1.5.3 Boxed Pigs

Actors: Two pigs are put in a box:

- Big Pig, dominant
- Piglet, subordinate.

Game:

- a lever at one end of the box dispenses food at the other end.
- So the pig that presses the lever must run to the other end to eat;
- but by the time it gets there, the other pig has eaten most, but not all, of the food.
- Big Pig is able to prevent Piglet from getting any of the food when both are at the food.

Assuming the pigs can reason like game theorists, which pig will press the lever?

Payoffs.

Six units of food are delivered:

- If Piglet presses the lever, then BP eats all 6 units; but
- if BP pushes the lever, then Piglet eats 5 of the 6 units before BP brushes him aside.
- If both press together, then Piglet, who runs faster, gets 2 units before BP arrives;
- running costs half a unit.

Decision:

- wait for the food, or
- press the lever & run for the food

The Boxed Pigs

		Piglet	
		Press	Wait
Big Pig	Press	3½, 1½	**½, 5**
	Wait	6, –½	0, 0

TABLE 1. The payoff matrix (Big Pig, Piglet)

A non-cooperative, positive-sum game,
with a Nash equilibrium.

Choices.

What is best for Piglet?

- ➢ He is better off to wait.

What is best for Big Pig?

- ➢ If Piglet presses, then BP gets:
  3½ if she presses or
  6 if she waits.
- ➢ If Piglet waits, then BP gets
  ½ if she presses or
  0 if she waits.
- ➢ So BP's *best response* differs depending on what she conjectures her rival will do.

How to resolve this dilemma?

Equilibrium.

If BP puts herself in the shoes of her rival, then BP realises that Piglet's best action is unambiguous: Wait.

If BP presumes Piglet is rational, then she knows she should use her best response to her rival's waiting: thus she presses.

Rational behaviour, therefore, indicates a surprising conclusion:

Big Pig presses the lever and Piglet gets most of the food.

Weakness, in this case, is strength!

Not a Prisoner's Dilemma.

Unlike the Prisoner's Dilemma, Boxed Pigs generates no conflict between individual rationality and collective rationality.

∴ The Nash Equilibrium is efficient in this game.

The outcome cannot be changed without making one of the players (Piglet) worse off.

The outcome may not be fair—the pig that does all the work gets the smaller share—but there is no alternative that the players unanimously prefer.

Market Analogy

e.g. Consider OPEC as an effective cartel:

Saudi Arabia was the “swing” producer, it would unilaterally act to keep oil prices high by reducing its production when one of the smaller member cheated and increased its production of oil.

Not through altruism, but—as with Big Pig—through the logic of the situation: the smaller producers took advantage of the common knowledge that the cartel would collapse unless the Saudis limited their production.

Saudi captured for itself a sufficiently large share of the benefits of the high prices that it was rationally willing to bear a disproportionate share of the cost of maintaining the cartel.

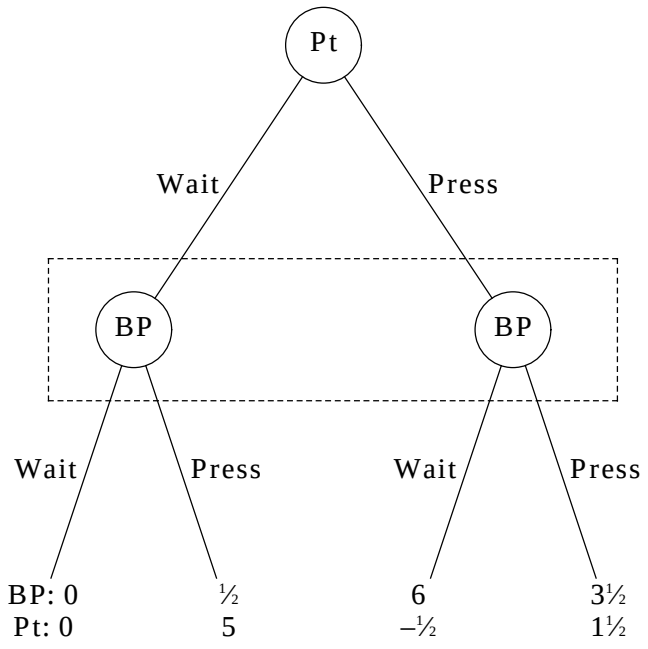

Figure 1. Boxed Pigs, Extensive-Form Game Tree

NB: dashed information set.: BP ignorant of Pt’s move
(or use a dashed line between nodes in the set)

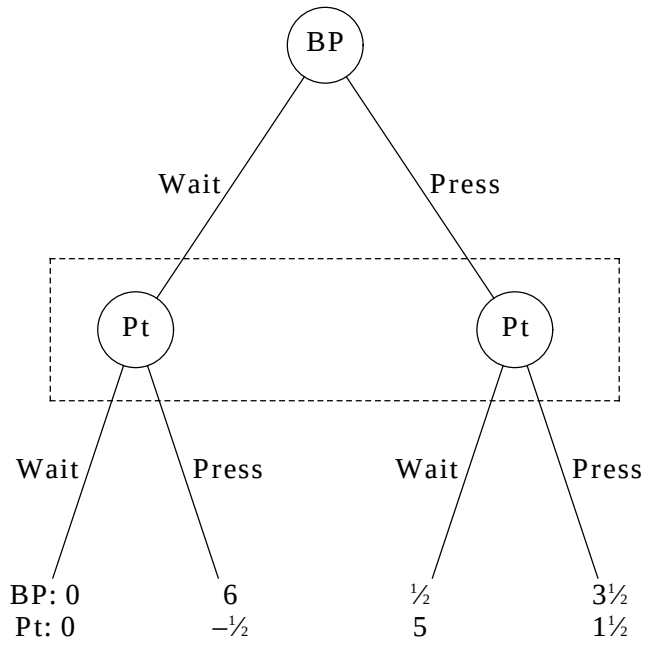

Figure 2. Boxed Pigs, Extensive-Form Game Tree

NB: dashed information set: Pt doesn't know what BP is doing. Whatever, Pt chooses Wait. Why?

1.5.4 A Grade Game

- Choose your grade for this assignment.
- Write your grade & your name on the paper.
- Fold the paper and return it.
- But your grade depends on whom you're matched with:

		The other student	
		HD	PS
You	HD	48%	90%
	PS	48%	63%

TABLE 2. Your payoff matrix

1.5.5 The Gift of the Magi

(from O. Henry's short story)

The players and the game:

Della and Jim were deeply in love and eager to make any sacrifice to get a really worthy Christmas present for the other.

- Della would sell her hair (to wig-makers) to get Jim a chain for his heirloom watch, and
- Jim would sell the watch to buy a comb for Della's beautiful hair.

The Gift of the Magi

		Jim	
		Sell watch	Keep watch
Della	Keep hair	**1, 2**	0, 0
	Sell hair	–2, –2	**2, 1**

TABLE 3. The payoff matrix (Della, Jim)

A non-cooperative, positive-sum game,

with two Nash equilibria.