➢ Lowest step in bidding: 20¢
(or multiples of 20¢)
➢ Auction lasts until the clock starts ringing.
➢ Highest bidder pays bid and gets $5 in return.
➢ Second-highest bidder also pays, but gets nothing.

Write down the situation as seen by
1. the high bidder, and
2. the second highest bidder.

What happened?

Escalation and entrapment

Examples? (See O’Neal’s paper in the package.)
1.4.2 Schelling’s Game

Rules:

➢ Single play, $4 to play: by writing your name on the slip
➢ Vote “C” (Coöperate) or “D” (Defect).
➢ Sign your ballot. (and commit to pay the entry fee.)
➢ If x% vote “C” and (100 – x)% vote “D”:
  • then “C”s’ payoff = \( \frac{x}{100} \times 6 \) – $4
  • then “D”s’ payoff = “C” payoff + $2
➢ Or: You needn’t play at all.
Schelling’s Game

Note: the game costs $4 to join.
Schelling’s Game

WHAT HAPPENED?
- numbers & payoffs.
- previous years?

Dilemma: \[
\begin{cases}
\text{coöperate for the common good} \\
\text{defect for oneself}
\end{cases}
\]

Public/private information

Examples?
1.4.3 The Ice-Cream Sellers

(See Marks in the Package)

\[
\begin{array}{ccc}
L & C & R \\
\wedge & \wedge & \wedge
\end{array}
\]

➢ Demonstration
➢ Payoff matrix
➢ Incentives for movement?
➢ Examples?
Modelling the ice-cream sellers.

We can model this interaction with a simplification: each seller can either:

- move to the centre of the beach (M), or
- not move (stay put) (NM).

The share of ice-creams each sells (to the total population of 80 sunbathers) depends on its move and that of its rival.

Since each has two choices for its location, there are $2 \times 2 = 4$ possibilities.

We use arrows and a payoff matrix, which clearly outlines the possible actions of each and the resulting outcomes.

What are the sales if neither moves (or both NM)? Each sells to half the beach.

What are the sales if You move to the centre (M) and your rival stays put at the three-quarter point?

What if you both move?

Given the analysis, what should you do?
The Ice-Cream Sellers

The other seller

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>NM</th>
</tr>
</thead>
<tbody>
<tr>
<td>You M</td>
<td>40, 40</td>
<td>50, 30</td>
</tr>
<tr>
<td>NM</td>
<td>30, 50</td>
<td>40, 40</td>
</tr>
</tbody>
</table>

**TABLE 1.** The payoff matrix (You, Other)

A non-cooperative, zero-sum game, with a **dominant strategy**, or dominant move.
Real-World Ice-Cream Sellers

Think of the beach as a product spectrum, each end representing a particular niche, and the centre representing the most popular product.

Demand is largest for the most popular product, but so is competition.

This simple model: a tendency to avoid extremes, especially with barriers to entry for new players.

Examples: — the convergence of fashions?
— the similarity of commercial TV and radio programming?
— the copy-cat policies of political parties?
— the parallel scheduling of Ansett and Qantas?

A twist: What if the centre is too far for some bathers (at the ends of the beach) to walk?

Then the tendency for the sellers to offer the same product (at the centre) is reduced, and they might differentiate their products.
1.4.4 The Prisoner’s Dilemma

(See Marks in the Package)

**Case: Telstra and Optus and advertising.**

David Ogilvy: Half the money spent on advertising is wasted; the problem is identifying which half.

Telstra and Optus independently must decide how heavily to advertise.

Advertising is expensive, but if one telco chooses to advertise moderately while the other advertises heavily, then the first loses out while the second does well.

Let’s assume if both Advertise Heavily then Telstra nets $70,000, while Optus nets $50,000.

But if Telstra Advertises Heavily while Optus Advertises Moderately only, then Telstra nets $140,000 while Optus nets only $25,000, and vice versa.

If both Advertise Moderately, then Telstra nets $120,000 and Optus nets $90,000.

What to do?
The Advertising Game

<table>
<thead>
<tr>
<th></th>
<th>Optus</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Heavy</td>
<td>Moderate</td>
</tr>
<tr>
<td>Telstra</td>
<td>70, 50</td>
<td>140, 25</td>
</tr>
<tr>
<td></td>
<td>25, 140</td>
<td>120, 90</td>
</tr>
</tbody>
</table>

**TABLE 2.** The payoff matrix (Telstra, Optus)

Ranking outcomes is OK: 4 = best → 1 = worst.
The Traditional, Symmetric Payoffs for the Prisoner’s Dilemma:

The Payoff Matrix:

➢ The Cheater’s Reward = 5
➢ The Sucker’s Payoff = 0
➢ Mutual defection = 2
➢ Mutual coöperation = 4

These are chosen so that: 5 + 0 < 4 + 4
so that C,C is efficient in a repeated game.
The Prisoner’s Dilemma

A need for:

* communication
* coördination
* trust
* or?

**Efficient Outcome:** there is no other combination of actions or strategies that would make at least one player better off without making any other player worse off.
**The Prisoner’s Dilemma**

The other player

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C</strong></td>
<td>4, 4</td>
<td>0, 5</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>5, 0</td>
<td><strong>2, 2</strong></td>
</tr>
</tbody>
</table>

**TABLE 3.** The payoff matrix (You, Other)

A non-cooperative, positive-sum game, with a dominant strategy.

Efficient at 

Nash Equilibrium at ____
1.4.5 The Capacity Game

Case: Du Pont's Titanium dioxide capacity.

Titanium dioxide is a whitener for paint, paper, and plastics.

In 1972 du Pont, with 34% of the U.S. market for titanium dioxide, announced additional capacity which would after six years result in its share rising to 65%.

Du Pont resisted others’ price rises, but slackening demand growth meant its plans were reduced in size and delayed.

But its preemptive strategy has been very profitable: it is now the global leader in titanium dioxide supply and its exclusive ilmenite production is the lowest-cost technology.

A Simpler Model:

Two firms each produce identical products and each must decide whether to Expand (E) its capacity in the next year or not (DNE).

A larger capacity will increase its share of the market, but at a lower price.

The simultaneous capacity game between Alpha and Beta can be written as a payoff matrix.
## The Capacity Game

<table>
<thead>
<tr>
<th></th>
<th>DNE</th>
<th>Expand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>$18, $18</td>
<td>$15, $20</td>
</tr>
<tr>
<td></td>
<td>$20, $15</td>
<td>$16, $16</td>
</tr>
</tbody>
</table>

### TABLE 4. The payoff matrix (Alpha, Beta)

A non-cooperative, positive-sum game, with a dominant strategy.

Efficient at ____

Nash Equilibrium at ____
**Equilibrium.**

At a **Nash equilibrium**, each player is doing the best it can, given the strategies of the other players.

We can use arrows in the payoff matrix to see what each player should do, given the other player’s action.

The Nash equilibrium is a self-reinforcing focal point, and expectations of the other’s behaviour are fulfilled.

The Nash equilibrium is not necessarily efficient.

The game above is an example of the Prisoner’s Dilemma: in its one-shot version there is a conflict between collective interest and self-interest.
1.5 Modelling Players’ Preferences

Without uncertainty or any dice-rolling, we need only rank the four combinations:

- best, good, bad, worst:

→ payoffs of 4, 3, 2, and 1, respectively, in a $2 \times 2$ interaction.
Complications

Larger numbers of possible actions:

harder to rank the larger number of outcomes
(with three actions there are $3 \times 3 = 9$),
but ranking sufficient.

(i.e. ordinal preferences, instead of asking “by how much is one outcome preferred to another?”)

Later, with mixed strategies (probabilistic or dice-throwing) and unpredictability:

- use probabilities over actions and
- the expected values of the possible outcome
- use cardinal measures over the amounts, usually dollar amounts, which are unambiguous, and the numbers matter!
1.6 More Interactions

1.6.1 Battle of the Bismark Sea

It’s 1943: Actors:

➢ Admiral Imamura: ordered to transport Japanese troops across the Bismark Sea to New Guinea, and

➢ Admiral Kenney: wishes to bomb Imamura’s troop transports.

Decisions/Actions:

➢ Imamura:
   — a shorter Northern route (2 days) or
   — a longer Southern route (3 days)

➢ Kenney: where to send his planes to look for Imamura’s ships; he can recall his planes if the first decision was wrong, but then loses one day of bombing.

Some ships are bombed in all four combinations. Kenney and Imamura each have the same action set — {North, South} — but their payoffs are never the same. Imamura’s losses are Kenney’s gains: a zero-sum game.
The Battle of the Bismark Sea

➢ Does any player have a dominant strategy?
➢ What is the most obvious way the game should be played?
The Battle of the Bismark Sea

<table>
<thead>
<tr>
<th></th>
<th>North</th>
<th>South</th>
</tr>
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<tbody>
<tr>
<td>North</td>
<td>2, -2</td>
<td>2, -2</td>
</tr>
<tr>
<td>South</td>
<td>1, -1</td>
<td>3, -3</td>
</tr>
</tbody>
</table>

**TABLE 5.** The payoff matrix (Kenney, Imamura)

A non-cooperative, zero-sum game, with an iterated dominant strategy equilibrium.

There is no other equilibrium combination: with all other combinations, at least one of the players stands to gain by changing his action, given the other’s action.

For Imamura, going N weakly dominates going S.

Neither player has a dominant strategy.
Players’ choices.

Neither player has a dominant strategy:

➢ Kenney would choose
   — North if he thought Imamura would choose North, but
   — South if he thought Imamura would choose South.
   — So Kenney’s best response is a function of what Imamura does.

➢ Imamura would choose
   — North if he thought Kenney would choose South, but
   — either if he thought Kenney would choose North.
   — For Imamura, North is weakly dominant.

And Kenney knows it and chooses North too.
Equilibrium.

The strategy combination (North, North) is an **iterated dominant strategy equilibrium**. (It was the outcome in 1943.)

(North, North) is a (Nash) equilibrium, because:

➢ Kenney has no incentive to alter his action from North to South so long as Imamura chooses North, and

➢ Imamura gains nothing by changing his action from North to South so long as Kenney chooses North.

➢ And neither player has a (strictly) dominant strategy.

➢ South is a (weakly) dominated strategy for Imamura.
A market analogue?

Two companies, K and I, trying to maximise their shares of a market of constant size by choosing between two product designs N and S.

K has a marketing advantage, and would like to compete head-to-head with I, while I would rather carve out its own niche instead of head-to-head competition.
1.6.2 The Battle of the Sexes

(A coordination game: video VHS v. Sony’s Betamax; now the competing standards for digital audio disks: SACD (Sony & Philips) v. DVD-A (Toshiba, Matsushita, Pioneer etc.))

The Players & Actions:

➢ a man (Hal) who wants to go to the Theatre and
➢ a woman (Shirl) who wants to go to a Concert.

While selfish, they are deeply in love, and would, if necessary, sacrifice their preferences to be with each other.

The payoff matrix (measuring the scale of happiness) is below.

What are all equilibria?
(Which pairs of actions are mutually best response?)
### The Battle of the Sexes

**TABLE 6.** The payoff matrix (Hal, Shirl)

A non-cooperative, positive-sum game, with two Nash equilibria.

<table>
<thead>
<tr>
<th></th>
<th>Theatre</th>
<th>Concert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theatre</td>
<td>2, 1</td>
<td>-1, -1</td>
</tr>
<tr>
<td>Hal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concert</td>
<td>-1, -1</td>
<td>1, 2</td>
</tr>
</tbody>
</table>
The Battle of the Sexes

There is no iterated dominant strategy equilibrium.
There are two Nash equilibria:
➢ (Theatre, Theatre): given that Hal chooses Theatre, so does Shirl.
➢ (Concert, Concert), by the same reasoning.

How do the players know which to choose?

(A coordination game.)
Players’ choices.

If they do not talk beforehand, Hal might go to the Concert and Shirl to the Theatre, each mistaken about the other’s beliefs.

Focal points?

Repetition?

Each of the Nash equilibria is collectively rational (efficient): no other strategy combination increases the payoff of one player without reducing that of the other.

There is a **first-mover advantage** in this sequential-move game.
Market analogue?

➢ An industry-wide standard when two dominant firms have different preferences but both want a common standard.

➢ The choice of language used in a contract when two firms want to formalise a sales agreement but prefer different terms.

➢ Bought a DVD player recently?
  DVD, CDV, MP3, CD, DVD-V, DVD+, etc.
  Emerging standards mean choice and decisions for early adopters.
1.6.3 The Ultimatum Game

➢ Your daughter, Maggie, asks for your sage advice.
➢ She has agreed to participate in a lab experiment.
➢ The experiment is two-player bargaining, with Maggie as Player 1.
➢ She is to be given $10, and will be asked to divide it between herself and Player 2, whose identity is unknown to her.
➢ Maggie must make Player 2 an offer,
➢ Then Player 2 can either:
   — accept the offer, in which case he will receive whatever Maggie offered him, or
   — he can reject, in which case neither player receives anything.
➢ How much should Maggie offer?
Maggie’s choices.

➢ Distinguish:
   ① the rationalist’s answer from
   ② the likely agreement in practice from
   ③ the just agreement.

The rationalist:

➢ Player 1 should offer Player 2 5¢ (the smallest coin).
➢ Player 2 will accept, since 5¢ is better than nothing.
➢ But offering only 5¢ seems risky, since, if Player 2 is insulted, it would cost him only 5¢ to reject it.
➢ Maybe Maggie should offer more. But how much more?

In-class exercise.
1.6.4 The Inheritance Game

The players:

➢ Elizabeth, an aged mother, wishes to give an heirloom to one of
➢ her several daughters.

The game:

➢ E. wants to benefit the daughter who values it most.
➢ But the daughters may be dishonest: each has an incentive to exaggerate
  its worth to her.
A second-price auction.

➢ so E. devises the following scheme:
   — asks the daughters to tell her confidentially (i.e. a sealed bid) their values, and
   — promises to give it to the one who reports the highest value
   — the highest bidder gets the heirloom, but only pays the second-highest reported valuation.

Will Elizabeth’s scheme (a Vickrey\textsuperscript{1} auction, or second-price auction) make honesty the best policy?

Yes.

\textsuperscript{1} The late Bill Vickrey shared the Nobel prize in economics in 1996.
Why? Thinking through the options.

Consider your reasoning as one of the daughters:

➢ three options: truthfulness, exaggeration, or understatement.
➢ The amount you pay is independent of what you say it’s worth,
➢ so the only effect of your report is to determine whether or not you win the heirloom, and hence what you must pay.
➢ Exaggeration: the possibility that you make the highest report when you would not otherwise have, had you been honest.

i.e., that the second-highest report, the one you now exceed, is higher than your true valuation.

But → that what you must pay (the second-highest report) is more than what you think the heirloom is worth.

Exaggeration not in your interest.

➢ Understating changes the outcome only when you would have won with an honest report;

but now you report a value lower than that of one of your sisters, so you do not win the heirloom.

Not in your interest either.
It works.

So the mother’s scheme works, and the truth is obtained—but at a price, to Elizabeth, the Mum.

E. receives a payment less than the successful daughter’s valuation, so this daughter earns a profit:

= her valuation – the 2nd-highest valuation.

= the premium the mother forgoes to induce honesty
Market Analogue?

Think: how can the neighbours who propose building a park overcome each household’s temptation to free-ride on the others’ efforts by claiming not to care about the park, when contributions should reflect the household’s valuation of the park?

How can the users of a satellite be induced to reveal their profits so that the operating cost of the satellite can be divided according to the profit each user earns?