Games Against Nature: Decision Making Under Uncertainty 1

Today's topics:

1. A Simple Decision
2. Introduction to Decision Analysis
3. Structuring the Decision
4. Influence Diagrams
5. The Glix Case

(See Dixit & Skeath, 2nd: pp. 222–228, 3rd: pp. 251–261.)

The Simplest Decision — Case 1

The simplest decision under uncertainty — calling a coin toss: you win $10 nothing.

— Highlights some concepts which are useful in more complex decisions.

Let's start with a volunteer and ask him/her some questions.

1. **Would you pay $1 for a ticket to play the game?**
2. **What's the minimum you'd sell the ticket for?**
3. **What's the maximum you'd pay for perfect information about the toss (from a clairvoyant)?**
4. **And for imperfect information?**

Everyone write down your answers to Questions 2 and 3.

Coin toss

______ values this game at ___¢.

______ values *Perfect Information* at $_____

______ values *imperfect information* at < $_____

Consistency Check

1. You sell the Ticket to the Lottery for your *Certain Equivalent,* or minimum selling price $\$X$.

   You walk away with $\$X$ for certain.

2. You buy Perfect Information about the coin toss for a maximum of $\$Y$. You then correctly call the toss and win the $10

   You walk away with for certain.

3. So, to be consistent:

Consistency Check ...

Minimum selling price
(The *Certain Equivalent*)
+
***Value of Perfect Information* (VPI)**
=
Maximum Payoff

But why?

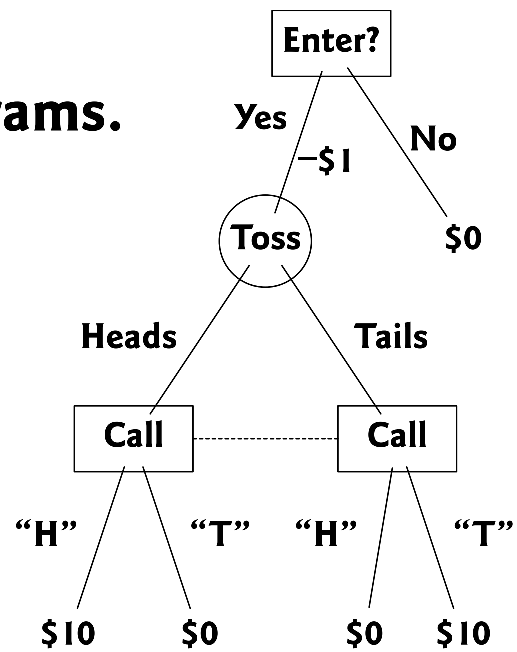
The Value of Imperfect Information
must be less than
the Maximum Payoff
minus
Minimum selling price
(The Certain Equivalent)

Calling the Toss

Concepts:

- **Uncertainty and probability**
- **Profit lotteries**
- **Decisions as allocations of resources**
- **Sunk cost — irretrievable allocations of resources**
- **Certain Equivalent — value of the lottery**
- **Information and probability**

- **Value of information**
- **Consistency in decision making**
- **Decisions versus outcomes**
- **What is meant by a *good decision*?**
- **Individual decisions, corporate decisions**
- **Decision trees:**
- **& Influence diagrams.**



Points to Ponder:

1. Probability is a state of mind (information)
2. Limit to the Value of Perfect Information
3. Limit to the value of imperfect information
   $<$ the Value of Perfect Information (VPI)
4. Consistency check:

   Certain Equivalent + VPI = maximum payoff.

5. $\therefore$ Value of imperfect information
   $<$ maximum payoff − Certain Equivalent

6. Risk averse or risk preferring or risk neutral?

Insights?

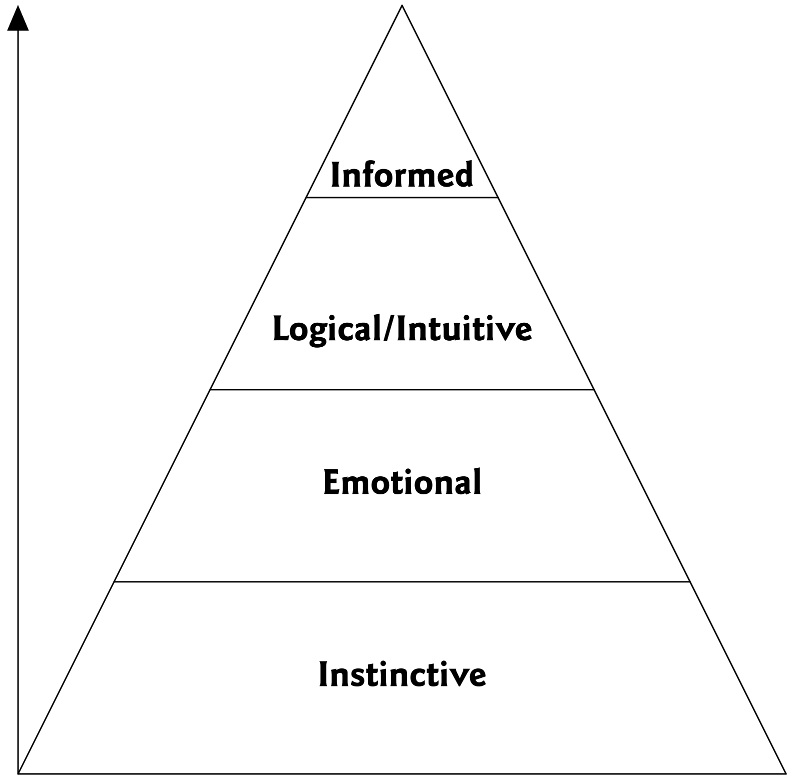
1. **The three elements of a decision:**
    - ***actions*: here call "Heads" or "Tails".**
    - ***events* are Nature's possible moves: here Heads or Tails.**
    - ***outcomes*: here either $10 for a correct call or nothing.**

2. **Her *attitude to risk*: the minimum she was prepared to sell the ticket for.**

3. **Her *value of information*: limited by the Value of Perfect Information, a function of the probabilities and payoffs.**

Decision Analysis: Games Against Nature

The Decision Response Hierarchy



The Decision Response Hierarchy

Moving up the hierarchy corresponds to increasing consciousness, clarity, and power.

Which response do you use most often in making decisions?

- ☐ **Informed**
- ☐ **Logical or Intuitive (learned)**
- ☐ **Emotional**
- ☐ **Instinctive**

Basic Concepts

A technique for helping make decisions, and avoiding pitfalls.

We discuss:

- **Formulating the issue.**
- **Identifying the alternative actions.**
- **Valuing the possible outcomes.**
  (Not merely in monetary terms.)
- **Encoding uncertainty.** $\rightarrow$ **probabilities**
- **Certain Equivalent (C.E.).**
- **The Value of Perfect Information. (VPI)**
- **The value of imperfect information.**
- **Utility and risk aversion.**
  The utility of a lottery is its expected utility

Why Is Decision-Making Difficult?

- **Uncertain consequences or outcomes.**
- **Conflicting objectives**
- **Competing projects or alternatives**
- **Being held accountable for outcomes**
- **Multiple decision makers**
- **Risk attitude**

Decision Analysis Addresses These Issues By:

- **Focussing on what *don't know* rather than refining what you already knew.**
- **Applying a *logically correct methodology* to consistently evaluate alternatives**
- ***Gaining insight* into the decision problem**

  — the numbers should always be subservient to the insights gained.

What Is Decision Analysis?

Decision analysis *is*:

> **A methodology based on a set of probabilistic frameworks which facilitates high-quality, logical discussions, leading to clear and compelling actions by the decision maker.**
>
> **— insights, not just numbers.**

Decision analysis is *not*:

- **A method for justifying decisions already made.**
- **Cost-benefit analysis**
- **A cookbook**

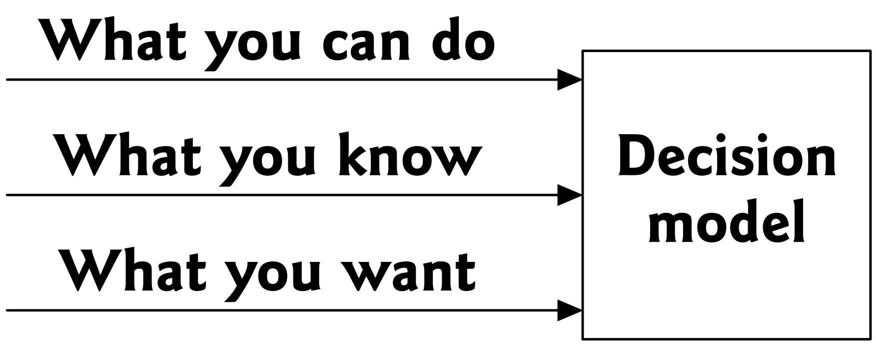
Decision analysis provides answers to questions such as:

- **How risky is this project?**
- **Which plan do we follow?**
- **Which assumptions are most important?**
- **What is the project's potential?**
- **Should we gather more data?**
  (and if so, what's the maximum we should pay?)

Beginning Principles:

The best you can do is to integrate in a logical manner:

- **What you *can* do.**
- **What you *know*. (Such as likelihoods and values)**
- **What you *want* or *value*. (Such as your risk attitude)**



The Decision Analysis Process.

Decision analysis is a three-stage, quality process. But if at any step in the process the decision becomes obvious, you should stop and make the decision.

1. ***Structuring:* Frame the Right Problem**
   - ➢ **Clarify the decision.**
   - ➢ **Raise and sort issues.**
   - ➢ **Generate creative alternatives.**
   - ➢ **Model the problem.**

The Decision Analysis Process — Stage 2

2. *Evaluation:* Use Logical Thinking
   - Discover what is important.
   - Apply an appropriate risk attitude.
   - Determine the maximum value of new information.

3. *Agreement:* Have Commitment to Action
   - Check for refinement.
   - Agree on course of action.
   - Implement course of action.

Decision analysis is a normative process.

The term *decision analysis* is becoming a broadly used term in many industries. While it can be used *descriptively*, here we use the *normative* meaning.

***Descriptive* decision analysis is a present-state approach, describing how things *are*.**

***Normative* decision analysis is a future-state approach, describing how things *should be*.**

Why Decision Analysis?

- ***Decision making*** **is at the heart of most technical, business and governmental problems, not to mention one's private life.**
- **Decision making requires the study of *uncertainty*. There are no sure things;**
  risk-taking is inescapable.
  - **How does uncertainty affect decision-making?**
  - **How can one make a rational or prudent decision (a "good" decision) without knowing exactly what consequences will follow?**

- **Think of risk-taking in terms of gambles or lotteries. Uncertainty can only be studied formally through *probability* theory, the only theory of uncertainty which has *this important property:***

  > **the likelihood of any event following the presentation of a sequence of points of data does not depend upon the order in which those data are presented.**

Probability is a state of mind, not things.

- **The Bayesian approach allows us to assign probabilities in once-off situations.**

  e.g. What is the value to you of a single toss of a coin: $100 if heads, nothing if tails?

  Define the *expected return* from the single toss to be the average return of a hypothetical series of many tosses: $\$100 \times ½ + \$0 \times ½ = \$50$. Treat unique events as if they were played over many times.

- **All *prior experience* must be used in assessing probabilities. (Coins are almost always fair; it's warm enough to go to the beach most weekends in March in Sydney.)**

Values plus probabilities.

➢ **Decision making requires the assessment of *values* as well as probabilities.**

Would you pay as much as \$50 to play in the once-off coin toss (for \$100)? Few people would; most people would pay a premium to reduce their risk: they are *risk averse*, and would sell their lottery ticket at something less than \$50; the lowest selling price is their *Certain Equivalent* (C.E.).

The *risk premium* equals the expected return less the certain equivalent, when selling.

***Risk aversion* can be defined and measured using *utility theory*.**

The utility of a lottery ...

- **Decisions can only be made when a criterion is established for choosing among alternatives.**

  The utility of a lottery is its expected utility.

  (by the definition of utility — remember this!)

- **The implications of the present for the future must be considered. What *discount rate* to use.**

- **Must distinguish between a *good decision* and a *good outcome*.**

  Prudent decision-making doesn't guarantee the desired outcome invariably, but should improve the odds.

The Value of Perfect Information?

- **Often we can, at a cost, reduce our uncertainty about Nature's future events (using market research, forecasting, statistical analysis). There must be a limit to what we should spend in these endeavours—how much is it?**

  = *The Value of Perfect Information.* (VPI)

  Note that if perfect information would not change our decision, then it's worth nothing to us because our expected payoffs wouldn't change with PI.

- **The value of *im*perfect information is less (of course).**

  Often we can, at a cost, buy more certainty about the future (pay an insurance premium, buy a hedge against future outcomes).

  What is a fair price to pay?

Summary of Introduction

We need a methodology to help us make difficult decisions. Decision analysis provides that methodology.

Decision analysis focuses on what we don't know, rather than on refining what we do know.

The best you can do is to integrate in a logical manner:

- **What you can do,**
- **What you know, and**
- **What you value.**

Decision analysis has three distinct stages — *Structuring*, *Evaluation*, and *Agreement*.

Structuring the Decision

Define the Problem and the Decision Criterion

To begin structuring the decision, we must first define the problem and the decision criterion.

- **What is the decision?**
- **Who is the decision maker?**
- **What is the decision criterion?**

— the decision criterion can be anything that allows the decisionmaker to quantitatively distinguish one alternative from another:

- **Net present value (NPV)**
- **Internal rate of return (IRR)**
- **Cash flow**
- **Goodwill/reputation**
- **Others**

Brainstorming

Once the problem has been defined, we need to brainstorm and sort issues.

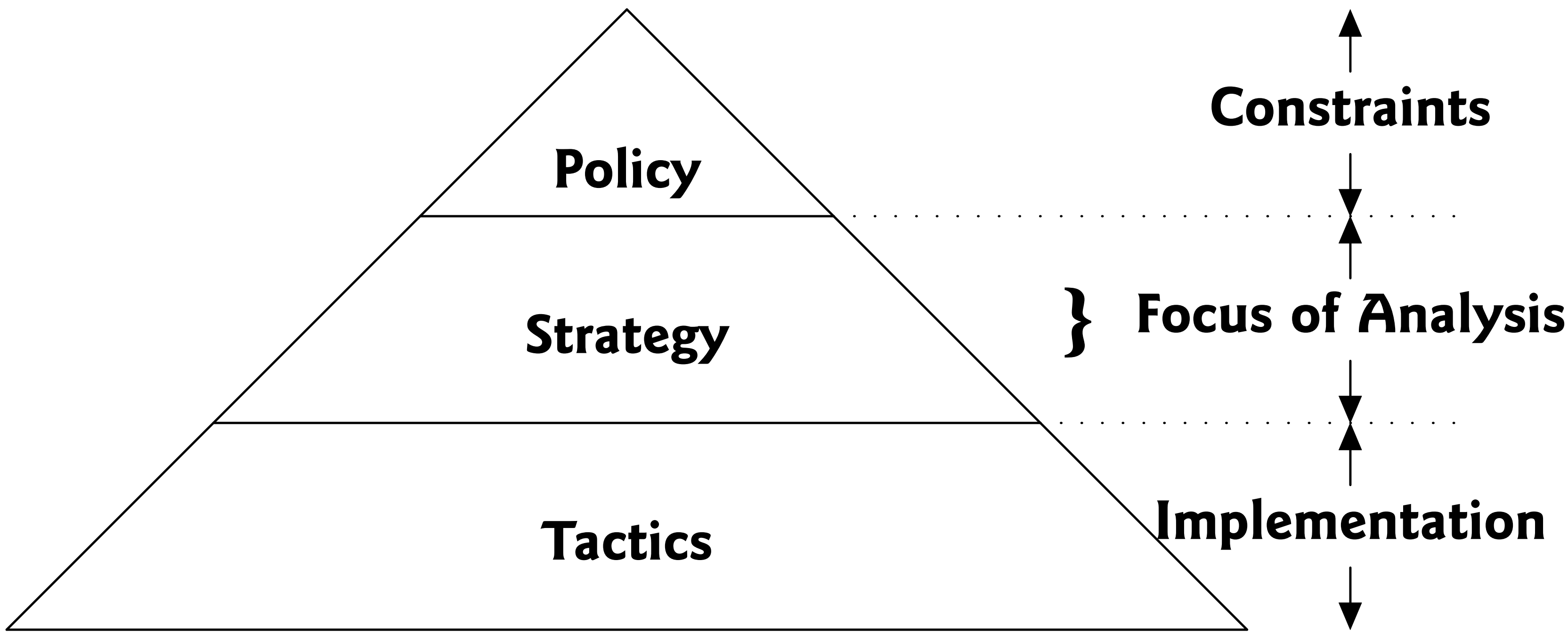
Raise issues.

Separate issues in order to begin problem framing.

Categorise the decisions using the *decision hierarchy*, to help identify the scope of the problem and to separate constraints and implementation from the focus of the analysis.

The Decision Hierarchy

The focus of decision analysis is at the strategic level.



Decision levels.

Policy **decisions are constraints.**

Strategy **decisions are the focus of our analysis here.**

Tactical **decisions are follow-on implementation decisions.**

Influence Diagrams

The next step is to model the decision making using influence diagrams.

Influence diagrams:

- **Provide a clear, graphical picture of a problem**
- **Show relationships and relevance**
- **Facilitate dialogue among team members with different backgrounds and interests**
- **Provide a means to compare alternatives**
- **Facilitate delegation**
- **Cannot have circular references or feedback loops**
- **Are not flow diagrams**

Plotting an Influence Diagram:

Uncertainties are chance events, ○

Decisions are controllable, □

Values are what you prefer, ◇

Arrows indicate relevance (not timing) $\rightarrow$

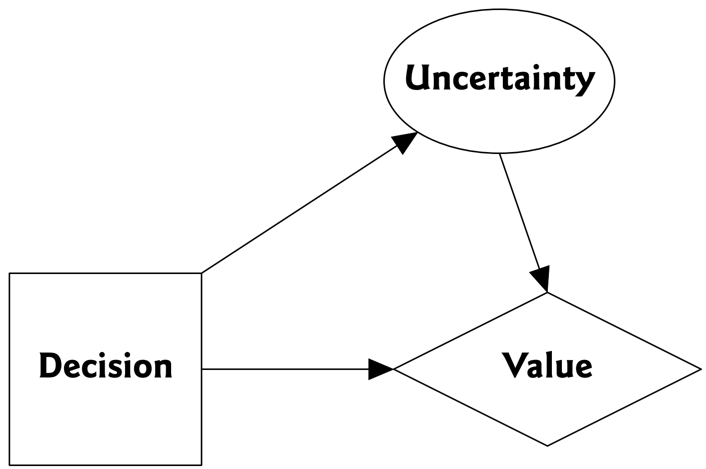
An arrow into an uncertain node ○ means relevance.

An arrow into a decision node □ means "is known".

An arrow into a value node ◊ means "functional".

Influence diagrams typically flow from decisions to uncertainties to value.

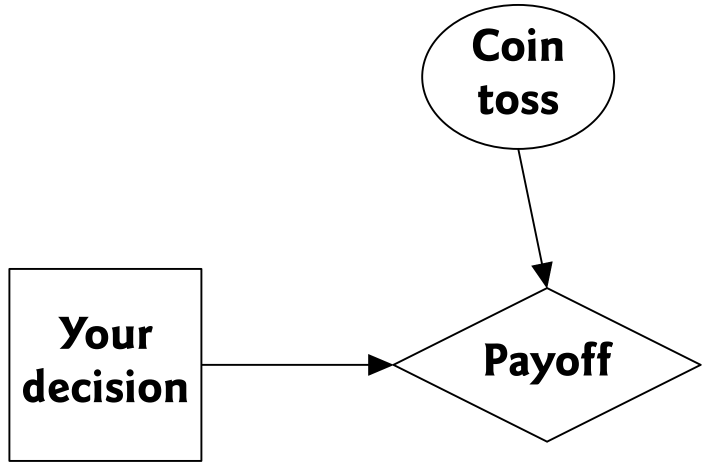
Arrows indicate relevance and show relationships.



**Be careful when adding an arrow: *influence diagrams are not flow charts;* arrows are not necessarily timings.
The lack of an arrow says more than having an arrow.**

In this I.D., the decision affects the uncertainty, as well as the outcome: e.g. eating and drinking to excess may lead to illness.

The influence diagram for the coin-tossing decision:

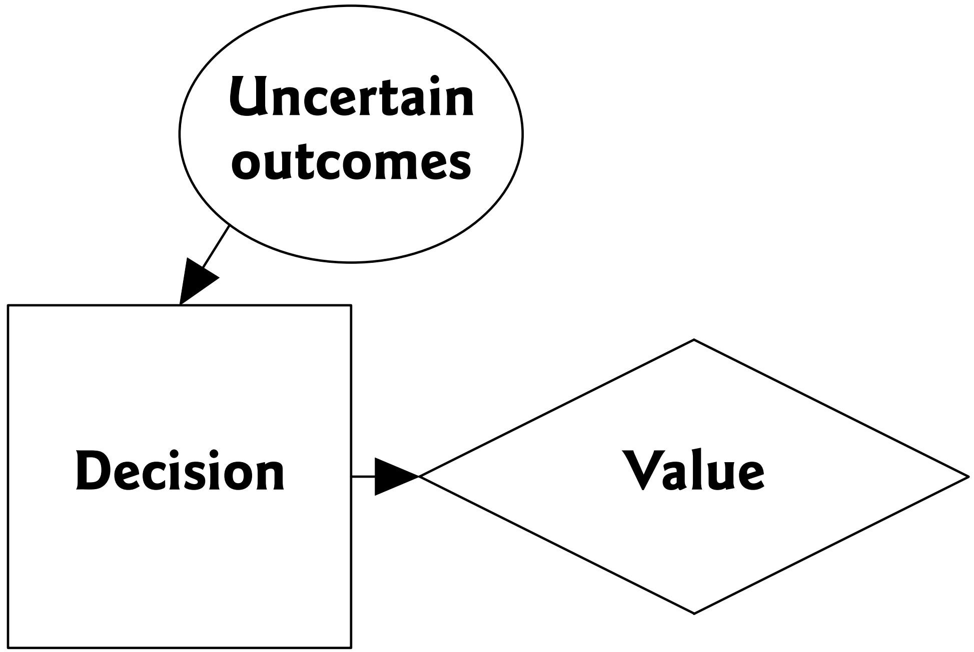


Since your decision (of whether to invest, and, if so, whether to call "heads" or "tails") does not influence the outcome of the coin toss, there is no arrow from the decision node to the chance node.

And since you will call the toss before you know the outcome, the arrow from the chance node goes to the payoff node.

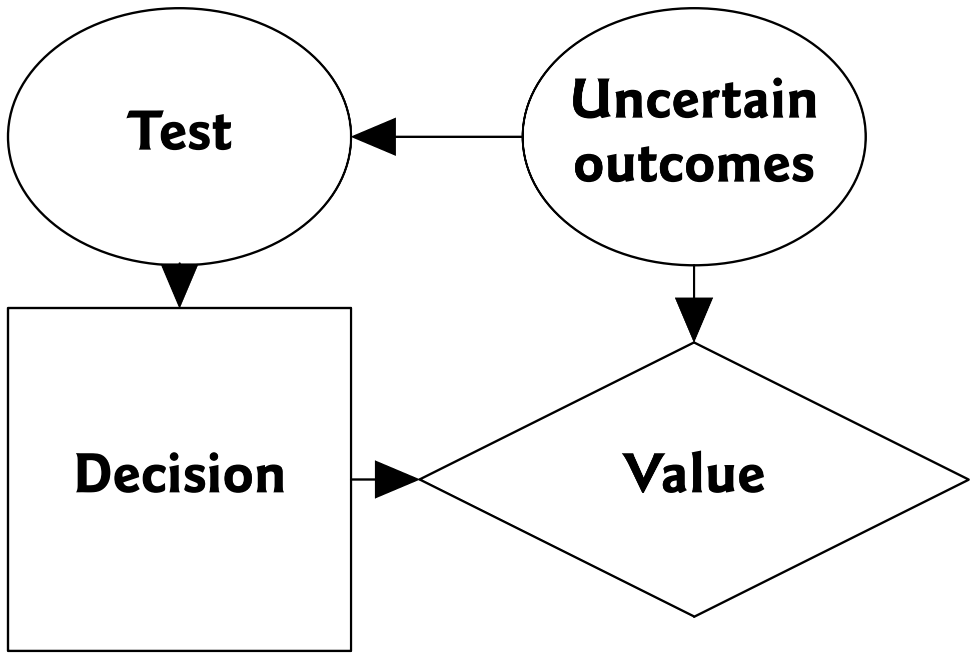
This is a very common Influence Diagram. Other examples?

Decision-making with a Clairvoyant (very unusual)



Before you make the decision you know the outcome of the future uncertainty, as shown by the arrow from the Uncertainty to the Decision. Both affect the Outcome. Note: there must have been a prior decision to seek the clairvoyant's knowledge.

Decision-making with Unreliable Information (usual)

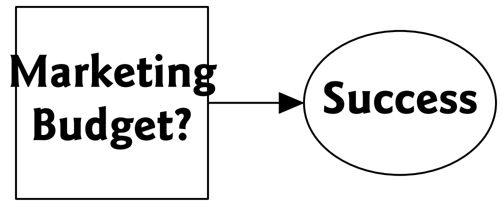


The Uncertainty affects the Test (in an uncertain way, since it's not 100% reliable), the result of which is known before you make the Decision. Note: there must have been a prior decison to undertake the test.

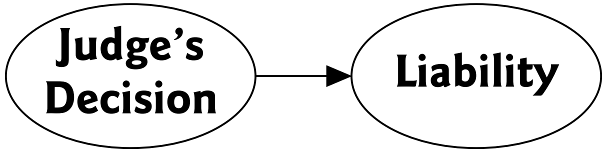
Three Types of Influence — 1. Probabilistic:

(See DATA 3.5 Manual extract in the Readings.)

1. ***Probabilistic Influence***



Decision about the Marketing Budget can *influence the probability* of success. If not, then no arrow.

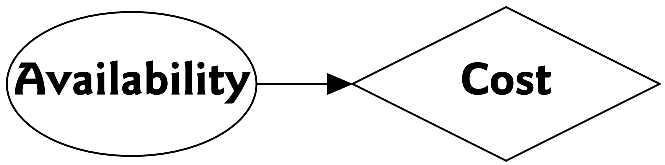


The probability of the defendent's liability *depends on* whether the judge will admit particular evidence. (Not necessarily a time flow.)

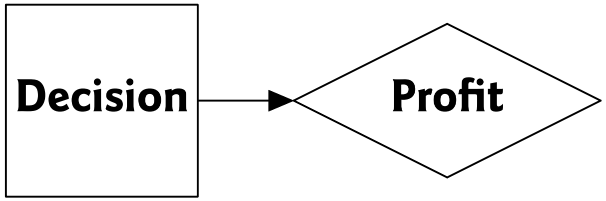
< >

Three Types of Influence — 2. Value:

2. *Value Influence*



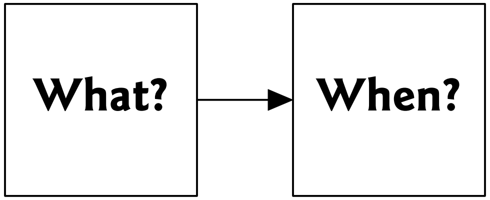
The manufacturing cost *depends on* the (unknown) local availability of an input.



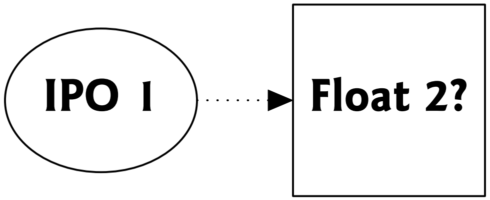
The manager's decisions *influence the profit* of a plant.

Three Types of Influence — 3. Structural:

3. ***Structural Influence***



What to make is *decided before* When to make it.

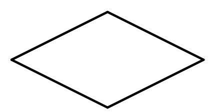


The outcome of floating firm 1 will be *known before* the decision of floating firm 2 is made. ∴ No arrow to the first (or only) decision: the uncertainty is already known or given at the time of decision.

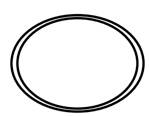
Influence Diagrams — Summary

An influence diagram provides a simple graphical representation of an uncertain decision problem. It contains at least three elements, linked with arrows to show the specific relationships among them:

- ***Decisions* are represented by squares □ or rectangles.**
- ***Chance events* (the uncertainty of which will be resolved before the payoff) are represented by circles ○ or ellipses.**
- ***Values* or payoffs are represented by diamonds:**



- ***Deterministic nodes* are represented by double ellipses::**



Influence diagrams.

Influence diagrams provide a snapshot of the decision environment at one point in time.

They are not flow charts or diagrams.

They cannot contain cycles.

The arrows must indicate how uncertainty is revealed (all will be revealed before the final payoff, but decisions are made with some uncertainty remaining).

Influence Diagrams — warning

Influence Diagrams are used to focus on decisions involving (future) uncertainty.

The Influence Diagram thus focuses on what might happen to *influence the final payoff* after the (first) decision has been made.

There is no point in plotting any pre-existing influences on the decision-maker that occur before the first decision is made; these are taken as given and (usually) excluded as incoming arrows into the first decison node (a rectangle).

Influence Diagrams are *not* used to examine the "influences" on the decision maker before the first decision is made.

The Glix Case:

The Gaggle Company has developed a new product — Glix.

While you think that Glix has great potential, you are unsure whether Glix will be profitable if brought to market.

Your decision: There are three alternatives facing you:

1. **launch Glix yourself**
2. **sell Glix to another company, or**
3. **licence Glix to all comers**

Issues about Glix.

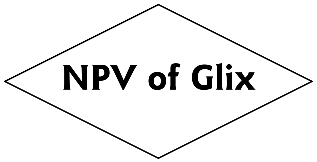
Issues about Glix include:

- **Market size**
- **Revenue**
- **Marketing costs**
- **Profit**
- **Share price**
- **Manufacturing costs**
- **Regulations**

Begin by clarifying the decision and the decision criterion, and work from right to left.

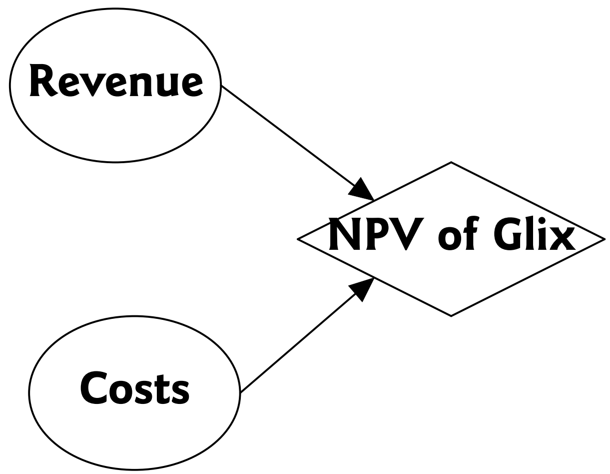
The *decision* is whether: to launch; to sell; or to license the Glix product.

Management have determined that net present value (NPV) is the correct decision *criterion*.



Launching Glix: What are the determinants of net present value?

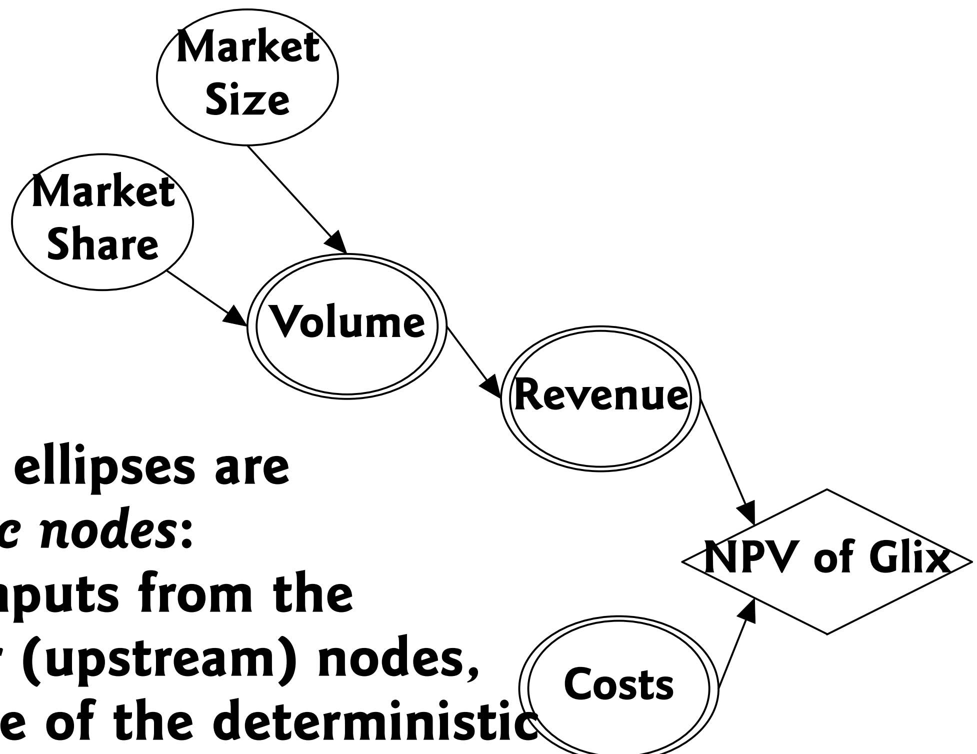
- Revenue
- Cost



What are the determinants of revenue?

What do you need to know the calculate revenue?

Consider the price of Glix to be $5.00 per kilo.

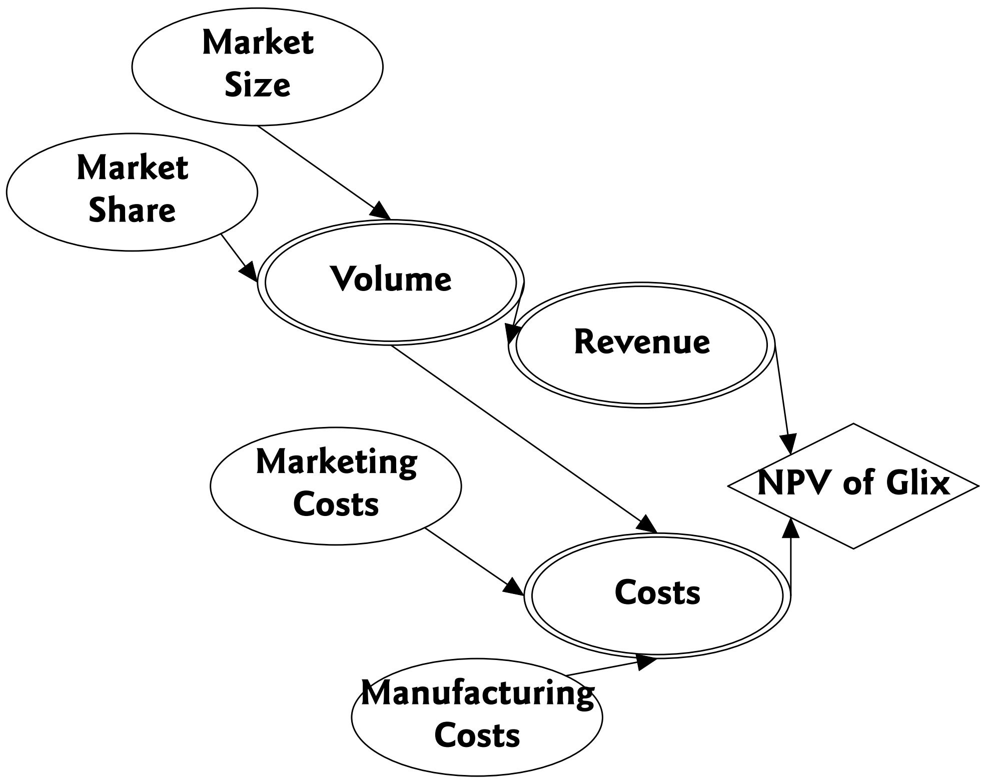


The double ellipses are *deterministic nodes*: given the inputs from the predecessor (upstream) nodes, the outcome of the deterministic node can be found immediately.

Next consider the determinants of cost.

What do you need to know to estimate the costs associated with Glix?

There is a factory in place that will only need minor modifications at a cost of $1,500,000.



Lastly, we need to add the decision.

Which key uncertainties does the decision influence?