These pages are at

http://www.agsm.edu.au/~bobm/teaching/SGTM/dlect16.pdf

9. Tender, Auctions, and Bidding In Competition

(See McMillan Ch.11)

9.1 Introduction

A friend of yours is the Chair of the Acne Oil Company. He occasionally calls with a problem and asks your advice. This time the problem is about bidding in an auction. It seems that another oil company has gone into bankruptcy and is forced to sell off some of the land it has acquired for future oil exploration. There is one plot in which Acne is interested. Until recently, Acne expected that only three firms would bid for the plot, and Acne intended to bid $10 million. Now they have learned that seven more firms would be bidding, bringing the total to ten.

The question is: should Acne raise or lower its bid?

What advice would you give? Raise? Hold? Lower?
(Write down your answer.)

More than two parties.

So far only two players (Burt & Sally). But negotiations often include three or more participants.

One of the main sources of bargaining power is the ability *to exploit competition*.

- ➢ How to take advantage of bidding competition among your potential trading partners.
- ➢ How to compete in a bidding competition.
- ➢ How can conspiracies of bidders seek to suppress competition among themselves.

How competition helps.

- ➢ Competition helps sellers to price items when buyers' willingness to pay is unknown (and perhaps even their identities too).
- ➢ Being faced with competition on the other side of the market is a source of bargaining power.
- ➢ Competition can be used to generate incentives for productive effort Tournaments: high rewards for pop stars, sports champions, CEOs.

  Can design new competitive mechanisms: e.g. electronic markets, when other markets work poorly, because of:

  - ✷ idiosyncratic and differentiated goods and services
  - ✷ multiple goods and synergies
  - ✷ ill-behaved buyers' preferences
  - ✷ need to match buyers and sellers.

Designer Markets

New markets designed for:

- selling spectrum licences
- designing railway timetables
- trading electricity and gas
- selling gambling licences
- devising long-term contracts for the supply of industrial chemicals.

Further new markets:

- B2B, ie, firms buying imputs from other firms

  the precuring firm could use a simultaneous auction mechanism to allow each seller to bid by component, and so reveal its economies of scope by the bundle of components for which it bid.

- sale of a multidivision firm

  simultaneous auction allows division-by-division bidding, with synergies or separate spin-offs.

Competition v. bargaining.

Competition is a good substitute for bargaining skill.

The price from competitive bidding on average > negotiated price.

Why?

A good bargainer is like an artificial competitor: his/her main power (the threat of witholding) in negotiation is similar to another bidder.

But a real bidder is more effective:

- $N+1$ bidders better than N bidders + minimum price
- with competition, the seller needs no info about bidders' valuations
- with competition, the seller needs no selling strategy (such as Take-it-or-leave-it), just to sell to the high bidder
- competition economises on knowledge, on competition, on commitment abilities

Auctions.

Auctions achieve three things:

- Determine the buyer
  (if efficient, the highest valuer)
- Determine the price, and
  (bounded above by the winner's valuation)
- Quickly sell the item

Different kinds of auctions:

- ➢ English ascending bid, open
  — houses, livestock, *objets d'art*, furniture
- ➢ Dutch descending bid, open (or "mine")
  — fish, flowers, perishables
- ➢ sealed-bid, closed
  — tendering, procuring
- ➢ second-price closed (Vickrey)
- ➢ spectrum auctions
  — electromagnetic frequency bands
- ➢ the new Anglo-Dutch auction
  — (See below)

9.2 Understanding Bidding Competition

e.g. Sally, the seller, has a unique, indivisible item to sell, to one of several potential buyers.

- ➢ Sally sets the rules that establish who gets it and for how much.
- ➢ Essence of bidding: the bidders value the item for sale differently, but no-one knows exactly how highly anyone else values it.
- ➢ If you, as one of the bidders, knew exactly how your rivals valued it, then your decision would be easy;

  if Sally knew which bidder valued the item most highly and for how much, she could bargain directly with that bidder.

Two sources of uncertainty.

Two sources of uncertainty about bidders' valuations:

1. **private-value case,** inherent differences among bidders, such as people bidding for an item (a bottle of 1892 Para port for drinking) for their own use, with no thought of reselling;
2. **common-value case,** when the item has a single, true value: winning would turn out to be equally rewarding for all, although just how rewarding is uncertain to any of the bidders at the time of bidding.

   Bidding for oil rights: forecast quantity of oil, quality of oil, price at the time of extraction and sale.

   Speculators for the '92 Para port will want to estimate its resale price when they're deciding how high to bid.

   In these cases, the bidders are trying to guess the same number — the true value of winning — with different pieces of incomplete information.

Different bidding behaviour.

Bidding behaviour will depend on the mix of sources of uncertainty:

- with private value, each bidder knows what the item is worth to him or her, but doesn't know its worth to others;
- with common value, each bidder guesses the true value, in ignorance of the others' guesses.

  With hindsight, all would agree on the value.

Corporate takeovers.

Corporate takeovers and the two sources of uncertainty.

Two kinds of takeovers:

1. the target of a **disciplinary** takeover: not realising its profit-making potential because of inefficient management; the raider believes that firings and new hirings and/or by altering the managers' incentives will improve the firm's profits and share price.

   Common value, with incomplete information.

2. in a **synergistic** takeover, the raiding firm sees specific gains from merging with the target firm: marketing, R&D, monopoly position, tax advantages. *Private value.*

   The most obvious is when a neighbour is bidding for a block of land: it may be more valuable for her than for an outsider. Is it in the neighbour's interest to conceal her interest in the property? Why?

Deciding what to bid.

Deciding a bid: decision making under uncertainty. Burt unsure of the value, unsure of others' valuations, so unsure of how high to bid to win.

Best way to bid?

Of interest too to Sally: in designing her selling strategy, must put herself in the bidders' shoes: look forward and reason back.

1. Sally might inform each of their rivals' bids, and allow revised bids. *An open-outcry, English (or ascending-bid) auction.* (A second-price auction.)
2. Sally might keep bids confidential. *A sealed-bid auction or tender.* (A first-price auction.)
3. *Or an open outcry Dutch (or descending-bid) auction.* (A first-price auction.)

- Exercise: Consider something you've sold or wanted to sell recently. Write down how you might have sold it differently.

9.3 Open English Auctions — (Second-price)

9.3.1 Private-Values

e.g. Sally is offering an undeveloped piece of land in an open, English auction. Bidders know their own valuations, but differ because of different planned uses of the land; have an idea of the ranges of values: a *private-values* case.

Best strategy: remain in the bidding until the high bid rises to your valuation, and drop out at higher bids, lest you pay more than the land is worth to you. A simple dominant strategy, which disappears with a sealed-bid.

In general Burt the winner makes a windfall, because pays less than the item is worth to him.

Because of the private valuations, Sally can't extract all of the gains from trade by offering it to the highest valuer with a take-it-or-leave-it.

The second-highest bid?

Since the high bid is marginally above the second-highest bid, *what determines the second-highest bid?*

- The greater the number of bidders, the smaller the difference between the highest and the second-highest, on average. So the more, the higher.
- The greater the spread of bidders' (private) valuations, the greater the difference between the highest and second-highest, on average. If there is wide disagreement about the item's worth, the winner may get it cheaply.

9.3.2 Common-Values

What if the bidders are speculators for resale later? All bidders are trying to guess the same number: the future market value. The *common-value* case.

Different information → different values. Factors as above, but more complicated.

e.g. A common-value, English auction.

Burt's rule: stay in the bidding until the high bid reaches your valuation, apparently as in the private-value.

But Burt can learn from others' bids, which provide indirect information of their valuations.

Valuable information.

Any extra information is useful to Burt:

— how aggressively others bid

— how many remain in the bidding

— when others apparently drop out of the bidding

may enable Burt to revise his estimate of the land's worth.

But if Burt wins, then he learns that no-one else thinks the land is worth at least what he is paying.

A reality check: Before he raises his bid, would he still value the item at the bid he's considering even if no-one else thought it was worth that much?

9.4 Sealed-Bid Auctions — (First-price)

Bidding requires a little more thought. Three risks to balance:

- ➢ risk of bidding much higher than the second-highest bid
- ➢ risk of losing a profitable opportunity by bidding below at least one other bidder
- ➢ (in a common-value auction) risk of bidding more than the item turns out to be worth.

Sealed bids.

e.g. Single-round of sealed bidding for exclusive rights to patent a new computer chip, when bidding firms differ in their value-added from the rights.

1. Assume Burt knows his opponents' values.

If his valuation is highest, then his best bid is slightly above the second-highest valuation: Burt guarantees winning with a windfall, at a bid less than his valuation.

The best bid.

2. More realistically, none of the bidders knows his competitors' valuations. What is Burt's lowest successful bid?

Burt begins by assuming his valuation is highest. (If not, then the presumption is costless because losing bidders pay nothing.)

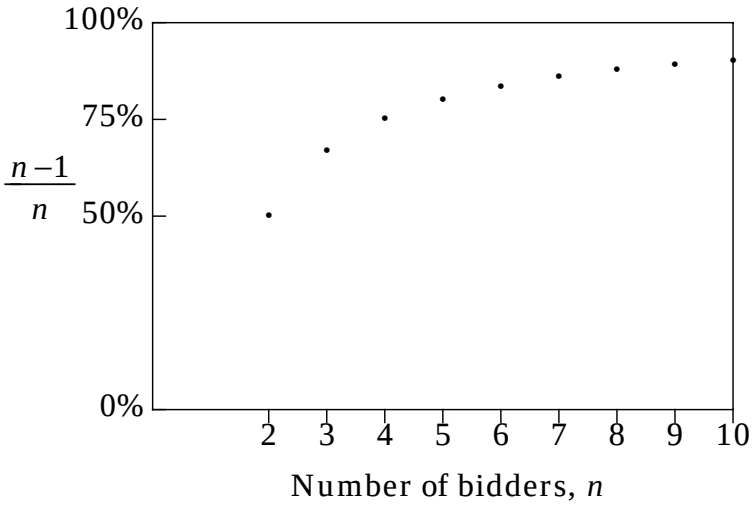
Burt doesn't know just how much lower the second-highest valuation is, but can estimate its most likely value, given the numbers of competitors and their range of valuations. (This is a skill.)

Burt submits a bid equal to the estimated second-highest valuation: bidding higher risks forgoing a windfall, lower risks not winning.

If Burt knows that each of his rivals values the chip rights at between zero and $10 million, with uniform distribution in this range, and Burt's rivals each perceived Burt's valuation lying in this range:

How competition matters.

McMillan shows that Burt should shade his bid, by bidding $\frac{n-1}{n} \times$ (his valuation),
where n is the total number of bidders, including Burt.



As the number of bidders rises, Burt's bid approaches his valuation.

Competition matters.

A small number of bidders will result, on average, in the winning bidder receiving a large windfall.

An extra bidder has a greater effect when there are few bidders.

e.g. US S&L auctions: mostly four or fewer bidders, and average windfall of $4 million.

Note:

the Vickrey, second-price auction

→ truth-telling

∴ the seller makes more revenue than when the bidders understate their values.

∴ What is your answer to your friend, Acne's chair?

9.5 Takeovers

(See Marks in Package)

Australian Corporations Law:
The market for shares must be:

- "efficient
- competitive, and
- informed."

Sealed auctions v. open English auction

Common value v. private value

(correlated value)

The three criteria.

Informed:

Bidding in open auctions.

Bidding in closed auctions.
— risks of bidding in closed auctions

Competition in auctions
— numbers of prospective bidders

Efficiency in auctions

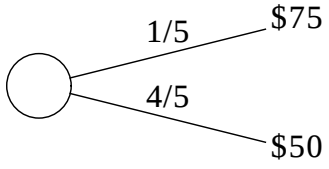
— no opportunity for arbitrage

— the highest valuer is the purchaser

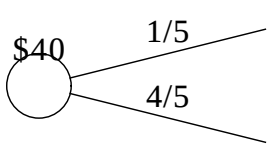
— can be measured by:
net gain to the buyer + seller revenue

9.5.1 Inefficiency

- Alan values item at $101
- Bill values item at (a distribution):



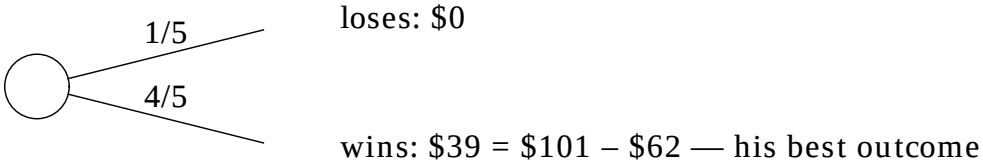
- What if Alan bids $51, so EV(bid = $51) = $40:



loses: $0

wins: $50 = $101 – $51

➢ What if Alan bids \$62, EV(bid = \$62) = \$31.20, and maximum gain = \$39:



no: Alan prefers \$40, and never bids above \$61

➢ So sometimes Bill wins with a bid between \$51 and \$75.

Inefficient. Plus: cost of bid preparation.

9.6 The Winner's Curse

(See Landsburg in the Package.)

A possibility in sealed-bid, common-value auctions.

e.g. Rights to drill in offshore oil leases: the winning bids can be huge, and much higher than the losing bids:

In March 1990, US\$590 million was bid in Gulf of Mexico. One single lease attracted a winning bid of US\$11.1 million; two losing bids over US\$8 million, and a third bid of US\$6 million. Much uncertainty: firms must consider: geological surveys, oil price forecasts, other tracts for bidding.

A class exercise.

Five people are invited to bid for a suitcase of money. Not permitted to look inside the suitcase, but each given a private estimate of $\$X$, the actual value of the amount, in thousands.

Estimates are $\$X-2$, $\$X-1$, $\$X$, $\$X+1$, $\$X+2$.

What if Burt is given an estimate of $10,000?

Thinking through the exercise.

If Burt knew all five estimates, then he could infer the value.

But he only knows that X could be between $8,000 and $12,000.

Burt knows that $10,000 is on average correct — an equal chance of being too high or too low — so he might choose to bid $10,000 less $1,000, to reap a $1,000 windfall if he wins.

But if all five bid their estimates less $1,000, then the winner is the person with the highest estimate, $\$X+2$, who will bid $\$X+1$, to make a loss of $1,000: the **winner's curse**.

Although, on average, the estimates are correct, the winner is not selected at random.

Winning conveys the bad news that the winner's estimate is the highest, and so too high.

Anticipate the Winner's Curse.

Burt could anticipate the winner's curse's effects beforehand, by presuming his is the highest estimate and so will win.

When incorrect, this presumption costs nothing since another bidder wins; when correct, the winner's curse is avoided.

If $10,000 is the highest estimate, then $$X$ is $8,000 and Burt should bid $7,000, for a windfall of $1,000.

If all others reason likewise and subtract $3,000 from their estimates, then Burt will make $1,000 when his is the highest estimate, and nothing at other times.

What to do.

In the face of the winner's curse, rational bidding requires discounting one's own estimate.

Holds too for less artificial auctions. Any actual common-value auction is more complicated.

Offshore oil rights: numbers of bidders? who? what geological information? consortia? A firm in action: exactly what potential for short-run/long-run profit improvement?

But to avoid the winner's curse, anticipate it.

So: presume your estimate is the highest, estimate what the second-highest must be, bid this amount, after correcting downwards for the possibility of the winner's curse (how many competitors expected? amount of uncertainty over item's true value?).

9.6.1 Winner's curse as explanation of 1980s' takeovers?

The share market as one "bidder", setting a going price; the takeover raider as the second bidder. Inexperienced raiders may have put too much weight on their own valuations and not enough on the market's.

Winner's curse when no competition:

the Alaskan oil pipeline, estimated at US$900 million in 1970, had cost US$7.7 billion in 1977; nuclear power stations; other large projects? (Olympics?)

Routine construction: cost estimates uncertain, especially with new technologies.

A bias towards the Winner's Curse?

Even if estimates are on average correct (as likely to be low as high), tendency for cost overruns if the decision-maker doesn't understand the winner's curse: a project will be accepted if PV of (B–C) is positive, and rejected otherwise, so a project with underestimated costs is more likely to go ahead, and cost overruns are likely.

Is the winner's curse real? Do people sometimes lose by overestimating values?

Perhaps, for unique one-offs.

Repeated auctions will allow bidders to learn from experience, as student bidding experiments reveal.

Oil companies have a powerful incentive not to make systematic errors in bidding, and statistical evidence suggests a normal rate of return from offshore oil tracts.

9.7 The Spectrum Auctions

After several false starts (see the 1993 simultaneous, single-round, sealed-bid auction for satellite-television licences in Australia), the FCC chose

a simultaneous ascending auction.

Proposed by game theorists.

- ➢ Multiple licences are open for bidding at the same time, and remain open so long as there is some bidding on any of the licences.
- ➢ Bidding occurs over rounds, with the results of each round announced to the bidders before the start of the next round.
- ➢ By computer, on-line.
- ➢ Many detailed rules (130 pages); most importantly, the activity rule.

Why simultaneous ascending auction?

The licences are interdependent: substitutes or complements.

Efficiency (assigning the licences to the firms most willing to use them) requires buying of multiple licences — the aggregation is determined by the competition.

Ascending bids allow bidders to see how highly their rivals value each licence and which aggregations they seek. Diminishes the winner's curse, leading to high bids.

Simultaneous bidding allows bidders to switch to back-up aggregation in the light of others' higher valuation.

Australian spectrum auctions, see
http://auction.aca.gov.au
NZ
http://auction.med.govt.nz/

9.8 The Seller's Strategies

Sally the seller must use the game-theoretical trick of putting herself in the bidders' shoes and understand how they would respond to alternative selling schemes.

Sally must make decisions without full knowledge too: she doesn't know exactly what the item is worth to the bidders, or who values the item most highly.

How can Sally make the bidding as competitive as possible? (For her, the more competition the better.)

More competitive bidding.

1. Encourage *extra bidders* to enter.
2. What about a minimum (reserve) price?
3. Open or sealed-bid auction?
4. Should Sally release any information she has relevant to valuing the item?

➢ The risk of a *minimum (reserve) price* is that all bids will fall short and the item will not sell,

➢ But a reserve price may force a bidder, Burt, to bid above what otherwise would have been necessary from the competition.

The expected gain from a higher bid can offset the risk of no sale.

Open auctions are informative.

From the winner's curse discussion, provided there is a common element to bidders' valuations:

> on average the winning bid in *an open auction* will be higher than in a sealed-bid, because of learning and revision of valuations.

In a pure private-value, should make no difference since bidders' valuations will not be revised given knowledge of others', irrelevant, valuations.

The more information Burt has, the less he rationally distrusts his own information, and so the less the winner's-curse correction he should apply in shading his bid below his valuation.

Open auctions are the most common.

Open auctions are the most common: up to 75% of the auctions in the world.

Is the US government using the wrong method for auctions offshore oil rights, if its aim is to maximise its return from the sales?

Open auction, or several rounds of a sealed-bid auction, with release of all bids each round?

Hence the Spectrum Auction
— with full information.

The seller's information.

In a common-value auction, the better the bidders' information, the more aggressive their bidding, and the less they fear the winner's curse.

∴ Sally should *reveal her information* about the true value of the item, to get higher bids on average.

Sometimes, Burt's valuation will fall with Sally's information, but on average should rise since he is more confident in his valuation and so less concerned about the risk of a winner's curse.

Sally must release all information, not just value-enhancing information. Establish her credibility.

e.g. Christie's and Sotheby's estimate in advance the price which artworks and antiques will fetch, as do other auction houses. This is an expensive process: high-priced expertise. Between 1980 and 1982, the average difference between the predicted and the actual sale price was less than 2.4%.

9.9 Does Price Measure Value?

"A cynic knows the price of everything and the value of nothing," (Oscar Wilde, *Lady Windemere's Fan*).

For auction markets, as we have seen, bidders understate their valuations, so auction prices understate value.

The greater the number of bidders, the closer the bids to valuations, so with sufficient bidding competition, the winning bid is close to the highest valuation.

So auction prices are very close to value.

But auctions: price → value.
With smooth competition, price is value.

Auctions and value.

Remember: Auctions are a way of doing two things:

- establishing the values of unique objects
- determining the new owners (the highest valuers, if efficient)

The 1892 Para port's value? Subjective opinions of self-acknowledged œnological experts? Or auction prices recently? Measuring the quality of a wine by what people are willing to pay for it produces different rankings from those announced by the wine columnists.

9.9.1 Airport Slots

Airport "slots" are necessary for planes to pick up and discharge passengers.

A shortage at busy airports, so slots are valuable, but how valuable?

With no market for slots when airport authorities used to bestow slots on persuasive airlines, no market measure of value.

How to value bankrupt Eastern's slots?

Valuing the slots.

Bankruptcy judge held an auction, cancelling previous agreements.

Uncontested negotiations had yielded total offers of US$155 million.

But auction prices totalled nearly US$260 million.

Three gates at LAX went for US$21.7 million (to Delta) after an initial offer of $6 million (from United).

The auction prices were higher because:

- ➢ the auction ensured that the high bidder was the airline that most highly valued the slot (efficient), and
- ➢ the presence of competing bidders meant that the winning bidder could not bid much less than the valuation of the highest bidder (see graph above).

9.10 Bidding Conspiracies

Competition is a source of bargaining power for Sally the seller. But Burt and the other buyers might *conspire to refrain from bidding, to reduce the price.*

In attempting to form and maintain a coalition, the bidders face a PD:

— if the collusion is effective, the high bid will be lower than with full competitive, but

— if any bidder values the object at more than its selling price, then he would be better off paying more and winning than not getting it at all. This incentive must be countered.

Repeated interaction provide an incentive for maintaining collusion/cooperation.

McMillan reports that British antique dealers threaten retaliation to prevent genuine competition: “When one of the members of a ring goes against his partners, or the ring falls out for one reason or another, then it works very much to the seller’s advantage, as vindictive competition leads to crazy prices.”

Minimising collusion

Open bidding is more susceptible to collusion than is once-off sealed bidding: bids are immediately known in open bidding, whereas sealed bidding means that deviation may be concealed until bidding is over, or for ever.

Retaliation must happen later, if at all.

If there is fear of collusion, then sealed bids are preferable to open bidding.

e.g., How will open bidding for Rugby League players result in players' getting more of the gains from trade? Will clubs collude to reduce players' gains?

See US baseball.

Competition among colluders

Conspirators seek an efficient solution: item awarded to the highest valuer, so that the pie to be divided is largest.

But what are others' valuations?

Asking is not enough: Burt has an incentive to overstate his valuation to his co-conspirators.

Well, conspirators in such diverse goods as fish, antiques, rare books, timber, and industrial machinery divide their ill-gotten gains in an especially apt way: they *hold an auction among themselves*, reports McMillan.

This succeeds since genuine, competitive bidding reveals information about bidders' valuations in auctions.

The conspirators share among themselves the difference between the price reached in the illicit auction and the price reached in the legitimate, uncompetitive auction. This difference represents the total profit from the conspirators' collusion: it would have gone to the seller if the bidding in the legitimate auction had been genuinely competitive.

McMillan details a collusive technique used in art and antique auctions.

9.11 Procurement Competition

With simple changes (substituting selling for buying, production cost for valuation, lowering price for raising it, and so on) the previous analysis becomes a model of a procurement competition, with a single buyer Burt and competing potential sellers.

The government wants to buy a new computer system from one of several qualified contractors; or a car manufacturer wants to source some of its components to another firm.

Procurement:

- Similar to the private-value case with bidding firms' production costs differ because of differences in wage rates, capital stocks, managerial expertise, etc.
- Similar to the common-value case since the firms are guessing about, say, a new technology that the winner will have to implement.

Conclusions

Replace Sally by Burt and the previous conclusions follow: Burt the buyer can stimulate competition by making it easy for new bidding firms to compete in selling.

Burt the buyer can also promote competition by narrowing any inherent differences in production costs by, for example, helping the selling bidders to adopt best-practice technology.

Burt the buyer can mitigate the sellers' risk of winner's curse — if there are common-value aspects — by accepting open bids and by releasing any information he has that would help predict production costs.

Further evidence:
one US study comparing production contracts for various items of military hardware that had first been awarded on a sole-source basis and were later opened up to competitive bidding found that the prices fell on average by an average of $1/8$.

Quality of performance may matter

The procurement game is more complicated than the selling games above: an antique is the same no matter who wins the bidding, but a computer from IBM is not the same as one from Fujitsu/Facom.

In procurement, competition is often over design as well as price: the *identity* of the winning bid matters, which means it is no longer a simple matter to compare bids, since the buyer must consider several *attributes*, not just price.

McMillan, Chapter 13, considers the Japanese case of a network of subcontractors. (See Lecture 20.)

9.12 Anglo-Dutch Auctions

Devised by Paul Klemperer at Oxford, see www.nuff.ox.ac.uk/economics/people/klemperer.htm

An ascending English (open) auction to determine the highest and second highest bidder; followed by one round of a sealed bid auction.

Why? To prevent

- collusive behaviour,
- predatory behaviour, and
- entry-deterring behaviour.

Ascending (open) and uniform-price auctions are particularly vulnerable to those problems.

The Ango-Dutch auction (a hybrid of the sealed-bid and ascending auctions) may often (but not always) perform better.

9.13 Fair Auctions?

"The essence of the auction problem is the unobservability of bidders' valuations." — McAfee & McMillan (1987)

Brams and Taylor (*Fair Division*, C.U.P., 1996) have proposed the following *two-stage auction:*

Stage 1: The players submit sealed bids, all of which are then opened and made public. No prior information about others' bids or valuations.

Stage 2: Each player chooses exactly one of any of the Stage 1 bids, his or her own or anybody else's bid.

Payoffs: If only one player makes the highest Stage 2 bid, that player wins. If a tie, the player with the highest Stage 1 bid wins. Pays the Stage 2 price.

Characteristics of the Fair Auction.

Like an English auction, bids can be revised; unlike an English auction, all bids revealed at once.

Like a sealed-bid auction, bids made simultaneously; unlike a sealed-bid auction, initial bids not (usually) decisive.

Rational to bid sincerely in Stage 1, as in a Vickrey auction, but bidders may bail out in Stage 2, and also can identify shills or confederates.

Minimises the risk of the Winner's Curse in common-value auctions.

9.14 Summary of Bidding

Can extend the recommendations beyond the case of formal auctions: since most business negotiations include competition, either explicitly or implicitly, and there is usually some alternative trading partner for one to turn to.

Extend to informal negotiations: open v. sealed-bid auctions becomes whether to inform the parties competing for your business of each other's best offer.

Stimulate competition.

Competition among your potential trading partners is a potent source of bargaining power: *stimulate competition*:

- by increasing the number of bidders, or
- by reducing the inherent differences among them (informing)
- informing bidders of their rivals' bids and releasing any information the seller has of the true value of the items

From the bidders' perspective, rational bidding involves remaining in the bidding until the price reaches the the bidder's own valuation (open auction), and guessing the valuation of the next-highest bidder and bidding this amount (sealed-bid auction).

The winning bidder earns a windfall from the difference between his or her own valuation and the next-highest valuation.

9.15 Appendix: Bidding in a Sealed-Bid Auction

Several bidders compete in a sealed-bid auction. The bidders know their own valuations, but they don't know each others' valuations: all they know is that the others' valuations lie somewhere between 0 and 1 with all intermediate values equally likely. What is each bidder's best bid?

Consider first the case of two bidders, Cain and Abelle. Abelle knows her own valuation is v_a and Cain knows his own valuation is v_c, but Cain doesn't know v_a and Abelle doesn't know v_c. Suppose Abelle conjectures that Cain follows the decision rule: bid some particular fraction λ of his valuation (λ is, so far, an arbitrary number lying between 0 and 1). Thus Abelle conjectures that Cain will bid an amount λv_c (although Abelle doesn't know what this is). Abelle believes, therefore, that she will win if she bids an amount p_a that is bigger than this: that is, $p_a > \lambda v_c$, or $v_c < p_a/\lambda$.

Given the distribution of possible values for v_c (between 0 and 1), the probability of Abelle's winning is p_a/λ. If Abelle wins, then her profit is the difference between the price and her valuation, $v_a - p_a$; if she loses, then she gets nothing. Thus on average her profit from bidding p_a is $(v_a - p_a)p_a/\lambda$. This expected return equals 0 when $p_a = 0$ and when $p_a = v_a$; since it is a quadratic and symmetrical, its maximum point is halfway between these values, at $p_a = v_a/2$.

This is the bid that Abelle rationally chooses, given her conjecture about Cain's decision rule. Is this a rational conjecture? It is if $\lambda = ½$, since, by the symmetry of the situation, Cain's best response to Abelle's using this strategy is to bid one-half of his own valuation. Hence Abelle rationally bids half her valuation; and Cain behaves similarly.

An increase in the amount of competition would, however, reduce this profit. If there are more than two bidders, say n bidders, then the optimal bidding rule is: bid a fraction $\frac{n-1}{n}$ of valuation. In other words, shade valuation by $1/n$th.

To solve for the case of any number of bidders, consider one bidder's decision. Suppose this bidder, Obi, conjectures that each of his $n-1$ rivals submits a bid that is some particular fraction β of valuation. If Obi bids p_1, then he will win the item if his bid is higher than all of the others', which occurs if $p_1 \geq \beta v_i$, for all $i = 2, \cdots, n$ (provided his conjecture about his rivals' decision rules is correct). That is, Obi wins if $v_i \leq p_1/\beta$, for all $i = 2, \cdots, n$. He therefore wins with probability $(\frac{p_1}{\beta})^{n-1}$.

Obi's net gain if he wins with the bid p_1 is $(v_1 - p_1)$, so his expected net gain is $(v_1 - p_1)(\frac{p_1}{\beta})^{n-1}$, which he maximises by setting his bid at $p_1 = \frac{n-1}{n} v_1$. What has been found is Obi's best response to the decision rule he arbitrarily presumes his rivals to be following. But, by symmetry, it is rational for each of his rivals to be using this decision rule, provided they set $\beta = \frac{n-1}{n}$. Thus the Nash equilibrium has each bidder, given a valuation of v, submitting a bid of $\frac{n-1}{n}$. Thus, as discussed above, the bidders choose their bids by shading their valuations by a fraction $1/n$.

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