Utility

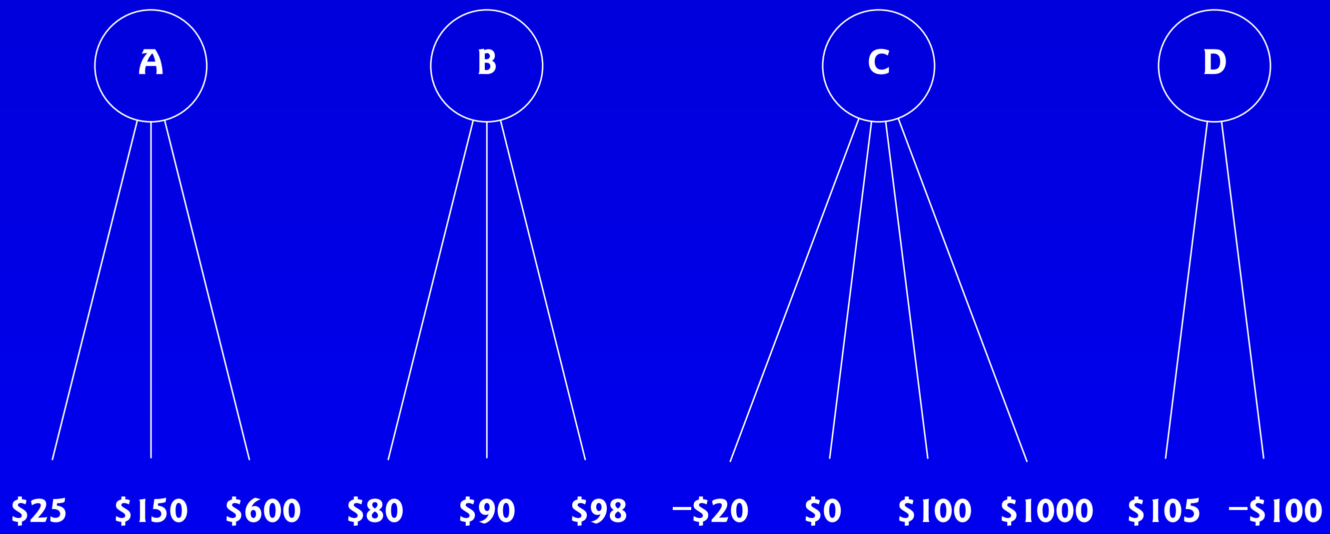
Topics:

1. Decisions Under Uncertainty
   - Certainty Equivalents
2. Expected Utility
3. Constant Absolute Risk Aversion
4. Eliciting Utility Functions
5. Choosing Among Lotteries
6. Appendix: Approximating a Certainty Equivalent
7. Appendix: Finance.

(See Dixit & Skeath: pp. 228–230, 300–303.)

1. Decisions with Uncertainty

Choose among the four lotteries with unknown probabilities on the branches: uncertainty —



(Write down your answer.)

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2a. *Hurwicz:* choose the lottery with the highest value of a weighted average of the minimum and maximum, say $\alpha \times$ lottery X's minimum payoff + $(1 - \alpha) \times$ lottery X's maximum payoff.

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has the advantage that it includes *all* payoffs, not just the extreme ones, but it imputes equal probabilities to each payoff's occurrence. (The *Laplace* criterion.)

Moreover, it also assumes a risk-neutral decision maker.

$\therefore$ would result in choice of lottery C.

4. **For lotteries with more common structure — say, a matrix R_{ij}, where lottery i and state of the world j result in payoff R_{ij} — we can use the *Savage rule of minimum regret* for the wrong decision:**
choose the lottery which minimises the maximum regret, where regret is the difference between the contingent outcome's payoff in the lottery you chose and the highest contingent outcome's payoff.

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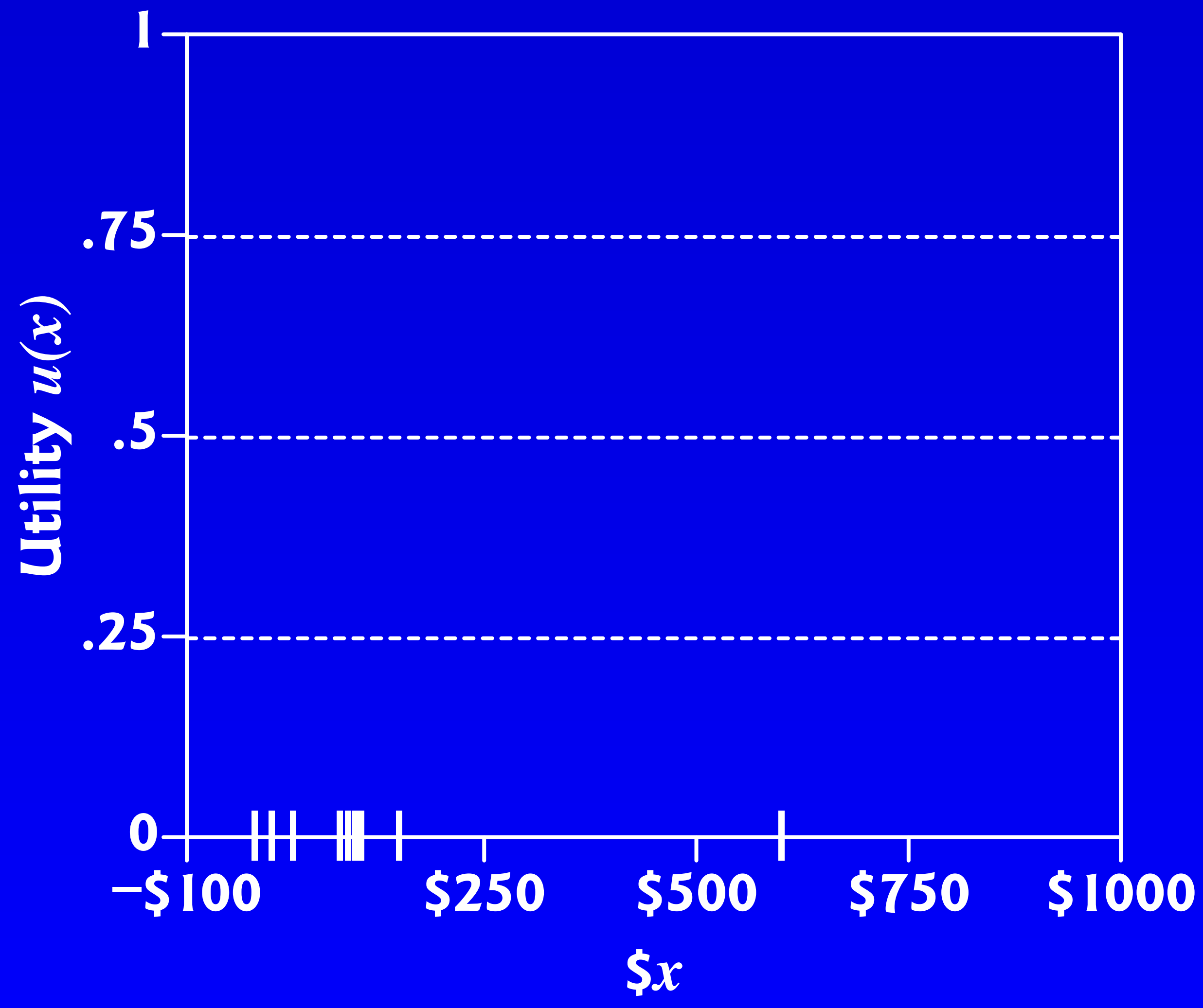
3. **convex (its slope is increasing), then the decision maker is *locally risk preferring.***

See
http://www.gametheory.net/Mike/applets/Risk/risk.html

Risk-averse, risk-neutral, and risk-preferring utility functions.



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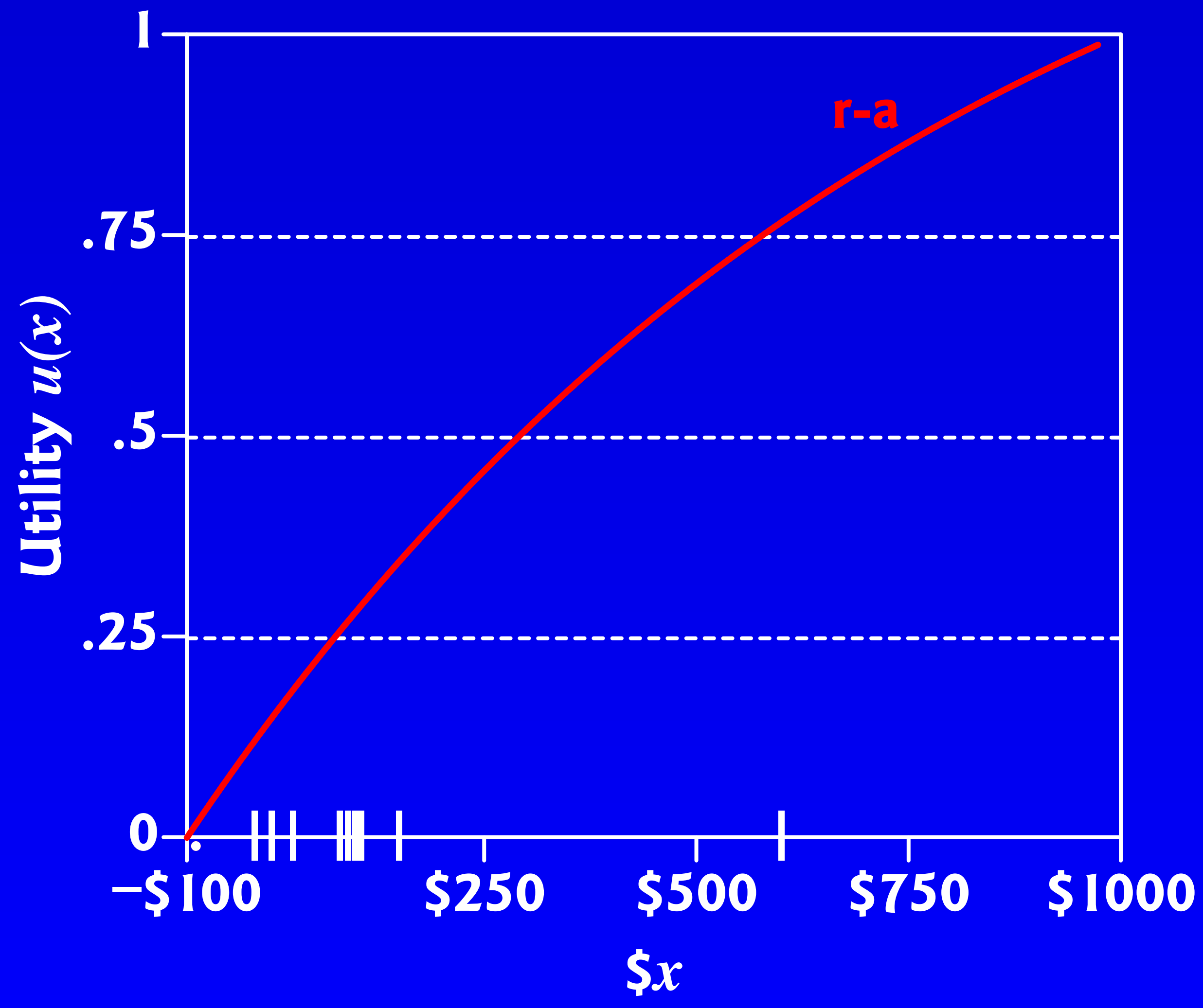

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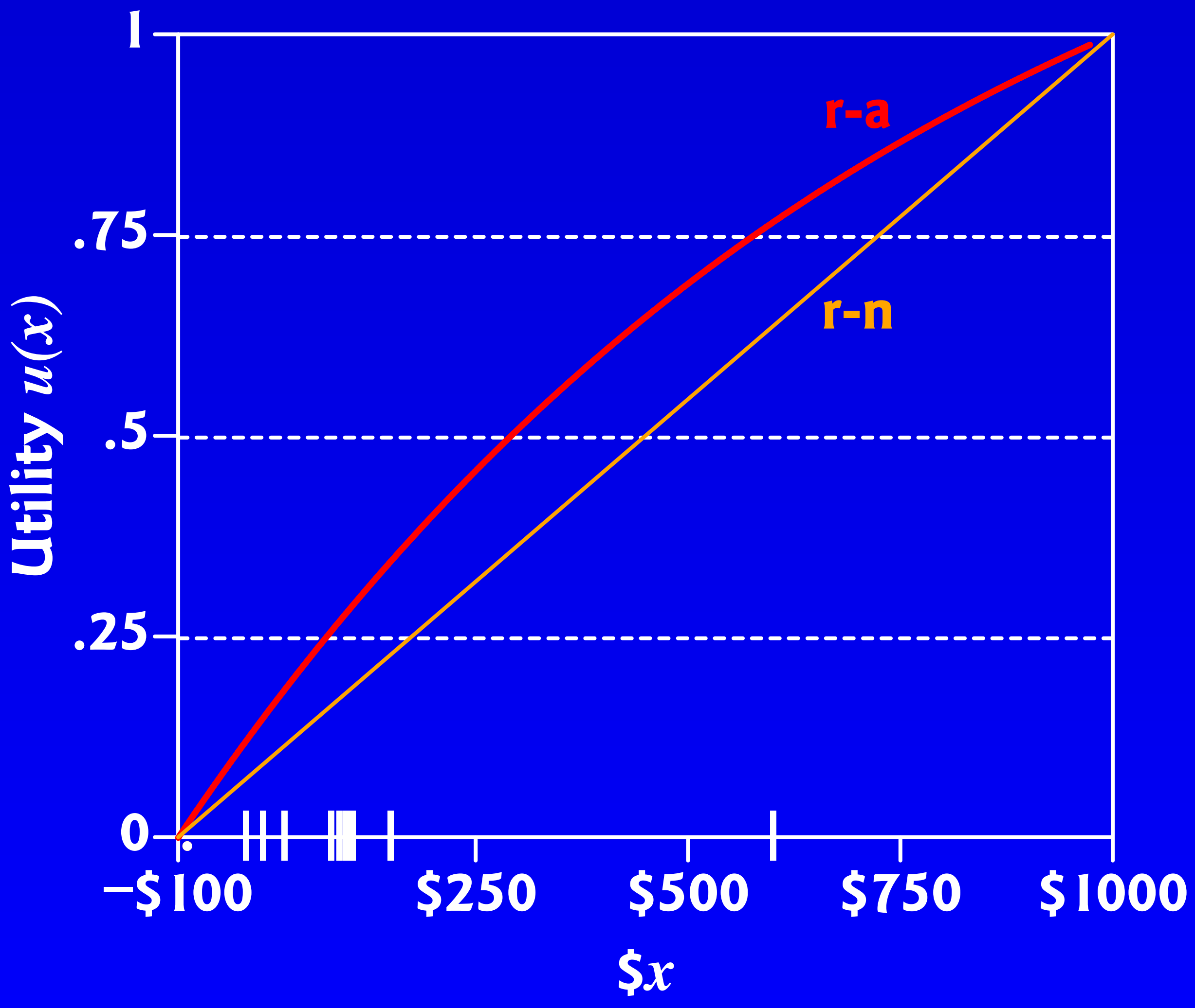

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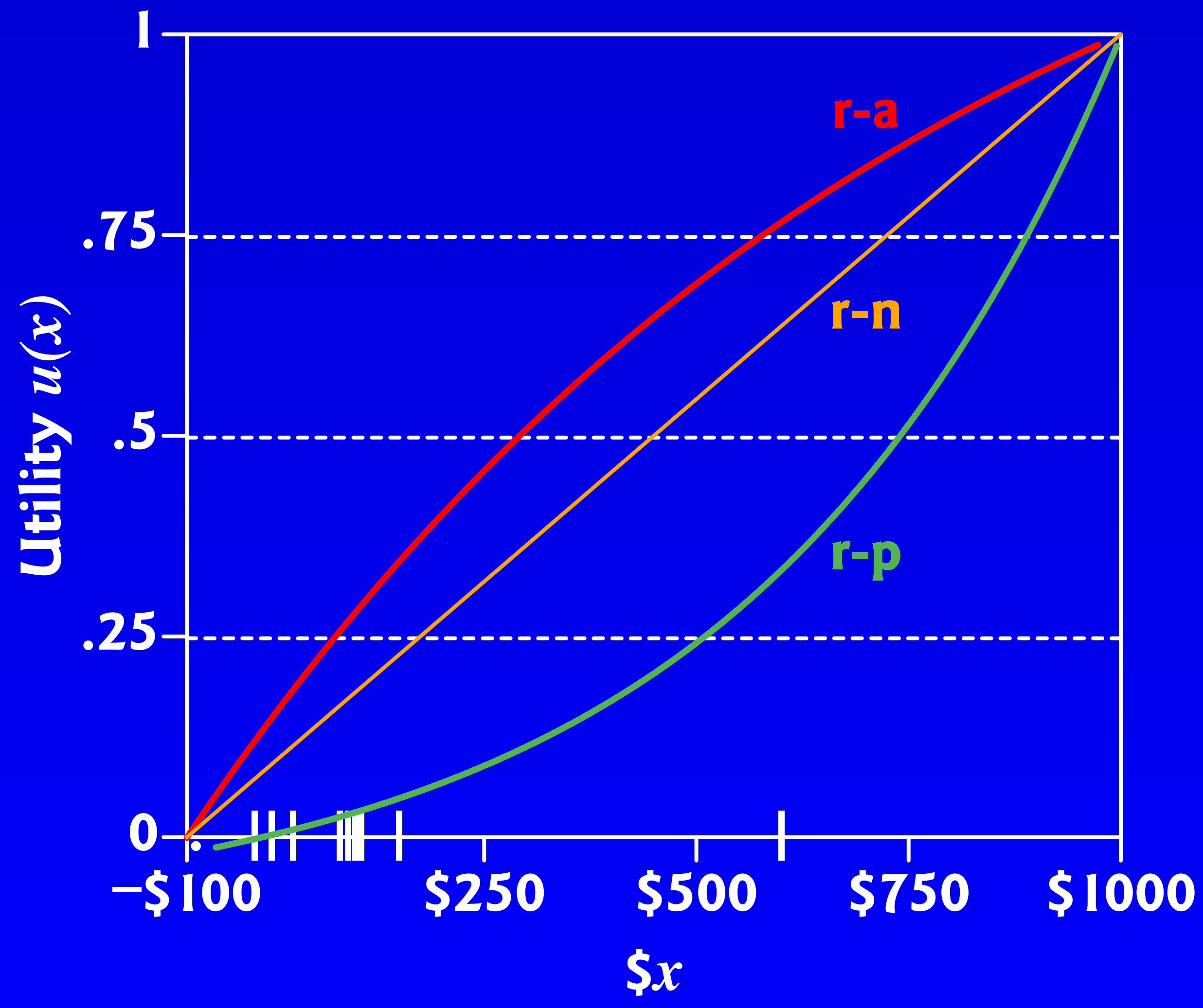
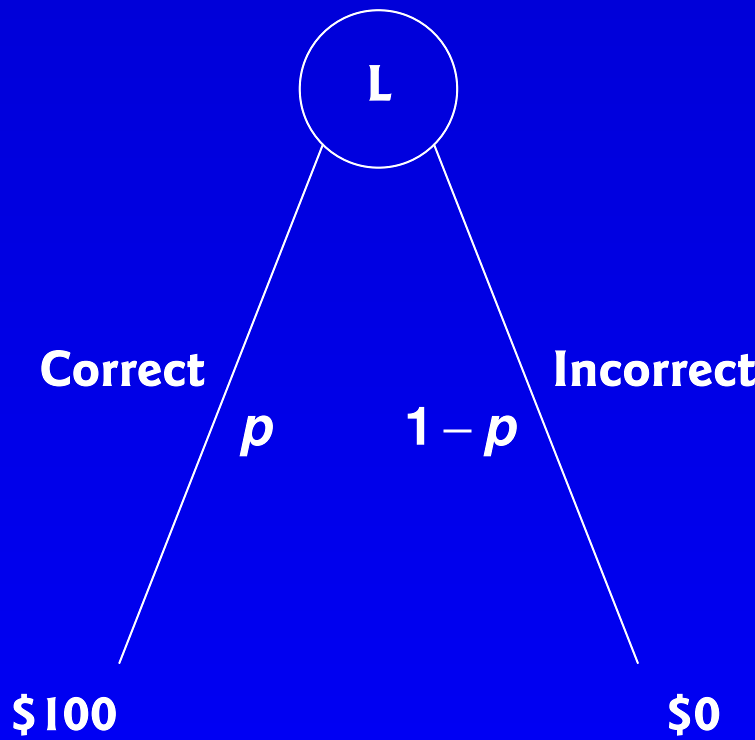


The Certainty Equivalent (C.E.) of a lottery.

The utility of a lottery = the utility of its C.E.



$\sim$ **\$______?**

a deal or opportunity or lottery

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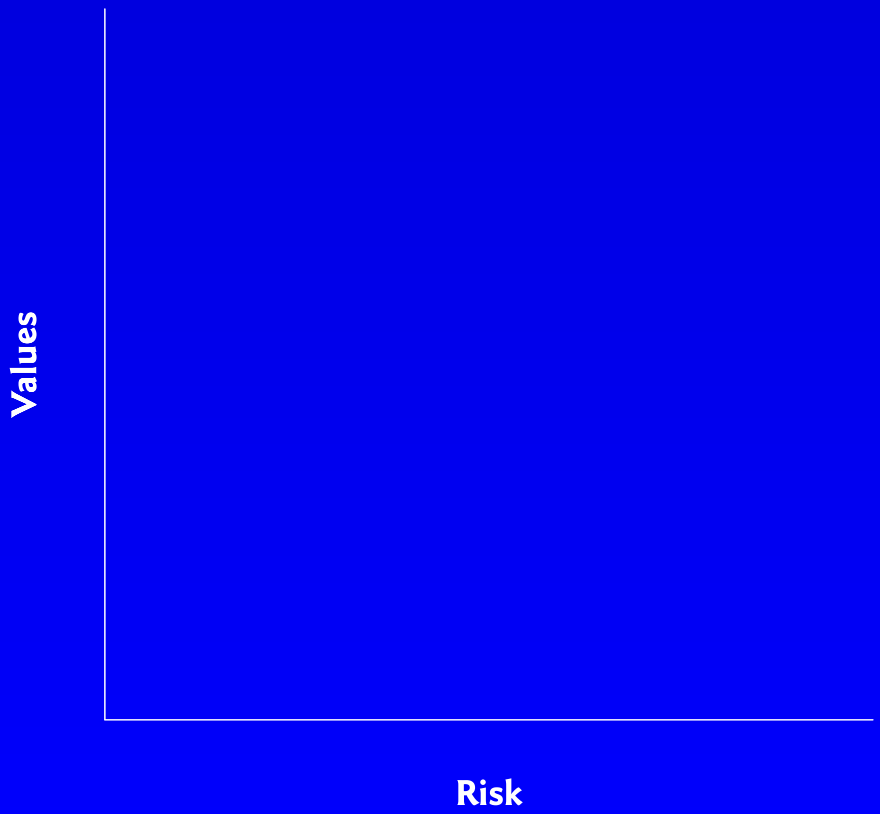
The difference between expected value (EMV) and the certain equivalent (C.E.) is your risk premium.



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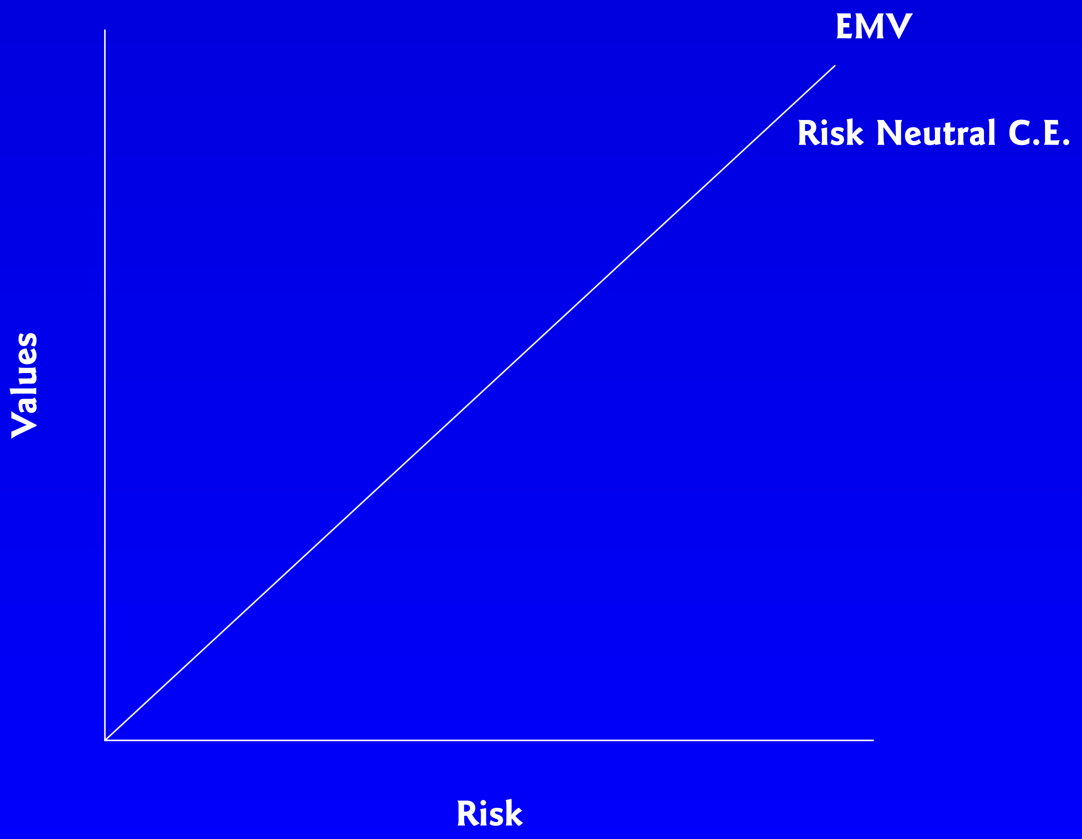

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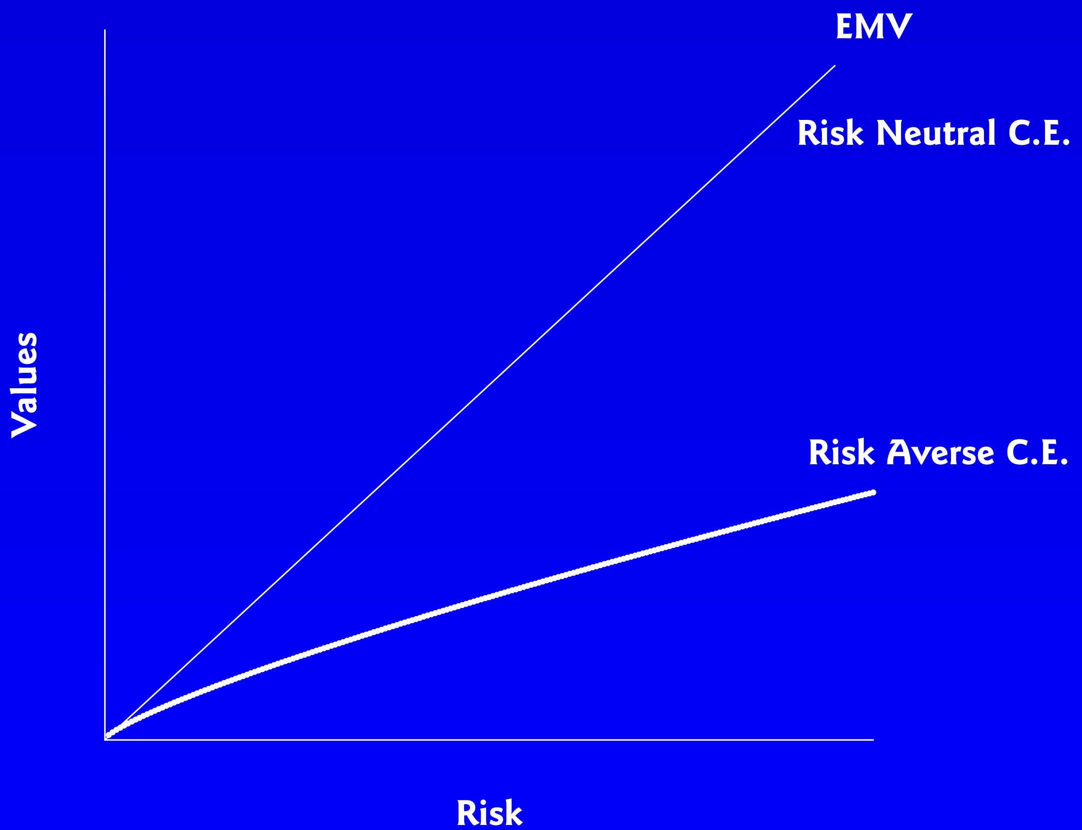

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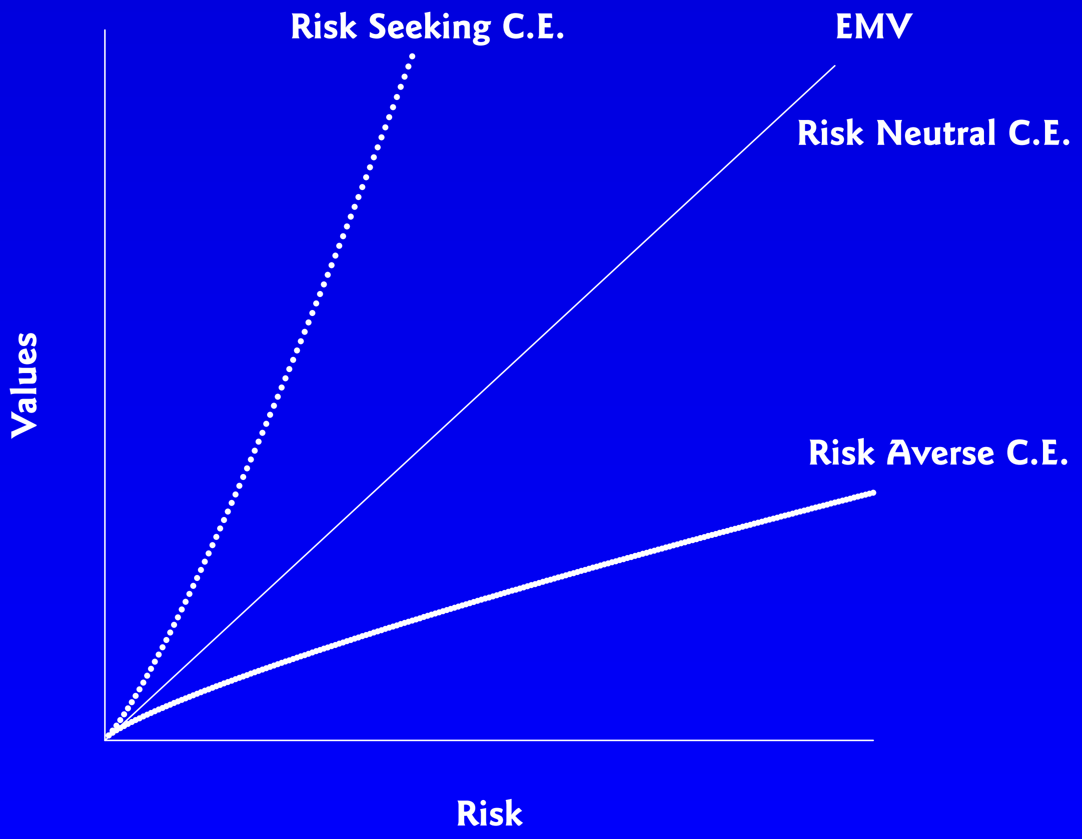

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According to Savage, this is the only ordering which satisfies five general conditions (or axioms) we'd like a good decision rule to satisfy:

Completeness and Transitivity,
Continuity,
Substitutability,
Monotonicity, and
Decomposability.

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- The lottery owner agrees to pay you an additional $100 regardless of outcome: your final payoffs will be $100 and $200 with equal probability.
- If you feel that your C.E. would now be $125 and reason consistently in all such situations, then you satisfy the Delta property.

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The linear and exponential utility curves are called *wealth-independent,* or constant-absolute-risk-aversion (CARA) functions.

3. Constant Absolute Risk Aversion

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Acceptance of the Delta property leads to the characterisation of risk preference by a single number, the *risk aversion coefficient.*

The reciprocal of the risk aversion coefficient is known as the *risk tolerance*, $R = \frac{1}{\gamma}$.

Assessing your Risk Tolerance R when your Utility is Wealth-Independent.

The exponential utility function is given by:

$$U(x) = a + b\,e^{-\frac{x}{R}},$$

where R is a parameter that determines how risk-averse the utility function is, the *risk tolerance*, and a and b are constants used to normalise the function.

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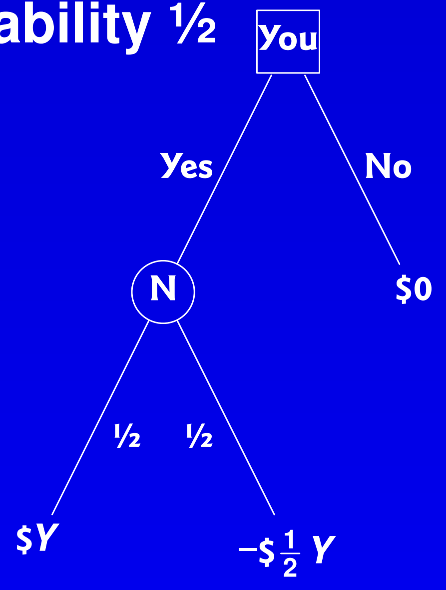
As we have seen, the exponential utility function is appropriate if (and only if) the individual's preferences satisfy the Delta Property of wealth independence.

A simple choice to obtain one's CARA Risk Tolerance

Win \$$Y$ with probability ½
Lose \$ $\frac{1}{2}Y$ with probability ½



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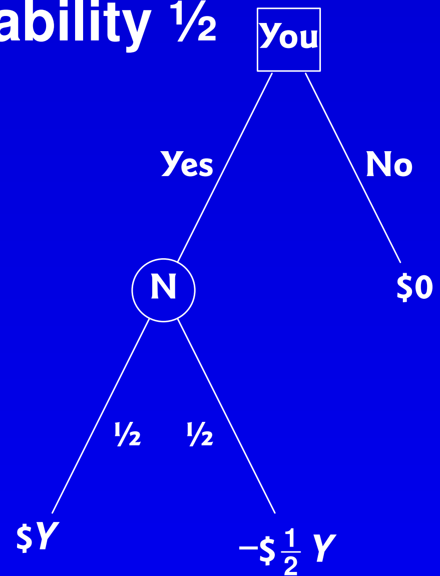

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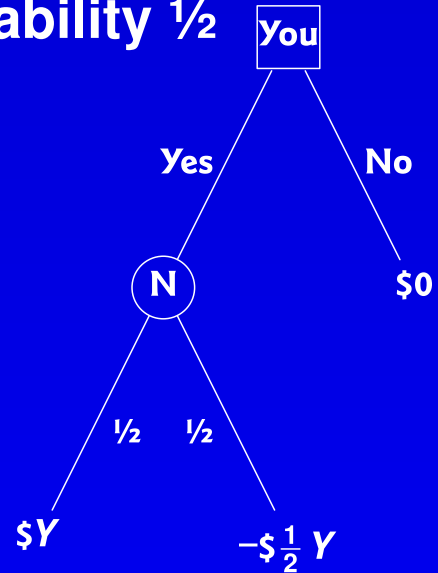

Q: What is the *maximum* size of Y at which you'd prefer doing nothing to having this lottery: the point at which you'd give the lottery ticket away?

This Y is approximately equal to your risk tolerance R in the exponential (wealth-independent) utility function.

(See Clemen, *Making Hard Decisions*, pp. 379–382.)

Two Types of Decision-Makers: Fred and Mary.

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Mary's utility curve can be derived by asking her the C.E. of a series of lotteries, as described below. Each of her answers determines the next lottery she confronts. In general, the lotteries (and so the utility curve elicitted) will be specific to a particular decision of Mary's.

Ron Howard's insights …

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- **15.7% of equity.**

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See: Howard R A (1988), Decision analysis: practice and promise, *Management Science*, 34, 679-695.

4. Eliciting Utility Functions

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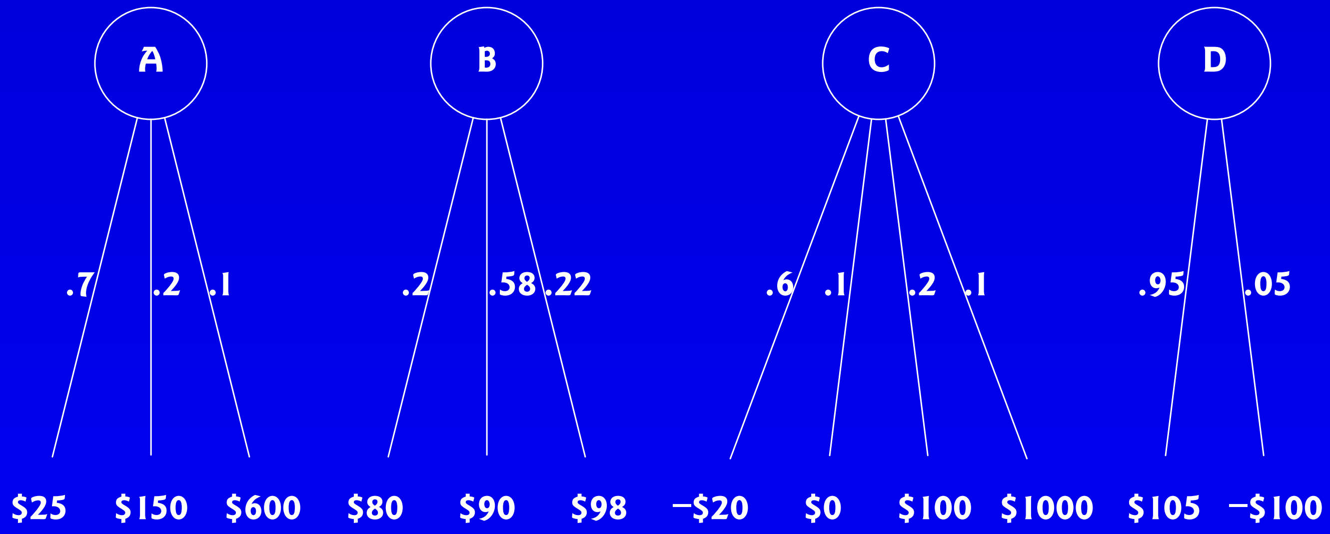

The probabilities are objectively determined:

> **the lotteries are all based on things like the spin of a smooth roulette wheel, etc.**

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We can assess Mary's utility function:
by making some judgements that are easier than those called for in a direct choice among the four gambles above.

Question 1: What is Mary's C.E. for the lottery L1:

- probability ½ of getting $1000
- probability ½ of owing $100

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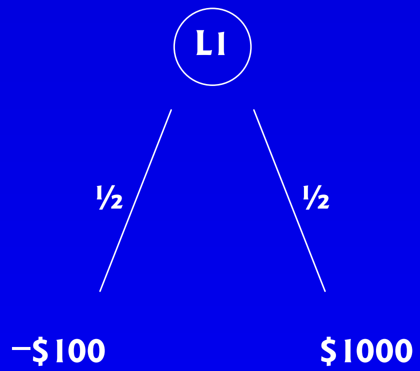
This gamble is selected so that its two prizes span all the prizes in the four gambles from which Mary must choose (set $u(\$1000) = 1$ and $u(-\$100) = 0$), and it gives probability ½ to each:



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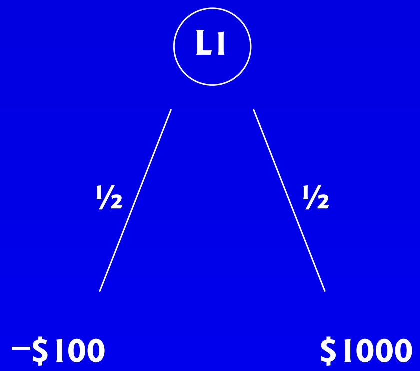

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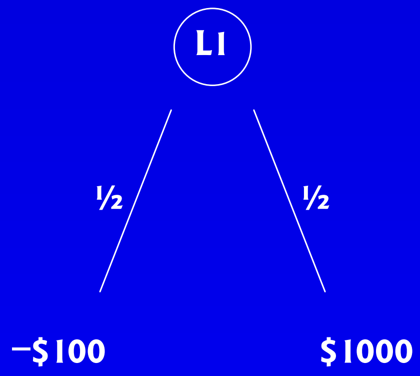

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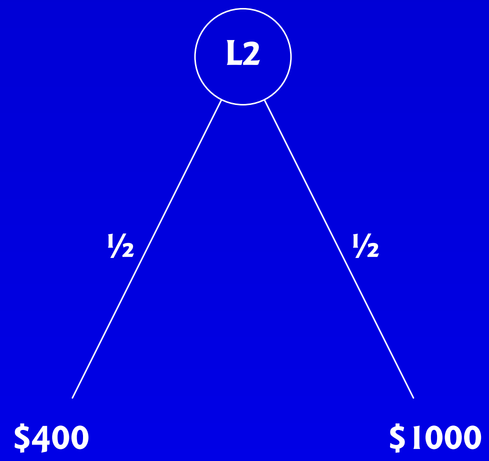
Mary's Answer 1: **she's indifferent between the gamble above and \$400 for sure.**

If $u(-\$100) = 0$, and $u(\$1000) = 1$, then $u(\text{L1}) = 0.5 = u(\$400)$, since the utility of a lottery equals its expected utility, by definition.

Question 2: **What is Mary's C.E. for the gamble L2:**



This gamble has:

- **top prize equal to the upper prize from the question, and**
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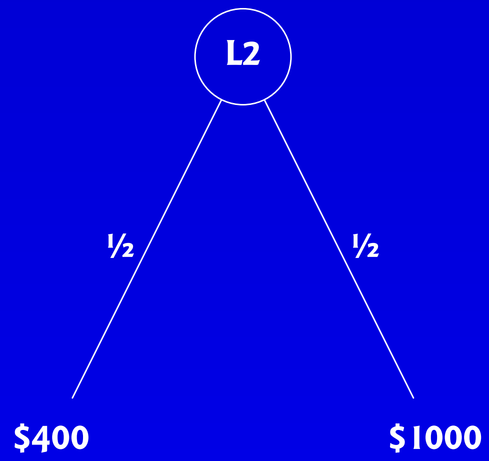
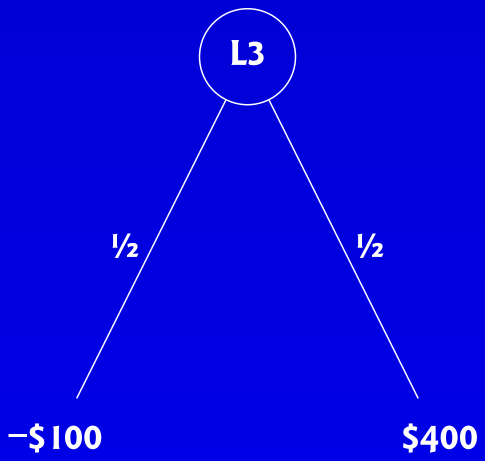


This gamble has:

- **top prize equal to the upper prize from the question, and**
- **bottom prize equal to Mary's previously assessed C.E.**

Mary's Answer 2: **Approximately $675.**

Question 3: **What is Mary's C.E. for the gamble L3:**



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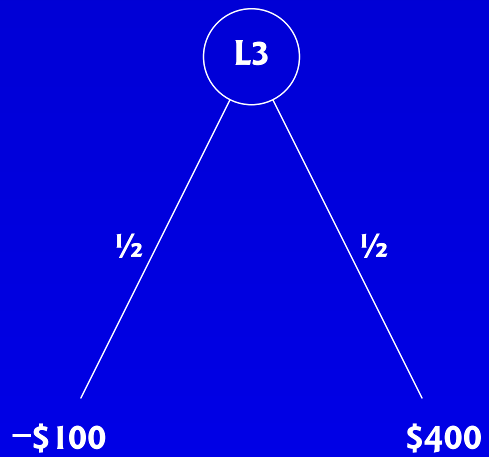

Mary’s Answer 3**: Approximately $100.**

Why these questions?

If Mary can answer these questions, then we'll have five points on Mary's utility function in the range −$100 to $1000:

arbitrarily assigning −$100 utility = 0 and $1000 utility = 1,

Why these questions?

If Mary can answer these questions, then we'll have five points on Mary's utility function in the range −$100 to $1000:

arbitrarily assigning −$100 utility = 0 and $1000 utility = 1,

then the answers reveal that

- **Mary's utility of $400 [*u*($400)] = 0.5,**
- **Mary's utility of $675 [*u*($675)] = 0.75, and**
- **Mary's utility of $100 [*u*($100)] = 0.25.**

Plotting Mary's Curve:

With these five values, we can rough in a pretty good approximation of Mary's utility function and compute her expected utilities for the four original gambles, making her choice accordingly.

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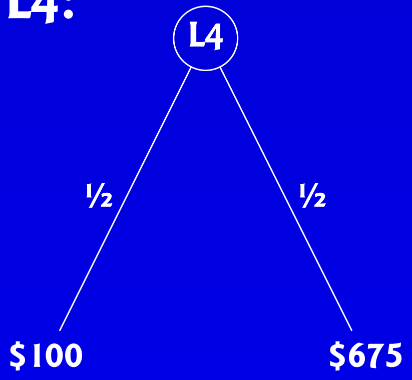
Even if our approximation is off, it is close to Mary’s “true utility” and her choice according to the approximation will be nearly as good as the best gamble using Mary’s “true utility”.

Mary’s making some judgement calls above, and she may not be doing so well.

Checking for consistency ...

The data above allow us to run consistency checks, such as: "What is Mary's C.E. for L4:"



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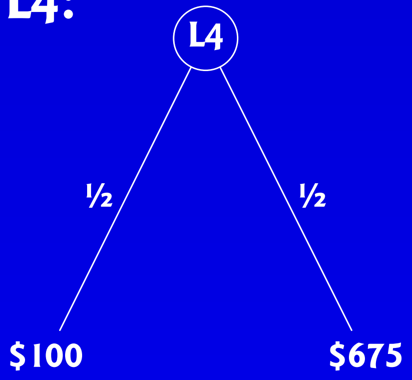

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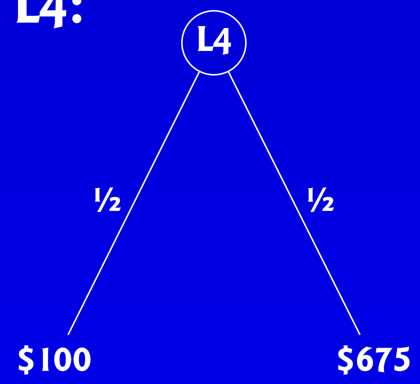

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Because this is a gamble whose prizes have utilities of 0.25 and 0.75 for Mary, so that it has expected utility of 0.5,

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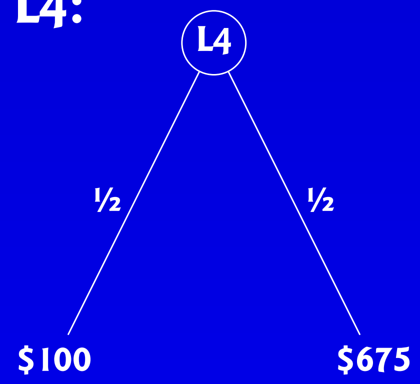

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and the certain amount of money that has utility 0.5 is $400, for her.

Mary's assessed C.E. for this gamble is approximately $375, but now we can return to Mary's original assessments and iterate so that we have five consistent values.

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Why is this procedure better than just choosing one of the original gambles?

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Is this benefit coming for free?

***No* — we also had to make a qualitative judgement that in this choice situation, the three axioms are good guides for choice behaviour.**

But because we know where the pitfalls in those axioms are, we are confident that, in this case, the axioms are a sound guide to behaviour.

The Utility Curve

How does eliciting the C.Es. of simple lotteries allow us to construct Mary's utility curve?

- Using the rule that the utility of a lottery is its expected utility,
- and setting $u(-\$100) = 0$ and $u(\$1000) = 1$, so that the utility function spans the possible payoffs,
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➢ **we see that Mary's utility of the first of the simple lotteries above (the lottery over −\$100 and \$1000, C.E \$400) is**

$$\tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 1 = 0.5;$$

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$$½ \times 0 + ½ \times 1 = 0.5;$$

- her utility of the second (over $400 and $1000, C.E. $675) is

$$½ \times 0.5 + ½ \times 1 = 0.75;$$

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$$\tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 1 = 0.5;$$

- **her utility of the second (over \$400 and \$1000, C.E. \$675) is**

$$\tfrac{1}{2} \times 0.5 + \tfrac{1}{2} \times 1 = 0.75;$$

- **her utility of the third (over −\$100 and \$400, C.E. \$100) is**

$$\tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 0.5 = 0.25;$$

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- **her utility of the third (over −\$100 and \$400, C.E. \$100) is**

$$\tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 0.5 = 0.25;$$

- **and her utility of the fourth (over \$100 and \$675, C.E. \$375) is**

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$$\tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 1 = 0.5;$$
- her utility of the second (over $400 and $1000, C.E. $675) is
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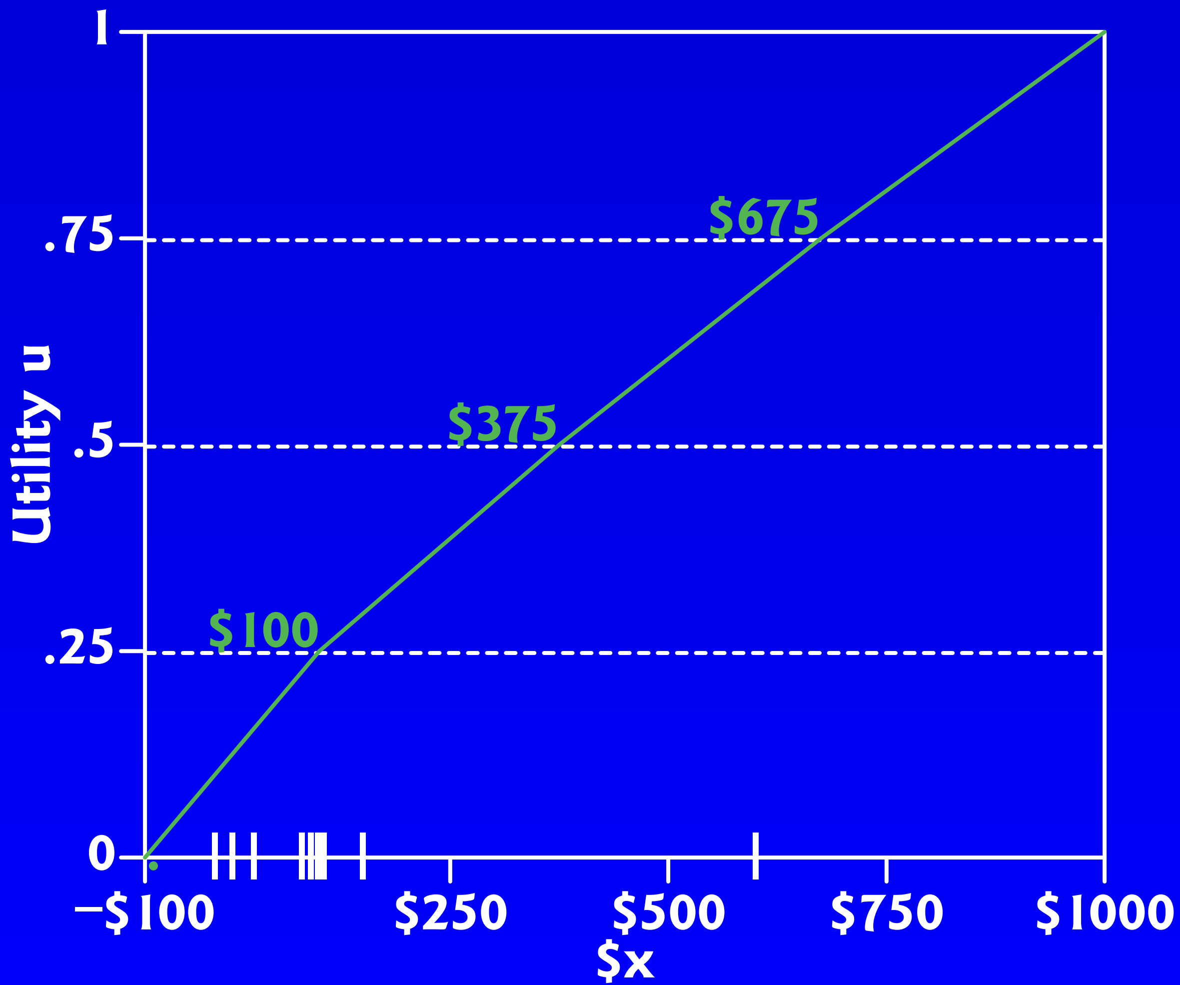
- her utility of the third (over −$100 and $400, C.E. $100) is
$$\tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 0.5 = 0.25;$$
- and her utility of the fourth (over $100 and $675, C.E. $375) is
$$\tfrac{1}{2} \times 0.25 + \tfrac{1}{2} \times 0.75 = 0.5.$$

The last three C.E.s ($675, $100, and $375) have been plotted against the lotteries' utilities (0.75, 0.25, and 0.5, resp.) on the following graph, and we've joined the five points with straight lines, to get an approximation for Mary's utility function. *Iterate.*

Utility Curves u = u(x): Mary's, Fred's normalised, and risk-neutral



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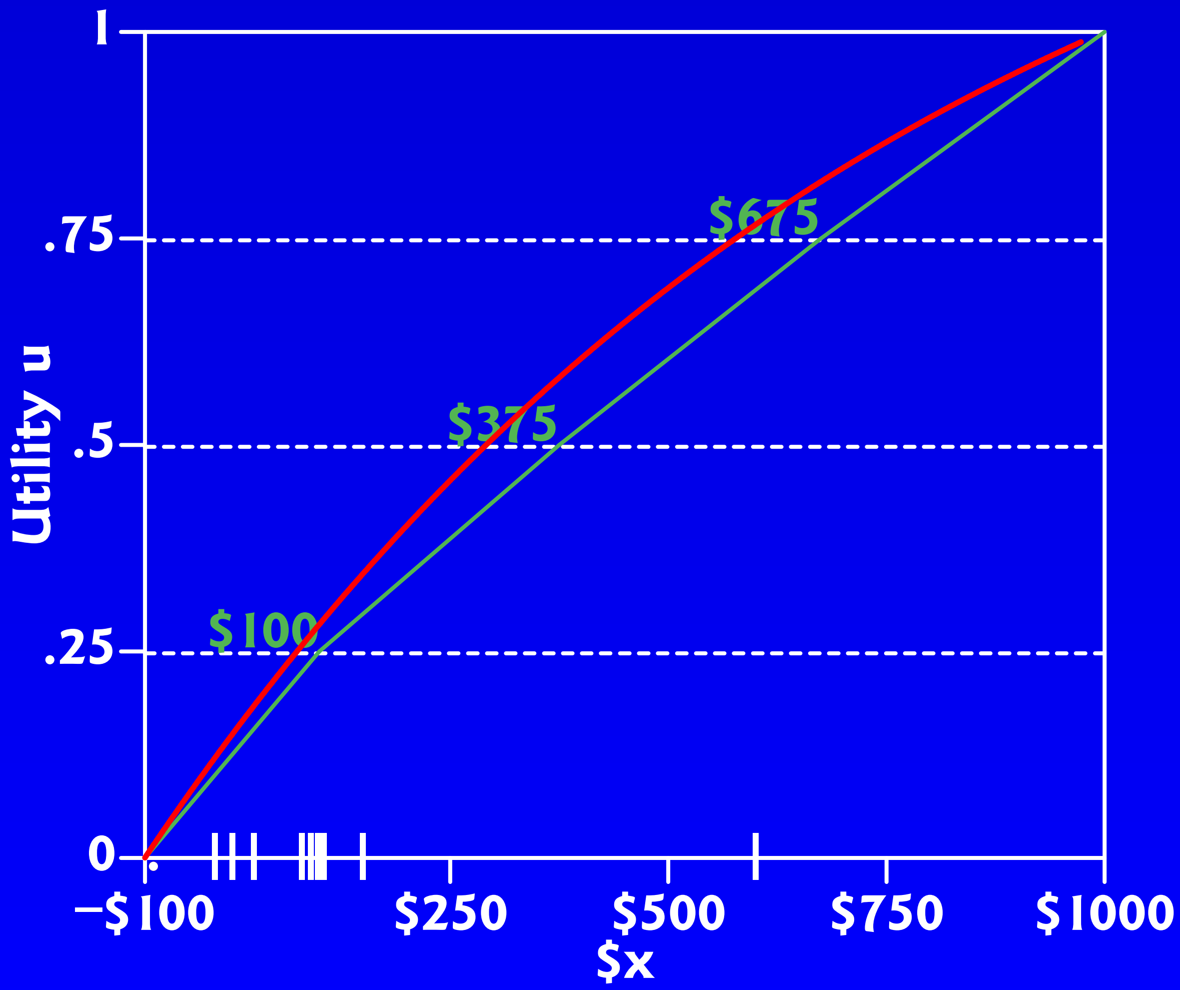

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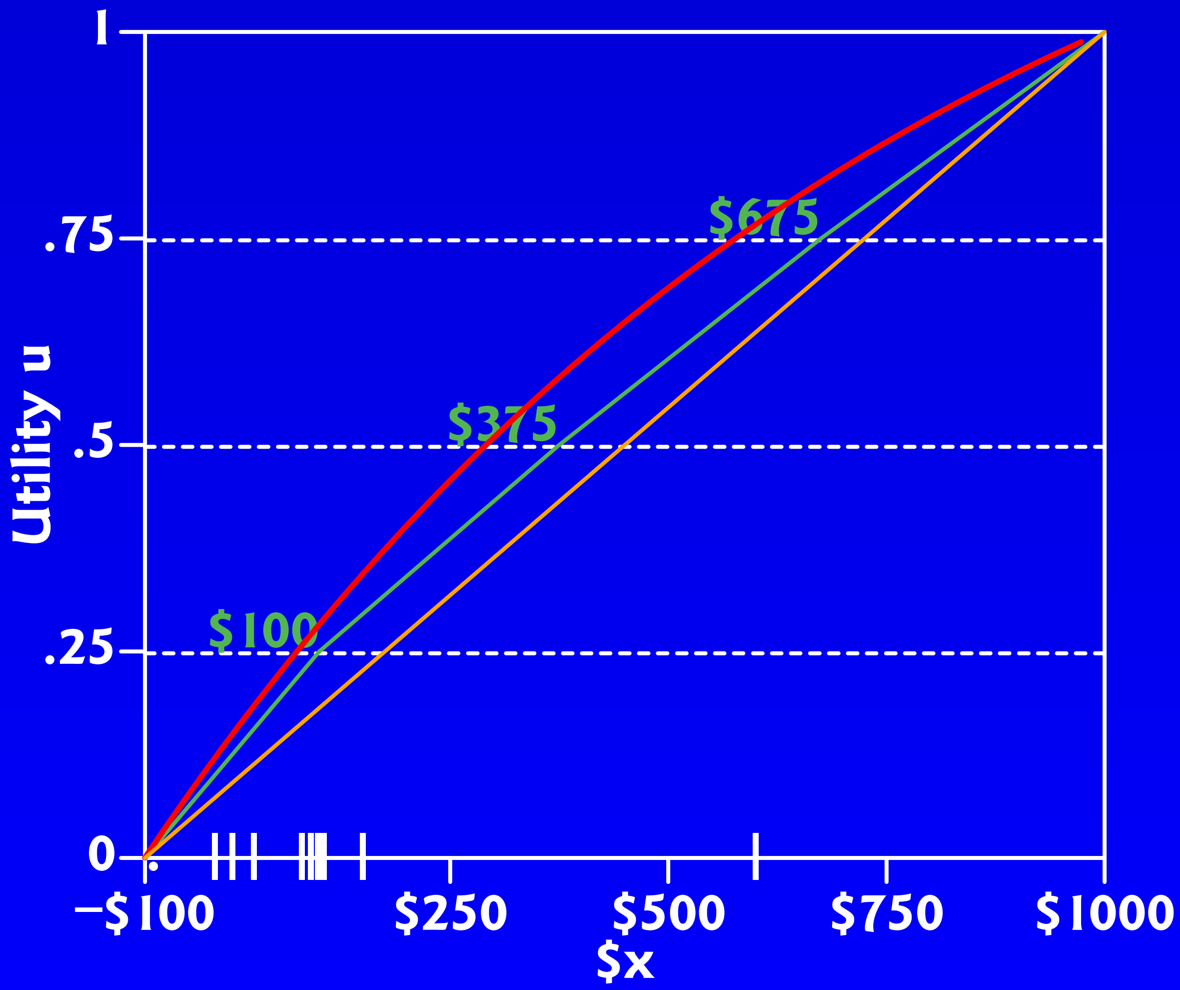

5. The Choice: Using the answers ...

We can use Mary's utility function as plotted, and Fred's utility function $u(x) = 1 - e^{-\frac{x}{900}}$, to calculate their utilities of the four lotteries, A, B, C, and D, of section 4.

Simply a matter of reading off Mary's utilities of the dollar payoffs of the lotteries, and calculating her expected utilities of the four lotteries.

Dollars	Mary's Utility	Fred's Utility
−$100	0.000	−0.118
−$20	.100	−0.022
$0	.125	0.0
$25	.156	.027
$80	.225	.085
$90	.238	.095
$98	.248	.103
$100	.250	.105
$105	.255	.11
$150	.295	.154
$600	.69	.487
$1000	1.0	.671

Mary's expected utilities …

∴ **Mary's utility of lottery A is:**

$$0.7 \times 0.156 + 0.2 \times 0.295 + 0.1 \times 0.69 = 0.237$$

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➢ **of lottery B:**

$$0.2 \times 0.225 + 0.58 \times 0.238 + 0.22 \times 0.248 = 0.238$$

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Mary's expected utilities …

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➢ **of lottery B:**

$$0.2 \times 0.225 + 0.58 \times 0.238 + 0.22 \times 0.248 = 0.238$$

➢ **of lottery C:**

$$0.6 \times 0.100 + 0.1 \times 0.125 + 0.2 \times 0.250 + 0.1 \times 1 = 0.223$$

➢

Mary's expected utilities …

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$$0.7{\times}0.156 + 0.2{\times}0.295 + 0.1{\times}0.69 = 0.237$$

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➢ of lottery C:

$$0.6{\times}0.100 + 0.1{\times}0.125 + 0.2{\times}0.250 + 0.1{\times}1 = 0.223$$

➢ and of lottery D:

$$0.95{\times}0.255 + 0.05{\times}0 = 0.248$$

So Mary would choose lottery D.

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$$0.7\times0.156 + 0.2\times0.295 + 0.1\times0.69 = 0.237$$

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➢ and of lottery D:

$$0.95\times0.255 + 0.05\times0 = 0.248$$

So Mary would choose lottery D.

We can see from the plot of her utility function that she's slightly risk averse.

Fred's expected utilities ...

Using Fred's utility function: $u(x) = 1 - e^{-\frac{x}{900}}$

$\therefore$ **Fred's utility of lottery A is:**

$$0.7 \times 0.027 + 0.2 \times 0.154 + 0.1 \times 0.487 = 0.098$$

➢

Fred's expected utilities …

Using Fred's utility function: $u(x) = 1 - e^{-\frac{x}{900}}$

$\therefore$ **Fred's utility of lottery *A* is:**

$$0.7 \times 0.027 + 0.2 \times 0.154 + 0.1 \times 0.487 = 0.098$$

➢ **of lottery B:**

$$0.2 \times 0.085 + 0.58 \times 0.095 + 0.22 \times 0.103 = 0.095$$

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$\therefore$ **Fred's utility of lottery** A **is:**

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➢ **of lottery B:**

$$0.2 \times 0.085 + 0.58 \times 0.095 + 0.22 \times 0.103 = 0.095$$

➢ **of lottery C:**

$$0.6 \times (-0.022) + 0.1 \times 0 + 0.2 \times 0.105 + 0.1 \times 0.671 = 0.086$$

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Fred's expected utilities …

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➢ **and of lottery D:**

$$0.95 \times 0.11 + 0.05 \times (-0.118) = 0.074$$

So Fred would choose lottery *A*.

(Remember: we're only interested in the relative utilities, not the absolute values, and we can't compare Mary's with Fred's utilities directly.)

The risk-neutral decision maker ...

The risk-neutral decision maker would choose the lottery with the highest expected dollar payoff.

- **The expected dollar payoff of lottery *A* is:**

$$0.7 \times \$25 + 0.2 \times \$150 + 0.1 \times \$600 = \$107.50$$

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- **The expected dollar payoff of lottery *A* is:**

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- **of lottery B:**

$$0.2 \times \$80 + 0.58 \times \$90 + 0.22 \times \$98 = \$89.76$$

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$$0.95 \times \$105 + 0.05 \times -\$100 = \$94.75$$

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- **and of lottery D:**

$$0.95\times\$105 + 0.05\times-\$100 = \$94.75$$

So the risk-neutral player would choose lottery C (or perhaps lottery A).

6. App: Approximating a Certainty Equivalent

Fred (whose Risk Tolerance R = 900 from the R.T. lottery on p.15) is considering the lottery L:

$$L = \begin{cases} \text{Win \$2,000 with probability 0.4} \\ \text{Win \$1,000 with probability 0.4} \\ \text{Win \$500 with probability 0.2} \end{cases}$$

6. App: Approximating a Certainty Equivalent

Fred (whose Risk Tolerance R = 900 from the R.T. lottery on p.15) is considering the lottery L:

$$\begin{cases} \text{Win \$2,000 with probability 0.4} \\ \text{Win \$1,000 with probability 0.4} \\ \text{Win \$500 with probability 0.2} \end{cases}$$

Its mean m = \$1,300, and standard deviation σ = \$600.

Variance = $\sum_{i=1}^{n}[x_i - m]^2 \text{ Prob.}(X = x_i) = \$600^2$ = \$360,000,

where the mean $m = \sum_{i=1}^{n} x_i \text{ Prob.}(X = x_i)$.

(Standard deviation σ = square root of the variance.)

Approximating a C.E. (cont.)

Since Fred's $R = 900$, then use the utility function:

$$U(x) = 1 - e^{-\frac{x}{900}}.$$

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Approximating a C.E. (cont.)

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$\therefore$ **$U(\$2{,}000) = 0.8916$, $U(\$1{,}000) = 0.6708$, and $U(\$500) = 0.4262$.**

$\therefore$ $U(L) = 0.8916 \times 0.4 + 0.6708 \times 0.4 + 0.4262 \times 0.2 = 0.7102$

$\therefore$

Approximating a C.E. (cont.)

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- ∴ **$U(\$2{,}000) = 0.8916$, $U(\$1{,}000) = 0.6708$, and $U(\$500) = 0.4262$.**
- ∴ $U(L) = 0.8916 \times 0.4 + 0.6708 \times 0.4 + 0.4262 \times 0.2 = 0.7102$
- ∴ **Fred's expected utility of this lottery is 0.7102, and**
- ∴ **his C.E. is \$1,114.71, since $U(\$1{,}114.71) = 0.7102$:**
- — **remember: the utility of a lottery is its expected utility, by definition.**

Approximating a C.E. (cont.)

Since Fred's R = 900, then use the utility function:

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∴ **$U(\$2{,}000) = 0.8916$, $U(\$1{,}000) = 0.6708$, and $U(\$500) = 0.4262$.**

∴ $U(L) = 0.8916 \times 0.4 + 0.6708 \times 0.4 + 0.4262 \times 0.2 = 0.7102$

∴ **Fred's expected utility of this lottery is 0.7102, and**

∴ **his C.E. is \$1,114.71, since $U(\$1{,}114.71) = 0.7102$:**

— **remember: the utility of a lottery is its expected utility, by definition.**

The C.E. can be approximated:

$$\text{C.E.} \approx \text{mean} - \frac{1}{2} \times \frac{\text{Variance}}{\text{Risk Tolerance}}$$

$$\text{C.E.} \approx \$1{,}300 - \frac{1}{2} \times \frac{360{,}000}{900} \approx \$1{,}100.$$

(Exact with a normal distribution.)

7. Appendix: Application to Finance

Consider a lottery on x described by the probability density function $f_x(.)$.

➢ Its C.E. , $\tilde{x}$, must satisfy the equation:

$$u(\tilde{x}) = \int f_x(x_0)\, u(x_0)\, dx_0.$$

Why? By the definition of utility, the utility of a lottery [$u(\tilde{x})$] equals its expected utility.

➢ Substituting the exponential form $u(x) = 1 - e^{-\gamma x}$, we can derive:

$$\tilde{x} = -\frac{1}{\gamma} \ln \overline{e^{-\gamma x}} = -\frac{1}{\gamma} \ln f_x^e(\gamma),$$

where $f_x^e(.)$ represents the exponential transform of the density function $f_x(.)$, and where $\overline{e^{-\gamma x}}$ is the mean of the function $e^{-\gamma x}$ for the lottery.

➢ The C.E. of any lottery is therefore the negative reciprocal of the risk aversion coefficient times the natural logarithm of the exponential transform of the variable evaluated at the risk aversion coefficient.
(So there!)

Finance (cont.)

➢ As γ approaches zero, this expression approaches $\bar{x}$: the C.E. of any lottery to a risk-indifferent individual is the expected value, $\bar{x}$.

$$\text{as } \gamma \to 0,\ \tilde{x} \to \bar{x}.$$

A constant-risk-averse decision maker (with a risk-aversion coefficient γ) is facing a *normal* (or Gaussian) lottery.[1] Then his C.E. to this lottery =

the mean minus a half γ times the variance, or

$$\tilde{x} = m - \tfrac{1}{2}\gamma\sigma^2$$

Hence a risk-averse individual will prefer the lottery with the lower variance σ^2, when both have the same expected value, or mean m. (See Finance.)