Combining Simultaneous and Sequential Games

Let’s mix and match our games, explore how games can change, and how we can model them:

1. Simultaneous and Sequential Together.
2. Changing the Order of Moves:
   - First-Mover Advantage
   - Second-Mover Advantage
   - Both-Mover Advantage
3. Trees for Simultaneous Games.
   - Subgame Perfect Equilibrium
1. Games with Both
I. Games with Both

Players: CrossTalk (CT) and GlobalDialog (GD)
1. **Games with Both**

Players: CrossTalk (CT) and GlobalDialog (GD)

Actions: each Invest $10 b in a separate fibre-optic network or not, simultaneously.
I. Games with Both

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Actions: each Invest $10 b in a separate fibre-optic network or not, simultaneously.

Neither invests: end of game.
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Only one invests: it must choose its price:
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  • price High (60 m customers, $400/cust rev), or
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  • price Low (80 m cust, $200/cust).
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Both invest: second simultaneous game:
1. Games with Both

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  • price High (60 m customers, $400/cust rev), or
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Both invest: second simultaneous game:
  • both price High (each 30 m cust, $400/cust),
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- price High (60 m customers, $400/cust rev), or
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Both invest: second simultaneous game:
- both price High (each 30 m cust, $400/cust),
- both price Low (each 40 m cust, $200/cust), or
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Actions: each Invest $10 b in a separate fibre-optic network or not, simultaneously.

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Only one invests: it must choose its price:
- price High (60 m customers, $400/cust rev), or
- price Low (80 m cust, $200/cust).

Both invest: second simultaneous game:
- both price High (each 30 m cust, $400/cust),
- both price Low (each 40 m cust, $200/cust), or
- one High and the other Low (High gets 0 cust, Low gets 80 m cust, $200/cust).
A Two-Stage Game (\(\$k\))

<table>
<thead>
<tr>
<th></th>
<th>Don’t Invest</th>
<th>Invest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Don’t</td>
<td>0, 0</td>
<td>0,</td>
</tr>
<tr>
<td>Invest</td>
<td>0</td>
<td>,</td>
</tr>
</tbody>
</table>

CT

GD
### A Two-Stage Game ($k$)

#### Payoff Matrix

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Don’t</td>
<td>0,0</td>
<td>0,0</td>
</tr>
<tr>
<td>Invest</td>
<td>0</td>
<td>0,14</td>
</tr>
</tbody>
</table>

#### Diagram

- **CT**: Don’t, Invest
- **GD**: High, Low
- **Payoffs**: 0, 0, 0, 14, 0, 0, 0, 6
A Two-Stage Game ($k$)

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CT

GD

High

14

Low

6
A Two-Stage Game ($k$)

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CT

GD

CT

High

Low

CT

High

Low

CT

High

Low

GD

High

Low

2, 2

−10, 6

6, −10

−2, −2
A Two-Stage Game ($k$)

```
\begin{array}{c|cc|c}
& Don't & Invest & GD \\
\hline
Don't & 0, 0 & 0, & \\
Invest & 0 & 6 & \\
\end{array}
```

```
\begin{array}{cc|c|c}
& \text{High} & \text{Low} & \\
\hline
\text{High} & 2, 2 & -10, 6 & \\
\text{Low} & 6, -10 & -2, -2 & \\
\end{array}
```
A Two-Stage Game ($k$)

CT
Don’t
Invest

GD
Don’t
Invest
0, 0
0,
0

High
Low
CT
High
Low
14
6

CT
High
Low
2, 2
6, −10
−2, −2

High
Low
CT
High
Low
14
6

High
Low
GD
Rolling Back the Game

CT’s payoff if it alone Invests and prices High = $14 \ b$

$= $400 \times 60 \ m - $10 \ b$

$= $24 \ b - $10 \ b$

$= $14 \ b$
Rolling Back the Game

CT’s payoff if it alone Invests and prices High = $14 b
= $400 \times 60 \text{ m} - $10 b
= $24 b - $10 b
= $14 b

If both Invest and price Low, each gets −$2 b
= $200 \times 40 \text{ m} - $10 b
= $8 b - $10 b
= −$2 b

Etc.
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Etc.

The second-stage pricing game is a PD: pricing Low is dominant.
**So what is the entire game?**

Hence, after rollback, the entire game is:

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<td>-2, -2</td>
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\[
\begin{array}{|c|c|c|}
\hline
 & \text{Don’t} & \text{Invest} \\
\hline
\text{Don’t} & 0, 0 & (0, 14) \\
\hline
\text{Invest} & 14, 0 & (2, -2) \\
\hline
\end{array}
\]

A Chicken! game.
So what is the entire game?

Hence, after rollback, the entire game is:

\[
\begin{array}{c|cc}
& \text{Don’t} & \text{Invest} \\
\hline
\text{Don’t} & 0, 0 & 0, 14 \\
\text{Invest} & 14, 0 & -2, -2 \\
\end{array}
\]

A Chicken! game.

What if one could move first? — see later.
A Subgame

The second-stage simultaneous pricing game at the bottom right of Page 3 is a complete game on its own.
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It is also a subgame of the full game.
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A multi-move game has as many subgames as it has decision nodes.

Later: subgame perfect equilibrium (SPE) and N.E., and the importance of credible strategies.
Sequential then Simultaneous

What if GD has already Invested $10 b and CT knows it? Or GD has made a credible commitment to Invest?
Sequential then Simultaneous

What if GD has already Invested $10 b and CT knows it? Or GD has made a credible commitment to Invest? CT now has to decide whether to Invest; then the pricing decision is made.
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\begin{array}{c|c|c|c|c}
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\hline
\text{High} & 2, 2 & -10, 6 \\
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\end{array}
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∴ CT Doesn’t Invest, or may try to get in first.
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2. Changing the Players’ Order of Moves

No Change In Outcome
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When *both or all players have dominant strategies*, there is no change in outcomes. There is no advantage in moving first, or second.
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The PD POM: Years of prison (Ned, Kelly):
2. Changing the Players’ Order of Moves

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The PD POM: Years of prison (Ned, Kelly):

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<tr>
<th></th>
<th>D Spill</th>
<th>C Mum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ned</td>
<td>8, 8</td>
<td>0, 20</td>
</tr>
<tr>
<td>Kelly</td>
<td>20, 0</td>
<td>1, 1</td>
</tr>
<tr>
<td>C Mum</td>
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2. Changing the Players’ Order of Moves

No Change In Outcome

When *both or all players have dominant strategies*, there is no change in outcomes. There is no advantage in moving first, or second. See the PD below.

The PD POM: Years of prison (Ned, Kelly):

Fewer years in prison are better: D,D is the N.E. (8,8).
**PD: Ned moves first:**

```
+---------+     +---------+
| Ned     |     | Ned     |
|         |     |         |
| C Mum   |     | D Spill |
|         |     |         |
| Kelly   |     | Kelly   |
|         |     |         |
| C       | D   | C       | D     |
| 1, 1    | 20, 0| 0, 20   | 8, 8  |
```
PD: Ned moves first:
PD: Ned moves first:

- Ned
  - C Mum
    - Kelly
      - C
        - 1, 1
      - D
        - 20, 0
  - D Spill
    - Kelly
      - C
        - 0, 20
      - D
        - 8, 8
PD: Ned moves first:

- Ned
  - C Mum
    - Kelly
      - C
        - I, I
      - D
        - 20, 0
  - D Spill
    - Kelly
      - C
        - 0, 20
      - D
        - 8, 8
PD: Ned moves first:
**PD: Ned moves first:**

Again: the N.E. is D,D or Spill,Spill, as with the simultaneous game.
**PD: Ned moves first:**

![Game Tree Diagram]

Again: the N.E. is D,D or Spill,Spill, as with the simultaneous game.
(No difference if Kelly moves first.)
**PD: Ned moves first:**

Again: the N.E. is D,D or Spill,Spill, as with the simultaneous game.
(No difference if Kelly moves first.)

∴ No first- or second-mover advantage in a PD.
2a. First-Mover Advantage

See the Capacity Game in Lectures 2 & 4, and
### 2a. First-Mover Advantage

See the Capacity Game in Lectures 2 & 4, and here: Chicken!

<table>
<thead>
<tr>
<th></th>
<th>Veer</th>
<th>Straight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alien Veer</td>
<td>Blah, Blah</td>
<td>Chicken!, Winner</td>
</tr>
<tr>
<td>Alien Straight</td>
<td>Winner, Chicken!</td>
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2a. First-Mover Advantage

See the Capacity Game in Lectures 2 & 4, and Here: Chicken!

<table>
<thead>
<tr>
<th>Alien</th>
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</tr>
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2a. First-Mover Advantage

See the Capacity Game in Lectures 2 & 4, and Here: Chicken!
2a. **First-Mover Advantage**

See the Capacity Game in Lectures 2 & 4, and Here: **Chicken!**

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Two N.E., but no easy way to coordinate on one or the other.
**Chicken: Alien Moves First:**
How do we solve this?

```
Alien
  +---+---+
  |   |   |
  | Straight | Veer |
  +---+---+
   |   |   |
   | Bomber | Bomber |
   +---+---+
     |   |   |
     | Veer | Veer |
     +---+---+
       |   |   |
       | Str | Str |
       +---+---+
         |   |   |
         | Die, Die | W, Ch! |
         +---+---+
           |   |   |
           | Ch!, W | Blah, Blah |
           +---+---+
```
Chicken: Alien Moves First: How do we solve this?
**Chicken: Alien Moves First:**
How do we solve this?
**Chicken: Alien Moves First:**
*How do we solve this?*

![Diagram](image_url)
Chicken: Alien Moves First:
How do we solve this?
Chicken: Alien Moves First: How do we solve this?

No longer two N.E.: only one: First Mover — go Straight, Second Mover — Veer.
Chicken: Alien Moves First:
How do we solve this?

No longer two N.E.: only one: First Mover — go Straight,
Second Mover — Veer.
∴ a clear first-mover advantage.
Chicken: Alien Moves First:
How do we solve this?

No longer two N.E.: only one: First Mover — go Straight,
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∴ a clear first-mover advantage.
(So: how to commit to Straight credibly?)
2b. Second-Mover Advantage
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The Tennis game between Venus serving and Serena receiving from Lecture 2 has the POM:

<table>
<thead>
<tr>
<th></th>
<th>DL</th>
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<tbody>
<tr>
<td>Venus</td>
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<td></td>
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<tr>
<td>DL</td>
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<td>80</td>
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**Serena**
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**Serena**
2b. Second-Mover Advantage

The Tennis game between Venus serving and Serena receiving from Lecture 2 has the POM:

\[
\begin{array}{c|cc}
 & DL & CC \\
\hline
DL & 50 & 80 \\
CC & 90 & 20 \\
\end{array}
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<tr>
<td>CC</td>
<td>90</td>
<td>20</td>
</tr>
</tbody>
</table>

Serena

Whichever action Venus chooses with her serve, and whichever action receiver Serena chooses in her court coverage, one or the other will regret the combination:
2b. Second-Mover Advantage

The Tennis game between Venus serving and Serena receiving from Lecture 2 has the POM:

\[
\begin{array}{c|cc}
\text{Venus} & \text{DL} & \text{CC} \\
\hline
\text{DL} & 50 & 80 \\
\text{CC} & 90 & 20 \\
\end{array}
\]

Whichever action Venus chooses with her serve, and whichever action receiver Serena chooses in her court coverage, one or the other will regret the combination: \( \therefore \) no N.E. (in pure strategies).
Venus the server moves first
Venus the server moves first

But if receiving Serena can pick serving Venus’s choice in time:
Venus the server moves first

But if receiving Serena can pick serving Venus’s choice in time:
**Venus the server moves first**

But if receiving Serena can pick serving Venus’s choice in time:
**Venus the server moves first**

But if receiving Serena can pick serving Venus’s choice in time:
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But if receiving Serena can pick serving Venus’s choice in time:
**Venus the server moves first**

But if receiving Serena can pick serving Venus’s choice in time:
**Venus the server moves first**

But if receiving Serena can pick serving Venus’s choice in time:

Serena has the second-mover advantage, and wins the shot half (50%) the time. Venus wins 50% too.
Serena the receiver moves first
Serena the receiver moves first

Or if Venus serving can pick Serena’s choice in time:
Serena the receiver moves first

Or if Venus serving can pick Serena’s choice in time:
Serena the receiver moves first

Or if Venus serving can pick Serena’s choice in time:
Serena the receiver moves first

Or if Venus serving can pick Serena’s choice in time:
Serena the receiver moves first

Or if Venus serving can pick Serena’s choice in time:

![Diagram](image)
Serena the receiver moves first

Or if Venus serving can pick Serena’s choice in time:

Receiver:

Serena

Server:

Venus

Venus

CC

DL

CC

DL

80, 20

20, 80

10, 90

50, 50
Serena the receiver moves first

Or if Venus serving can pick Serena’s choice in time:

Venus has the second-mover advantage, and wins the shot most (80%) of the time.
2c. Both Players May Do Better
2c. **Both Players May Do Better**

The Macro game from Lecture 2 has the POM:

<table>
<thead>
<tr>
<th>Gov't</th>
<th>Balanced</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3, 4</td>
<td>1, 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4, 1</td>
<td>2, 2</td>
<td></td>
</tr>
</tbody>
</table>
### 2c. Both Players May Do Better

The Macro game from Lecture 2 has the POM:

<table>
<thead>
<tr>
<th>Gov’t</th>
<th>Balanced</th>
<th>Deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>3, 4</td>
<td>4, 1</td>
</tr>
<tr>
<td>High</td>
<td>1, 3</td>
<td>2, 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RBA</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>3, 4</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>1, 3</td>
<td>2, 2</td>
</tr>
</tbody>
</table>
2c. *Both Players May Do Better*

The Macro game from Lecture 2 has the POM:

<table>
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<th>Gov't</th>
<th>Balanced</th>
<th>RBA</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deficit</td>
<td>3, 4</td>
<td>1, 3</td>
<td>2, 2</td>
</tr>
<tr>
<td></td>
<td>4, 1</td>
<td></td>
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</table>
2c. *Both Players May Do Better*

The Macro game from Lecture 2 has the POM:

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<tr>
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<td>Low</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3, 4</td>
<td></td>
<td>1, 3</td>
</tr>
<tr>
<td></td>
<td>4, 1</td>
<td>←</td>
<td>2, 2</td>
</tr>
</tbody>
</table>
2c. Both Players May Do Better

The Macro game from Lecture 2 has the POM:

\[
\begin{array}{c|cc}
 & \text{Low} & \text{High} \\
\hline
\text{Balanced} & 3, 4 & 1, 3 \\
\text{Gov't} & 4, 1 & 2, 2 \\
\text{Deficit} & & \\
\end{array}
\]
2c. *Both Players May Do Better*

The Macro game from Lecture 2 has the POM:

<table>
<thead>
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<th>Balanced</th>
<th>3, 4</th>
<th>1, 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deficit</td>
<td>Low</td>
<td>4, 1</td>
<td>2, 2</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>1, 3</td>
<td>2, 2</td>
</tr>
</tbody>
</table>

The Gov’t has a dominant strategy of Deficit, which the RBA knows, and ∴ chooses High interest rates.
2c. *Both Players May Do Better*

The Macro game from Lecture 2 has the POM:

<table>
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<th>RBA</th>
<th>High</th>
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<tr>
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<td>3, 4</td>
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</tr>
<tr>
<td></td>
<td>High</td>
<td>4, 1</td>
<td>2, 2</td>
</tr>
</tbody>
</table>

The Gov’t has a dominant strategy of Deficit, which the RBA knows, and \( \therefore \) chooses High interest rates.

Yields payoffs of 2 (the second-worst outcome) for each.
But What If The RBA Moves First?

The game tree (4 = best, 1 = worst):

```
              RBA
             /    \
            /      \
           High    Low
          /      /  \
Gov't    Gov't     \
Deficit Bal Def Balanced
  2, 2   3, 1   1, 4  4, 3
```
But What If The RBA Moves First?

The game tree (4 = best, 1 = worst):

RBA
/   \
|    |
/    \
High  Low
/ \   / \ 
Gov’t Gov’t
/ / \
Deficit Bal Def Balanced
/ \
2, 2 3, 1 1, 4 4, 3
But What If The RBA Moves First?

The game tree (4 = best, 1 = worst):
But What If The RBA Moves First?

The game tree (4 = best, 1 = worst):

RBA

Gov’t

Government

Low

High

Deficit

Balanced

3, 1

2, 2

4, 3

1, 4

X

X

X
But What If The RBA Moves First?

The game tree (4 = best, 1 = worst):

- **RBA**
  - High
    - Deficit
      - 2, 2
    - Bal
      - 3, 1
  - Low
    - Deficit
      - 1, 4
    - Balanced
      - 4, 3
But What If The RBA Moves First?

The game tree (4 = best, 1 = worst):

The RBA knows that the Gov’t will go into Deficit, come what may, and so chooses High interest rates, yielding the RBA 2 instead of 1.
But What If The RBA Moves First?

The game tree (4 = best, 1 = worst):

The RBA knows that the Gov’t will go into Deficit, come what may, and so chooses High interest rates, yielding the RBA 2 instead of 1. As in the simultaneous game.
But if the Gov’t moves first:
But if the Gov’t moves first:

The game tree is:
But if the Gov’t moves first:

The game tree is:
But if the Gov’t moves first:

The game tree is:
But if the Gov’t moves first:

The game tree is:

```
  Gov’t
   /   \
  /     \
/       /
RBA   RBA
  /   /  \
/     /   \
Low  Low

2, 2  4, 1  1, 3  3, 4
```

Deficit  Balanced
But if the Gov’t moves first:

The game tree is:
But if the Gov’t moves first:

The game tree is:

The chosen combination of strategies is \{Balanced, Low\}:
But if the Gov’t moves first:

The game tree is:

The chosen combination of strategies is \{Balanced, Low\}: this is the Rollback Equilibrium (R.E.), and, surprisingly, yields a better outcome for both players than does \{Deficit, High\}.
How?

But Balanced is a *dominated* strategy for the Gov’t:
**How?**

But Balanced is a *dominated* strategy for the Gov’t: how is it part of the R.E. in the sequential-move game in which the Gov’t moves first?
How?

But Balanced is a *dominated* strategy for the Gov’t: how is it part of the R.E. in the sequential-move game in which the Gov’t moves first?

The Gov’t knows that:

*If it*
How?

But Balanced is a \textit{dominated} strategy for the Gov’t: how is it part of the R.E. in the sequential-move game in which the Gov’t moves first?

The Gov’t knows that:
\begin{itemize}
  \item if it chooses Deficit, the RBA will choose High; and
  \item if it
\end{itemize}
**How?**

But Balanced is a *dominated* strategy for the Gov’t: how is it part of the R.E. in the sequential-move game in which the Gov’t moves first?

The Gov’t knows that:

- if it chooses Deficit, the RBA will choose High; and
- if it chooses Balanced, the RBA will choose Low.
How?

But Balanced is a *dominated* strategy for the Gov’t: how is it part of the R.E. in the sequential-move game in which the Gov’t moves first?

The Gov’t knows that:

- if it chooses Deficit, the RBA will choose High; and
- if it chooses Balanced, the RBA will choose Low.

The Gov’t prefers \{Balanced, Low\} to \{Deficit, High\} — all those mortgages! — so Gov’t gets its second-best outcome, and the RBA its best outcome.
How?

But Balanced is a dominated strategy for the Gov’t: how is it part of the R.E. in the sequential-move game in which the Gov’t moves first?

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The Gov’t must know that the RBA has the flexibility to respond (with Low).
How?

But Balanced is a *dominated* strategy for the Gov’t: how is it part of the R.E. in the sequential-move game in which the Gov’t moves first?

The Gov’t knows that:
- if it chooses Deficit, the RBA will choose High; and
- if it chooses Balanced, the RBA will choose Low.

The Gov’t prefers \{Balanced, Low\} to \{Deficit, High\} — all those mortgages! — so Gov’t gets its second-best outcome, and the RBA its best outcome.

The Gov’t must know that the RBA has the *flexibility* to respond (with Low).

*(See also the Capacity Game Revisited, Lecture 4, pp. 6–8, where a dominated move — Large — is also chosen in R.E.)*
3. Trees for Simultaneous Games
3. Trees for Simultaneous Games

The simultaneous Tennis game as a tree:

```
Venus
  /   \  \
CC    DL

Serena     Serena
|       |
|   CC   |
|       |
20, 80 90, 10 80, 20 50, 50
```
3. Trees for Simultaneous Games

The simultaneous Tennis game as a tree:

![Diagram of the tennis game as a tree.]

The dotted box is an *Information Set:* Serena can’t tell which of the two decision nodes she’s at since she doesn’t (yet) know how Venus will serve (CC or DL) and so she cannot do CC at one and DL at the other — *there can only be 1 action per Info Set.* (DSkR p.194)
Information Sets

Serena must choose without knowing what Venus has picked: Serena doesn’t know which decision node she’s at.

Use a dotted box around the relevant decision nodes to indicate her lack of specific information.
Information Sets

Serena must choose without knowing what Venus has picked: Serena doesn’t know which decision node she’s at.

Use a dotted box around the relevant decision nodes to indicate her lack of specific information.

Information Sets could also be called “ignorance sets,” since the player doesn’t know what’s happened, or where she is exactly in the game tree.
**Information Sets**

Serena must choose without knowing what Venus has picked: Serena doesn’t know which decision node she’s at.

Use a dotted box around the relevant decision nodes to indicate her *lack of specific information*.

Information Sets could also be called “ignorance sets,” since the player doesn’t know what’s happened, or where she is exactly in the game tree. An alternative convention is to join the decision nodes with a dotted line:
Information Sets

Serena must choose without knowing what Venus has picked: Serena doesn’t know which decision node she’s at.

Use a dotted box around the relevant decision nodes to indicate her *lack of specific information*.

Information Sets could also be called “ignorance sets,” since the player doesn’t know what’s happened, or where she is exactly in the game tree. An alternative convention is to join the decision nodes with a dotted line:

```
Serena --Serena
```
**Information Sets**

Serena must choose without knowing what Venus has picked: Serena doesn’t know which decision node she’s at.

Use a dotted box around the relevant decision nodes to indicate her *lack of specific information*.

Information Sets could also be called “ignorance sets,” since the player doesn’t know what’s happened, or where she is exactly in the game tree. An alternative convention is to join the decision nodes with a dotted line:

So a *strategy*: a complete plan of action, specifying the move a player would make at each Information Set (instead of each decision node) when the rules of the game specify that it is her turn to move.
4. Matrices for Sequential Games
4. Matrices for Sequential Games

Use the Macro game tree, where the Gov’t moves first. (Instructions for *delegation* of RBA action.)
4. Matrices for Sequential Games

Use the Macro game tree, where the Gov’t moves first.
(Instructions for delegation of RBA action.)

Four possible strategies for the RBA:
4. Matrices for Sequential Games

Use the Macro game tree, where the Gov’t moves first. (Instructions for delegation of RBA action.)

Four possible strategies for the RBA:
first, ① & ③: always H;
4. Matrices for Sequential Games

Use the Macro game tree, where the Gov’t moves first. (Instructions for delegation of RBA action.)

Four possible strategies for the RBA: first, ① & ③: always H; second, ② & ④: always L;
4. Matrices for Sequential Games

Use the Macro game tree, where the Gov’t moves first. (Instructions for *delegation* of RBA action.)

![Game Tree Diagram]

Four possible strategies for the RBA:
- first, ① & ③: always H;
- second, ② & ④: always L;
- third, ④ & ①: L if B & H if D; and
4. Matrices for Sequential Games

Use the Macro game tree, where the Gov’t moves first. (Instructions for delegation of RBA action.)

**Gov’t**
- Deficit
  - RBA
    - High
    - Low
- Balanced
  - RBA
    - High
    - Low

Four possible strategies for the RBA:
- first, ① & ③: always H; second, ② & ④: always L; third, ④ & ①: L if B & H if D; and fourth, ③ & ②: H if B & L if D.
As a payoff matrix:
As a payoff matrix:

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>B</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov’t</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<th>B</th>
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</thead>
<tbody>
<tr>
<td>Bal</td>
<td>L if B &amp; H if D</td>
<td>H if B &amp; L if D</td>
<td>L always</td>
<td>H always</td>
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<tr>
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<td></td>
<td></td>
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</table>
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<table>
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<tr>
<th></th>
<th>L if B &amp; H if D</th>
<th>H if B &amp; L if D</th>
<th>L always</th>
<th>H always</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>Bal</td>
<td>3, 4</td>
<td>1, 3</td>
<td>3, 4</td>
<td>1, 3</td>
</tr>
<tr>
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<td>2, 2</td>
<td>4, 1</td>
<td>4, 1</td>
<td>2, 2</td>
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</tbody>
</table>

*Note: R, B, A represent different strategies.*
### As a payoff matrix:

<table>
<thead>
<tr>
<th></th>
<th>L if B &amp; H if D</th>
<th>H if B &amp; L if D</th>
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**R**

**B**

**A**
As a payoff matrix:

<table>
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4: best; 1: worst

Gov’t’s possible strategies: Balanced or Deficit.
As a payoff matrix:

Gov’t’s possible strategies: Balanced or Deficit.

RBA has four possible strategies: always High; always Low; Low if Balanced and High if Deficit (L if B & H if D); High if Balanced and Low if Deficit (H if B & L if D).
As a payoff matrix:

Gov’t’s possible strategies: Balanced or Deficit.

RBA has four possible strategies: always High; always Low; Low if Balanced and High if Deficit (L if B & H if D); High if Balanced and Low if Deficit (H if B & L if D).

The last two columns are as if the game were simultaneous, but in the first two columns RBA’s decision depends on Gov’t’s.
Using arrow or cell-by-cell, find two NE:

1.
Using arrow or cell-by-cell, find two NE:

1. {Balanced, L if B & H if D} with payoffs (3,4), found by rollback on page 17 above, and

2.
Using arrow or cell-by-cell, find two NE:

1. \{Balanced, L if B & H if D\} with payoffs (3,4), found by rollback on page 17 above, and
2. \{Deficit, H always\} with payoffs (2,2).
Using arrow or cell-by-cell, find two NE:

1. {Balanced, L if B & H if D} with payoffs (3,4), found by rollback on page 17 above, and
2. {Deficit, H always} with payoffs (2,2).

Why are there two N.E. in this analysis of the sequential game but only one using rollback (R.E.) (p.16)?
Using arrow or cell-by-cell, find two NE:

1. \{Balanced, L if B & H if D\} with payoffs (3,4), found by rollback on page 17 above, and
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N.E. when: neither player gains from moving, given the other’s strategy.
Using arrow or cell-by-cell, find two NE:

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Why are there two N.E. in this analysis of the sequential game but only one using rollback (R.E.) (p.16)?

N.E. when: neither player gains from moving, given the other’s strategy. But R.E. asks: what would the player do, at each decision node (or Info Set)?
Using arrow or cell-by-cell, find two NE:

1. \{Balanced, L if B & H if D\} with payoffs (3,4), found by rollback on page 17 above, and

2. \{Deficit, H always\} with payoffs (2,2).

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“H always” is not optimal at one decision node:
Using arrow or cell-by-cell, find two NE:

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“H always” is not optimal at one decision node: if Gov’t chose Balanced, then RBA chooses Low (4 preferred to 2). So R.E. can’t include “H always”.

Using arrow or cell-by-cell, find two NE:

1. \{Balanced, L if B & H if D\} with payoffs (3,4), found by rollback on page 17 above, and
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But the N.E. doesn’t do the R.E. test: “H always” and Deficit are mutual best responses (“H always” equal to “L if B & H if D”).
Using arrow or cell-by-cell, find two NE:

1. \{Balanced, L if B & H if D\} with payoffs (3,4), found by rollback on page 17 above, and
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SPE, however, excludes non-credible strategies.
Subgame-Perfect Equilibrium SPE
Subgame-Perfect Equilibrium SPE

R.E. requires that all players make their best choices in every subgame of the larger game, whether or not along the equilibrium path down the tree.
Subgame-Perfect Equilibrium SPE

R.E. requires that all players make their best choices in *every* subgame of the larger game, whether or not along the equilibrium path down the tree.

Strategies are complete courses of action: for each and every decision node of the tree, on or off the equilibrium path.
**Subgame-Perfect Equilibrium SPE**

R.E. requires that all players make their best choices in *every* subgame of the larger game, whether or not along the equilibrium path down the tree.

Strategies are complete courses of action: for each and every decision node of the tree, on or off the equilibrium path.

The 2nd N.E. {Gov’t Deficit, then RBA High} is on the equilibrium path.
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∴ “H always” lacks *credibility*: it is not in the R.E. and ∴ is not in SPE.