

# Combining Simultaneous and Sequential Games

Let's mix and match our games, explore how games can change, and how we can model them:

1. **Simultaneous and Sequential Together.**
2. **Changing the Order of Moves:**
  - **First-Mover Advantage**
  - **Second-Mover Advantage**
  - ***Both-Mover Advantage***
3. **Trees for Simultaneous Games.**
4. **Matrices for Sequential Games.**
  - **Subgame Perfect Equilibrium**

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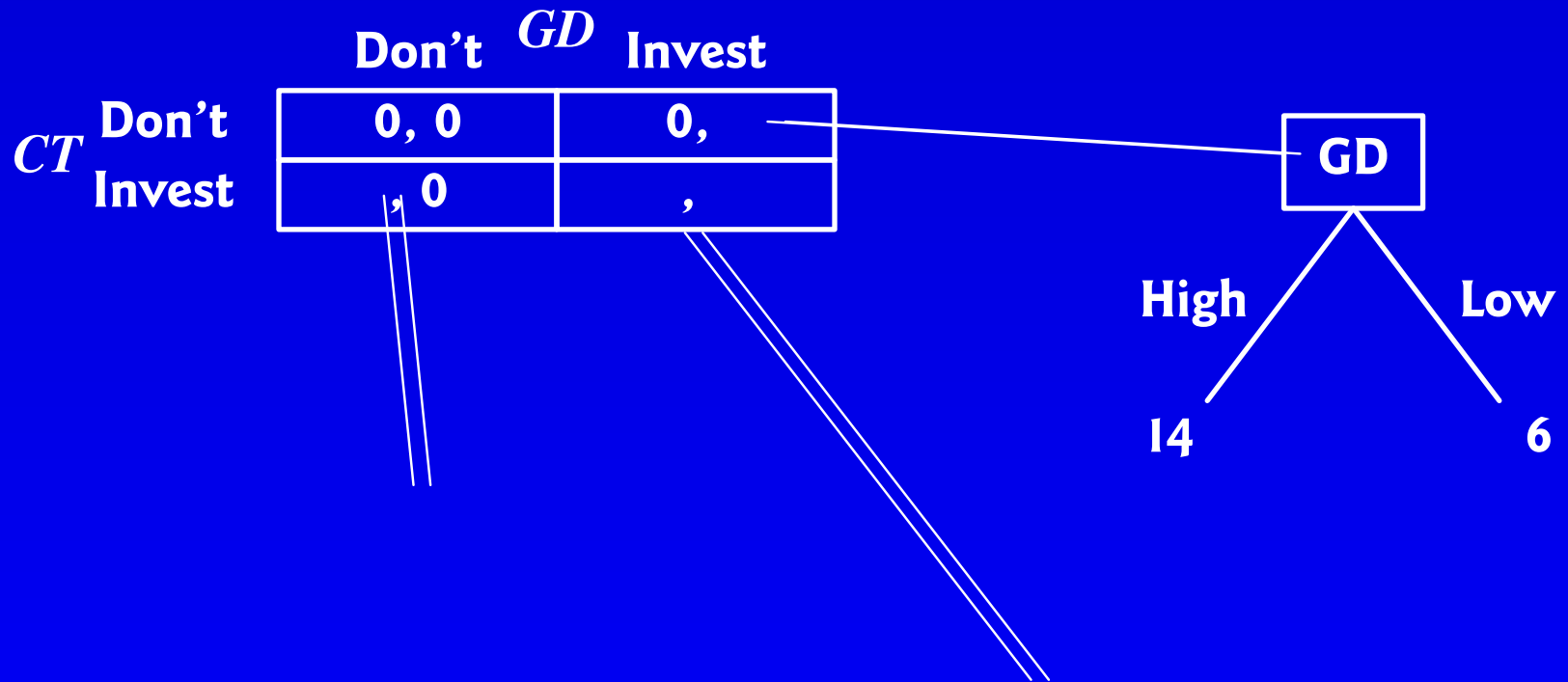
- **both price High (each 30 m cust, \$400/cust),**
- **both price Low (each 40 m cust, \$200/cust), or**
- **one High and the other Low (High gets 0 cust, Low gets 80 m cust, \$200/cust).**

## A Two-Stage Game (\$k)

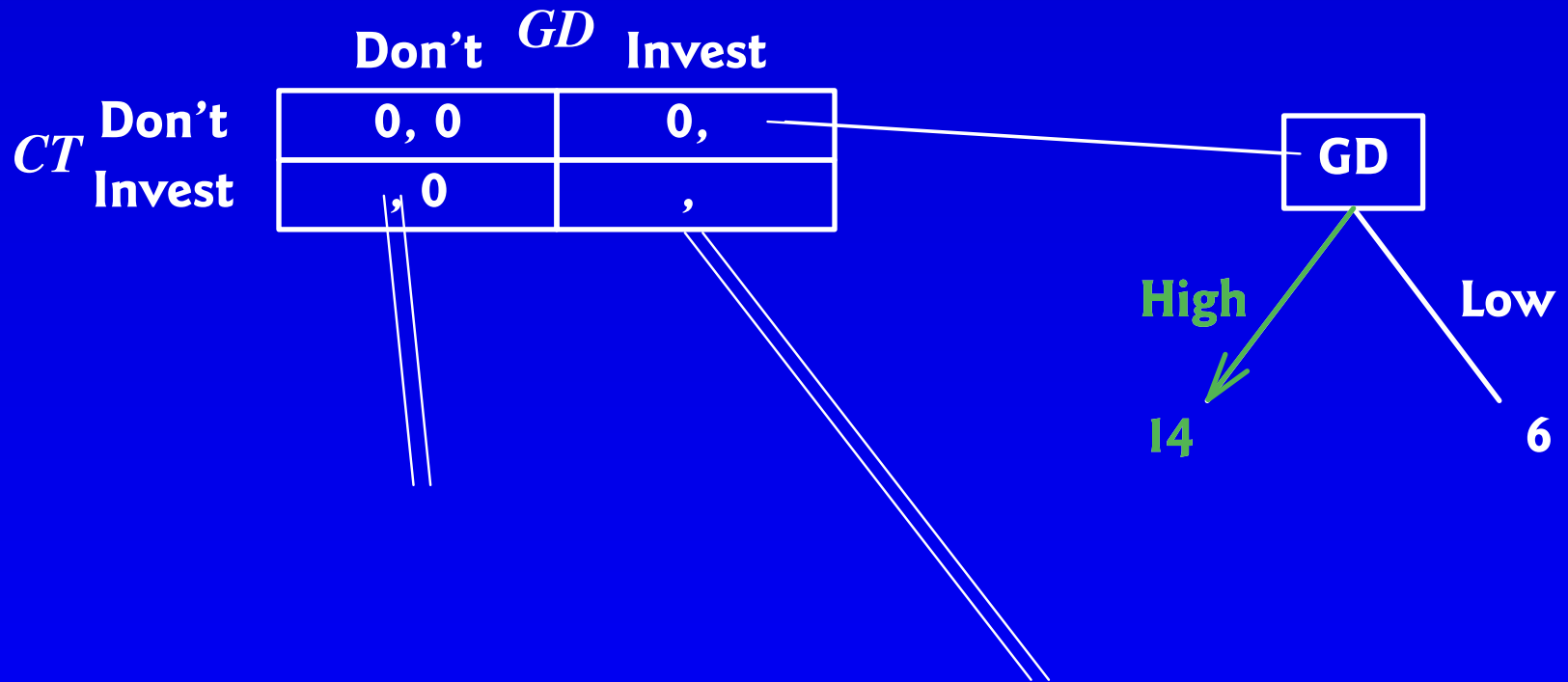
*CT*

		Don't	<i>GD</i>	Invest
Don't	Don't	0, 0		0, —
	Invest	, 0		,

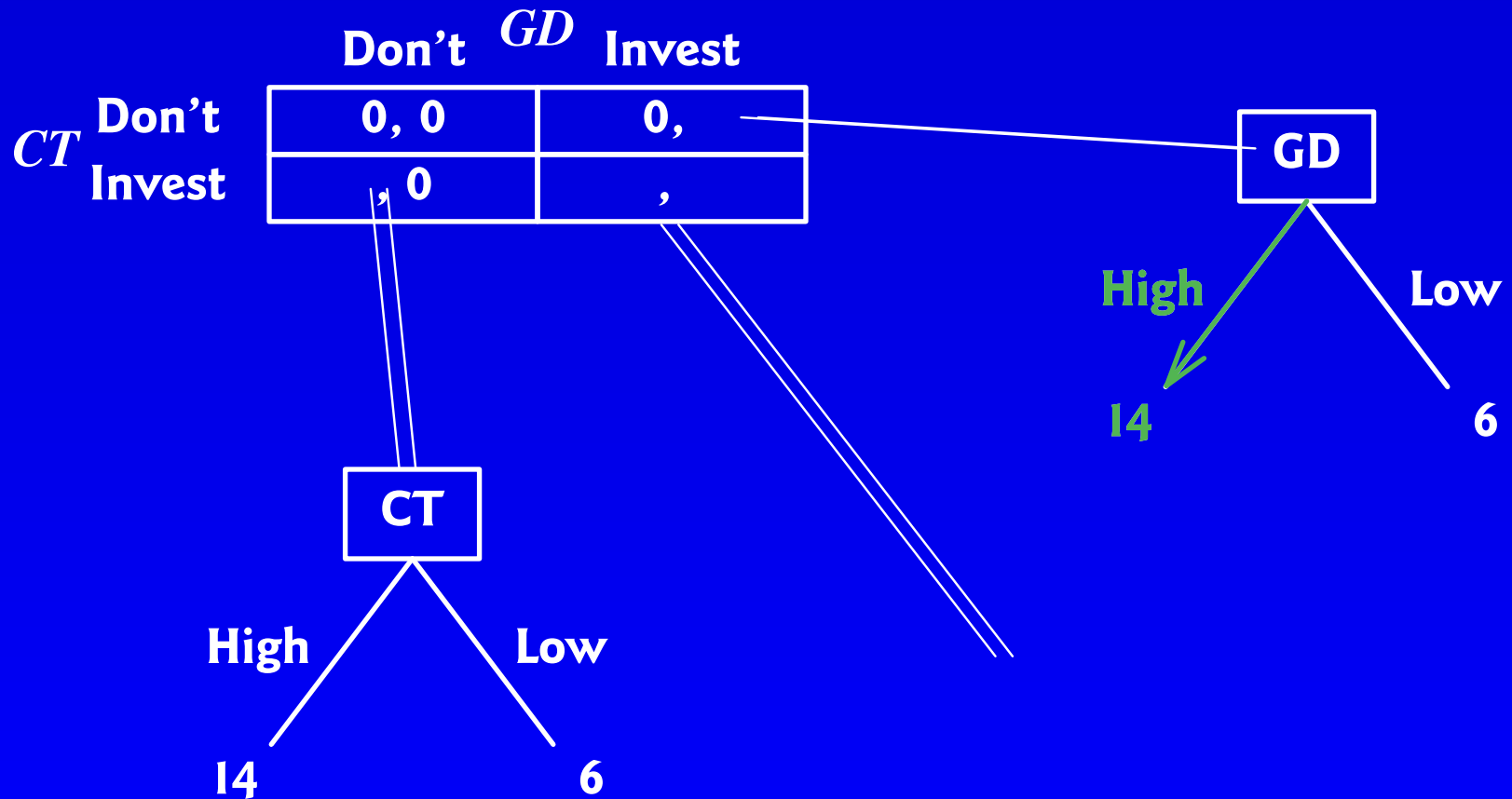
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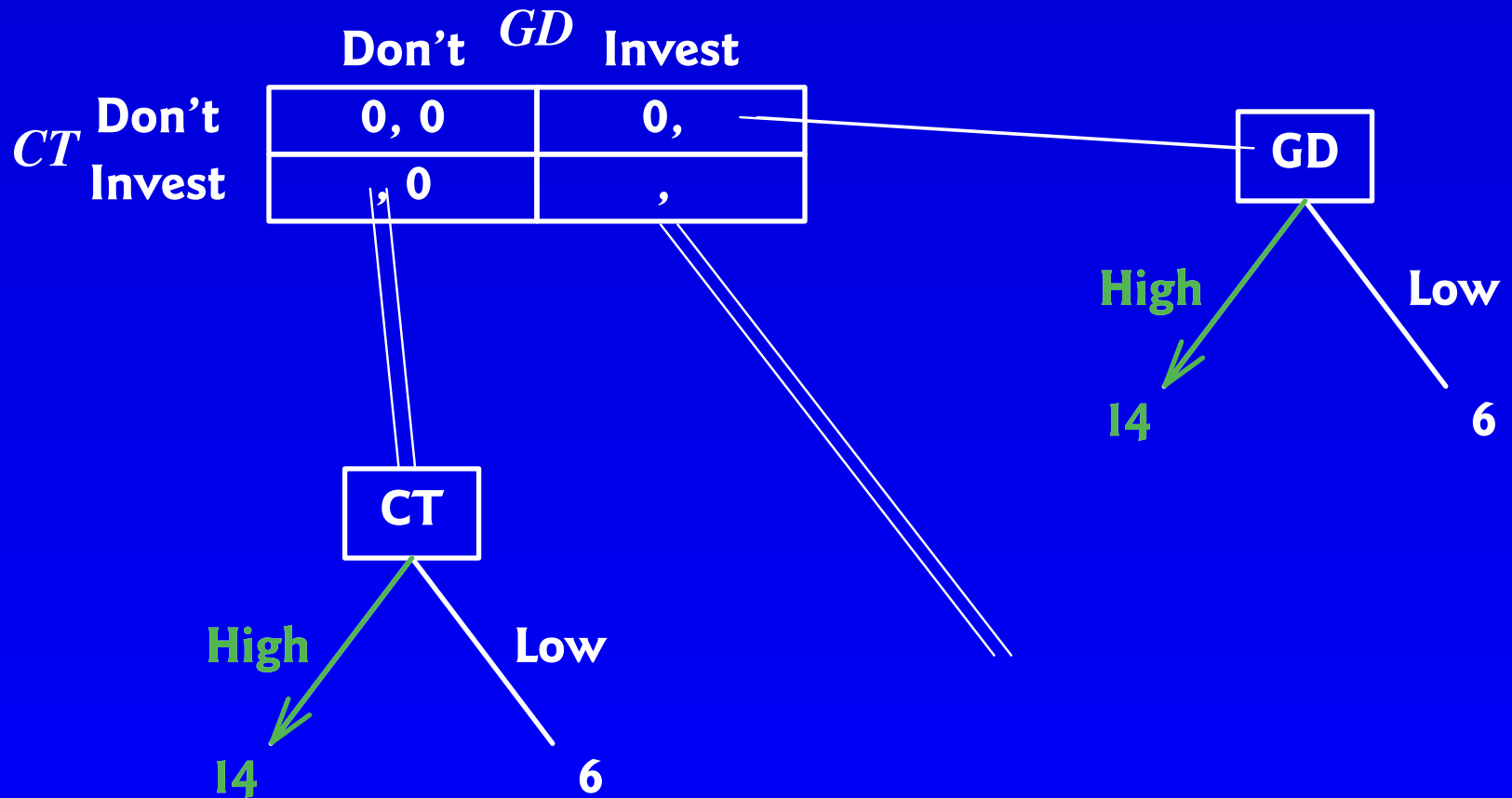


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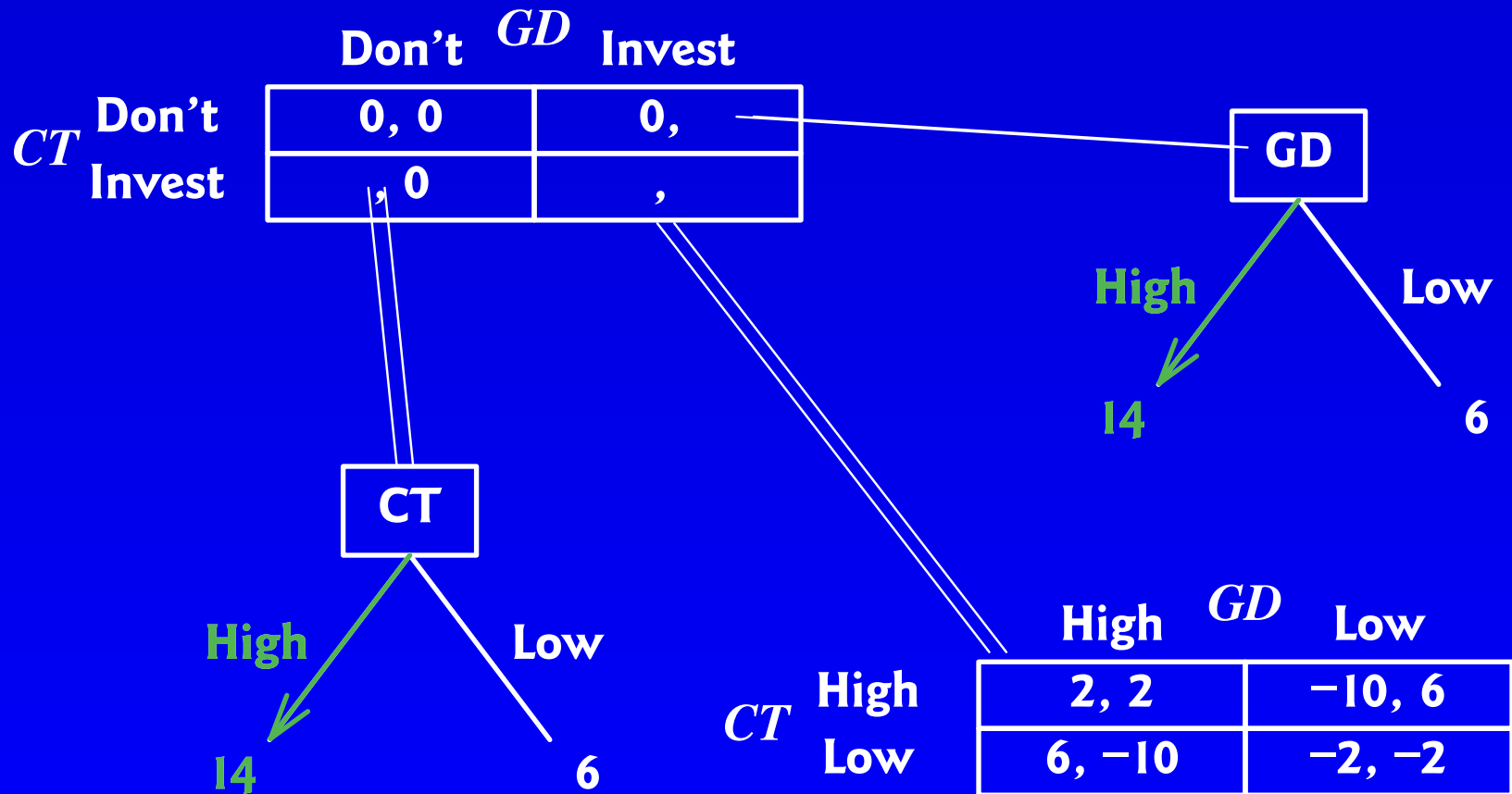




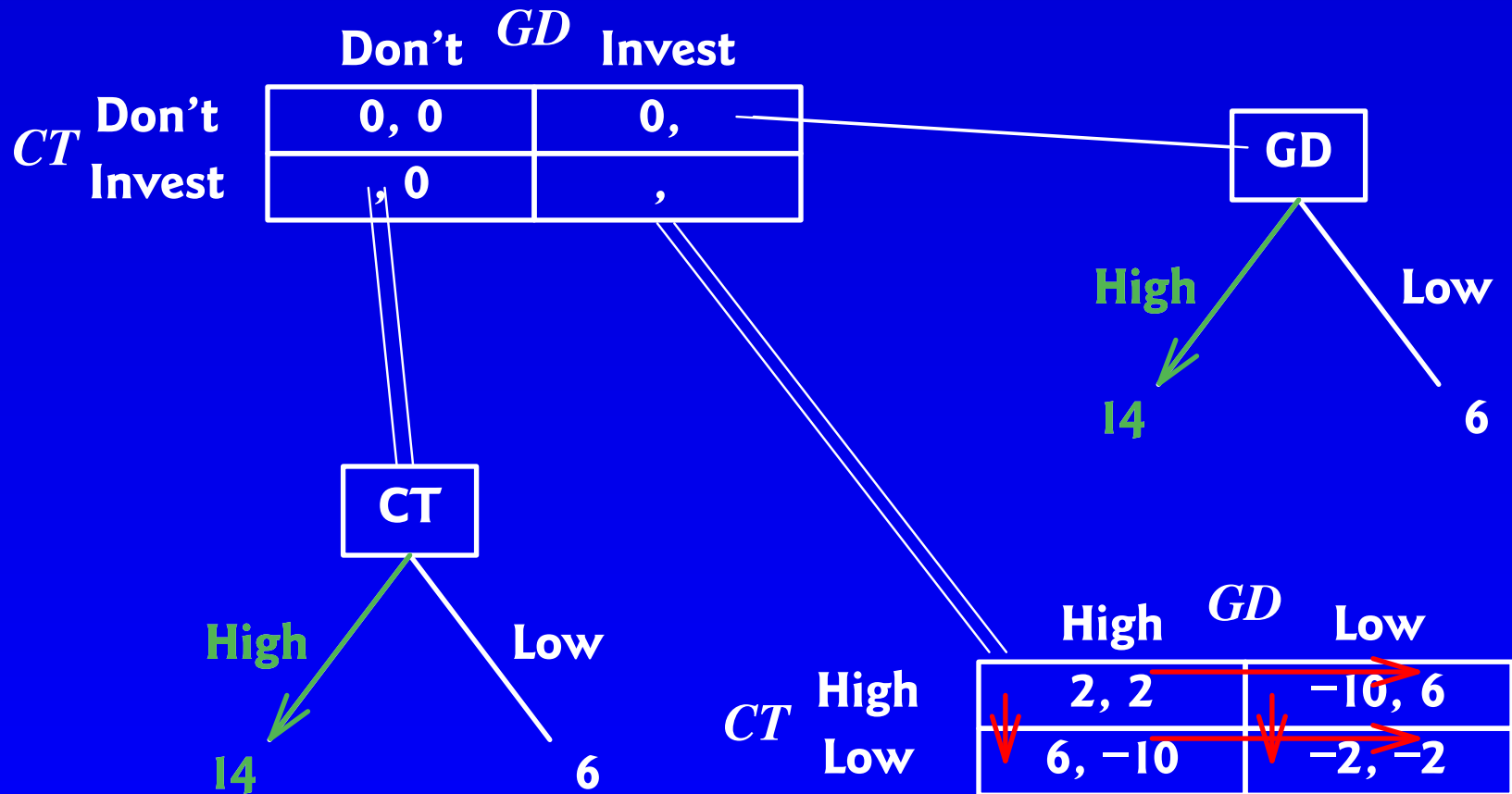
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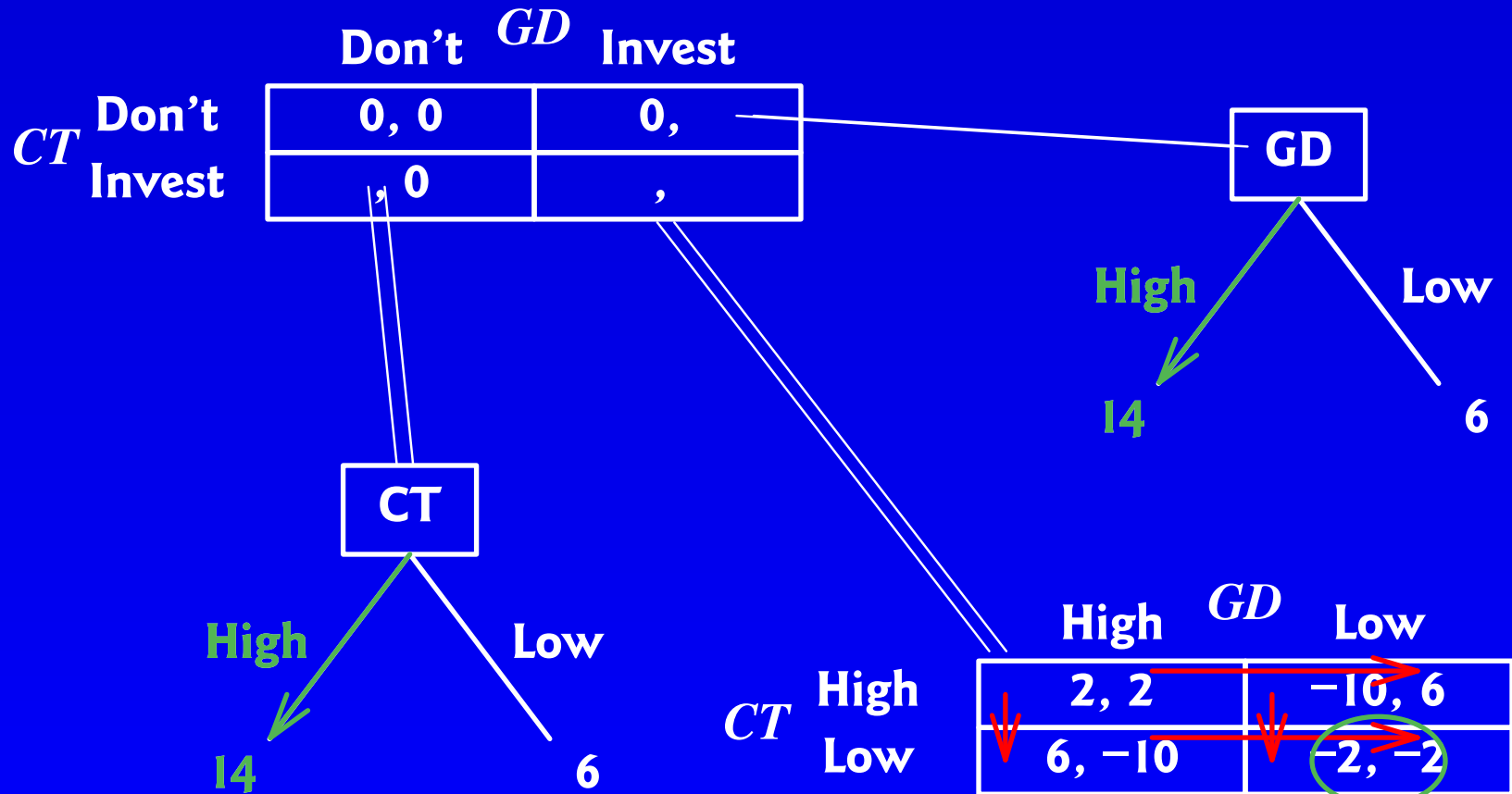
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## Rolling Back the Game

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**Etc.**

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Etc.

The second-stage pricing game is a PD: pricing Low is dominant.

## So what is the entire game?

Hence, after rollback, the entire game is:

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		Don't	Invest
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What if one could move first? — see later.

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**Later:** subgame perfect equilibrium (SPE) and N.E., and the importance of *credible strategies*.



## Sequential then Simultaneous

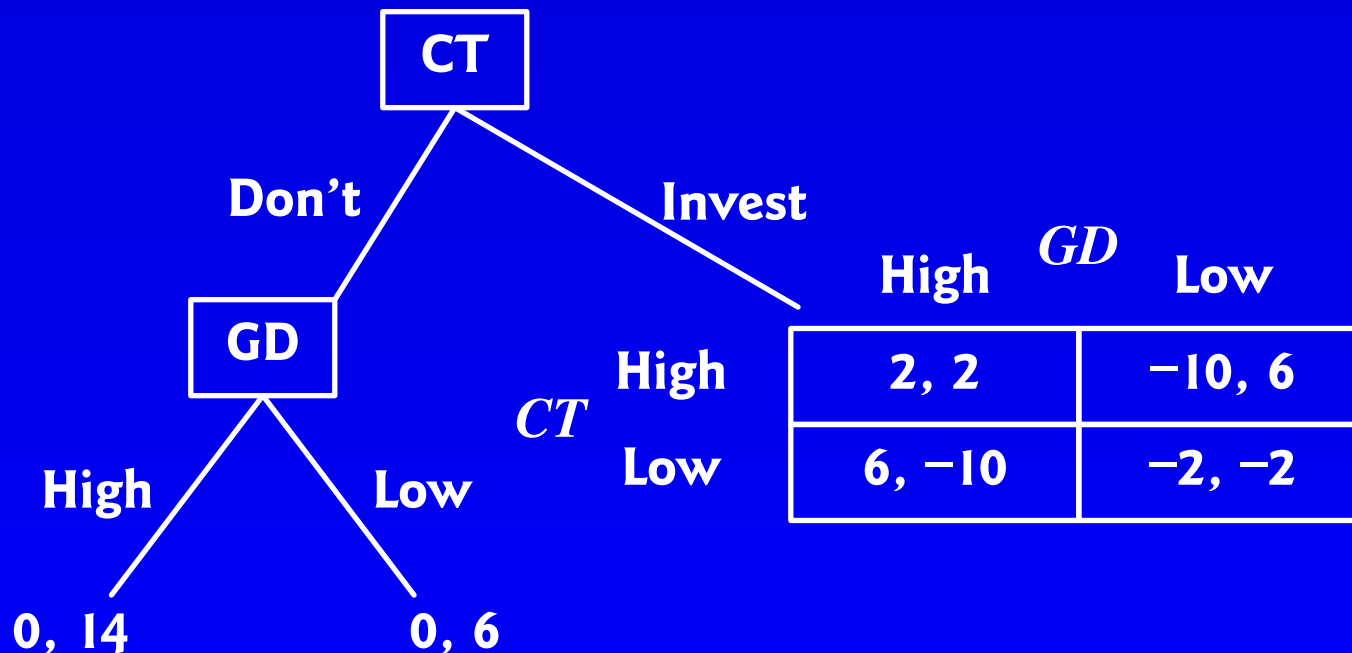
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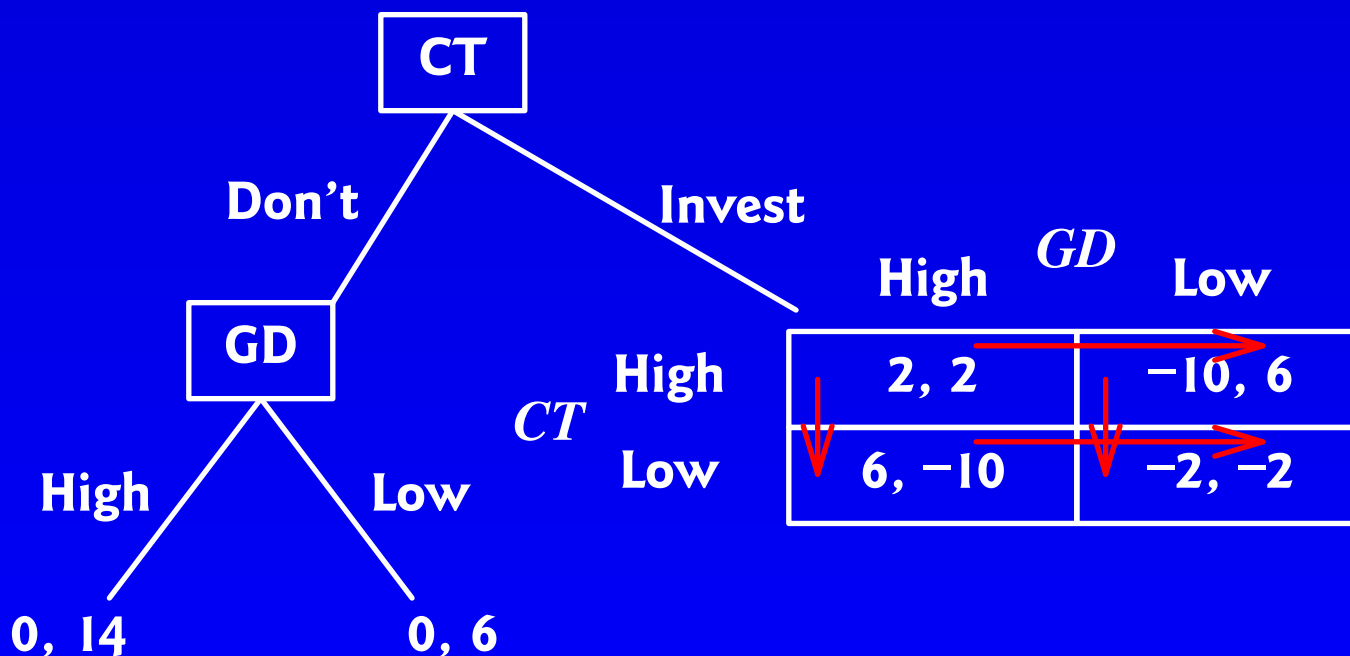
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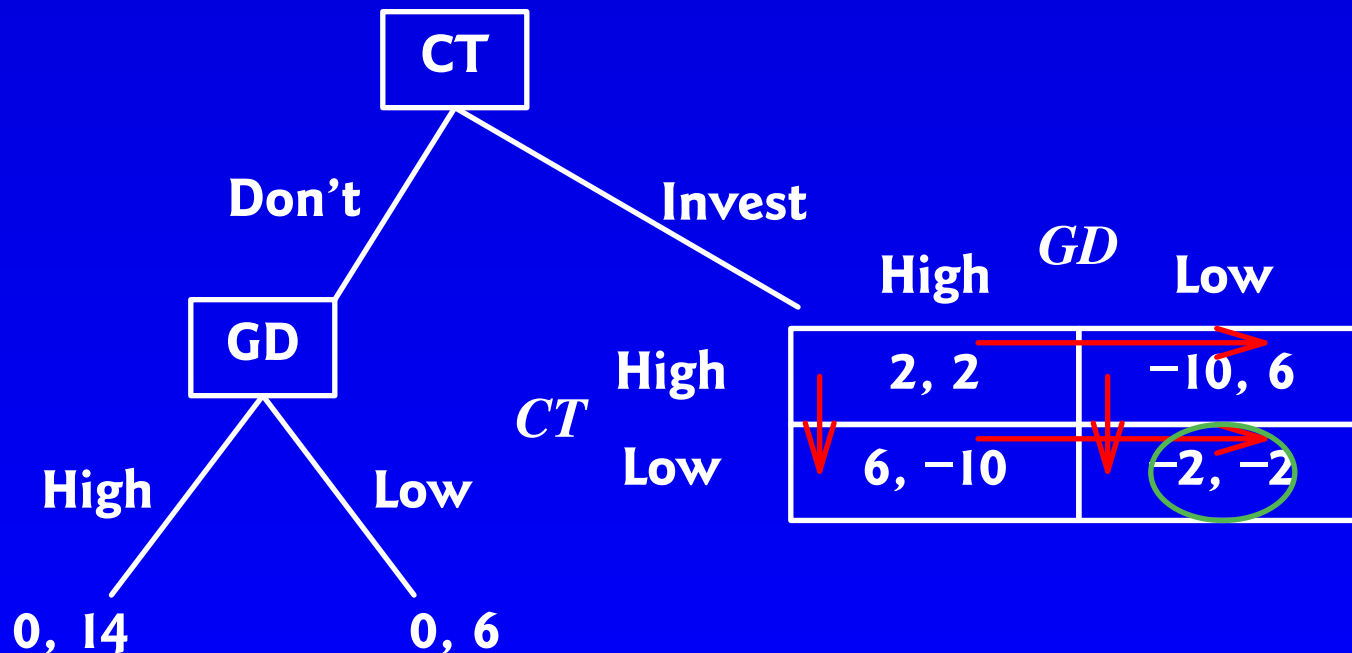
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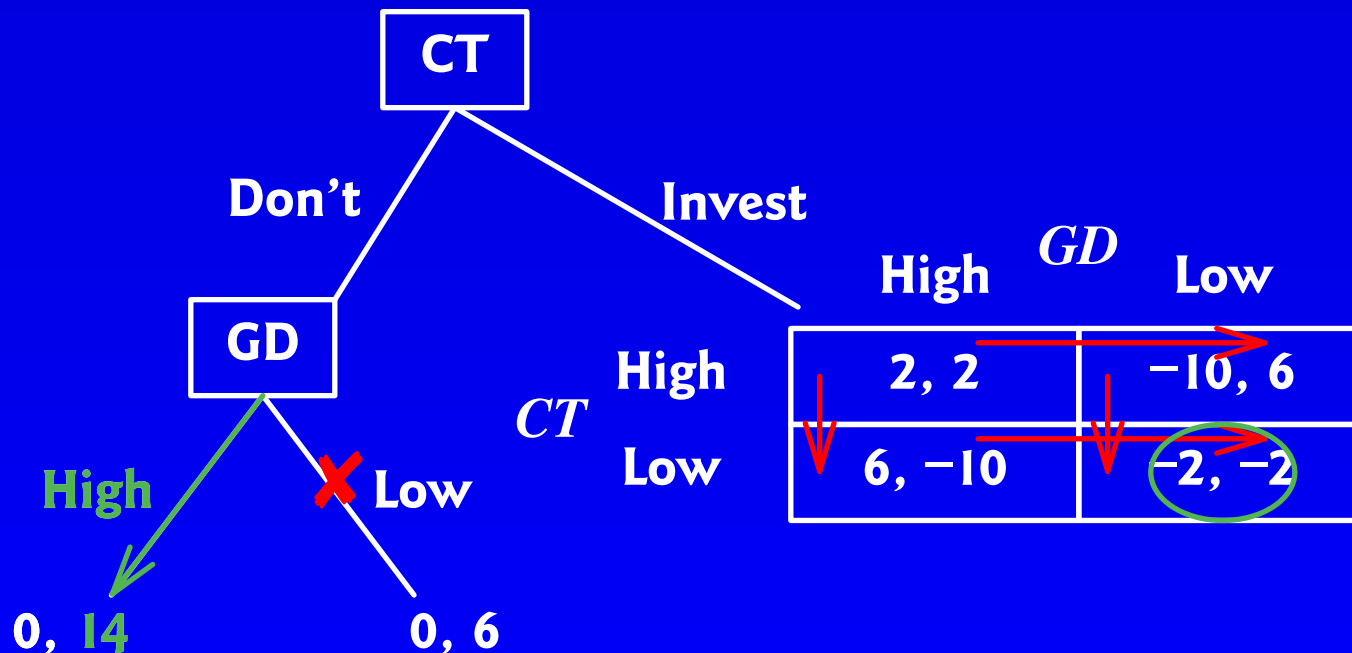
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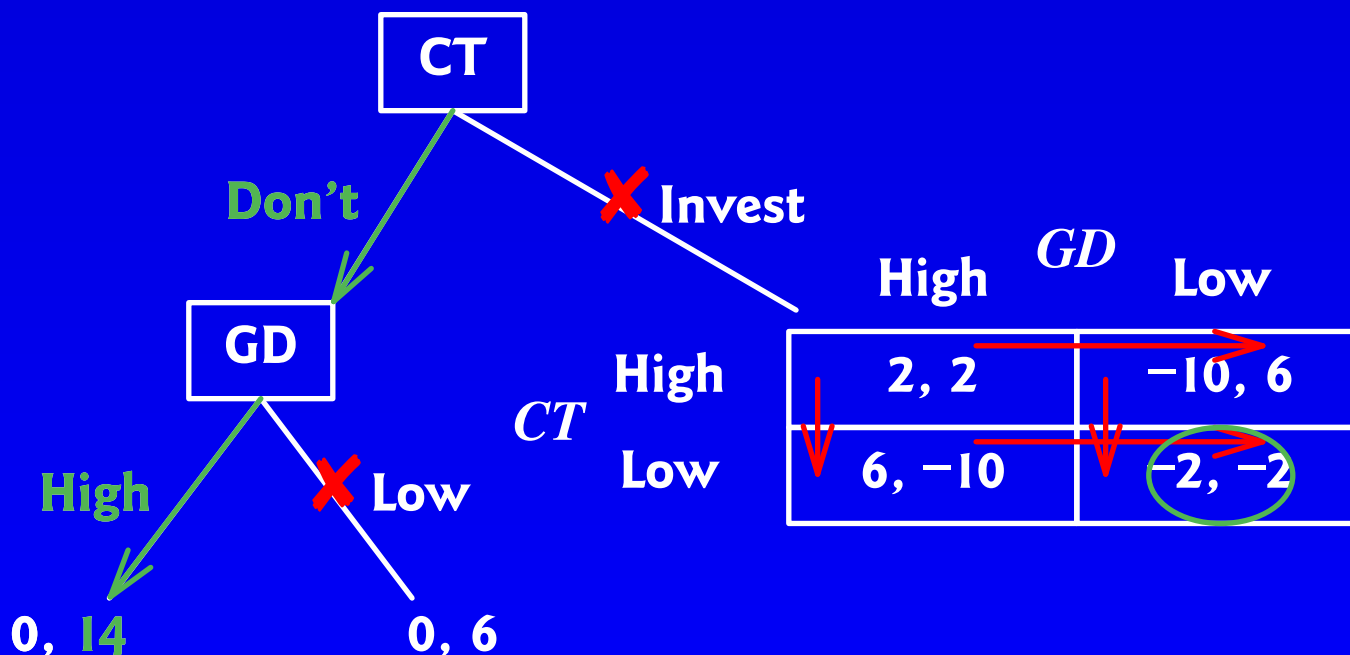
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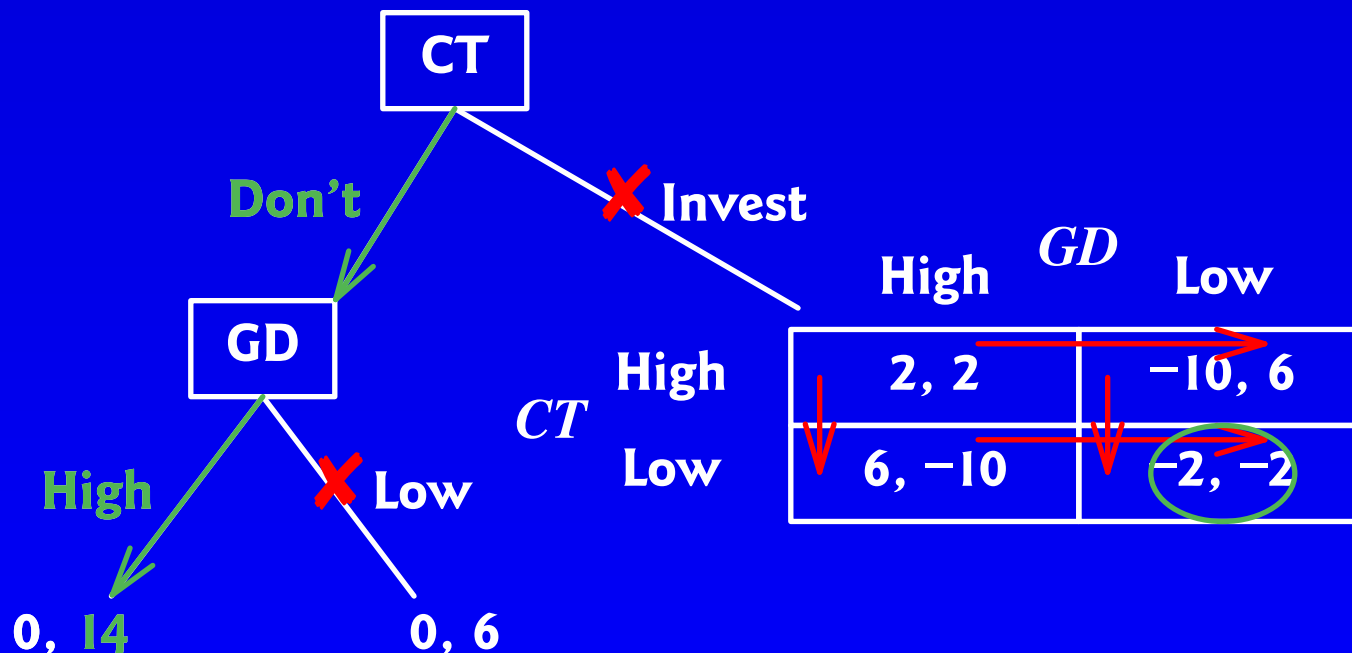
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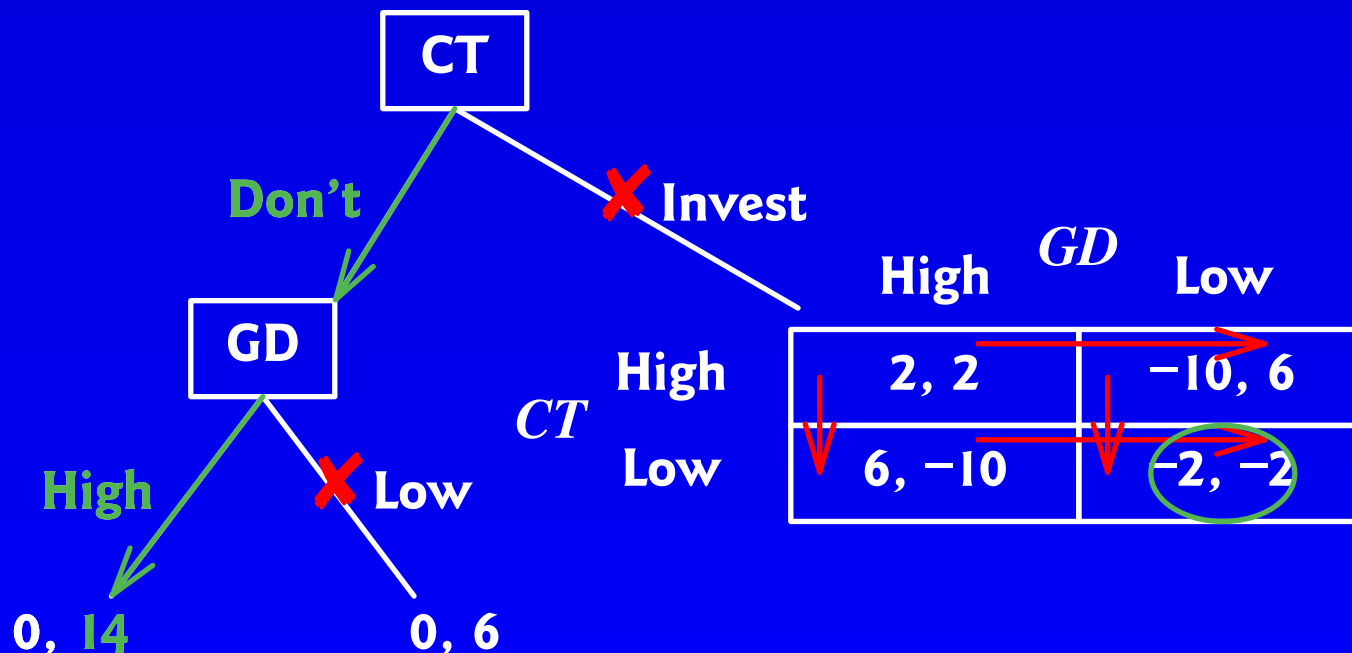


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 See Lect. 14: *strategic moves*, and exA: *Hold-Up*.

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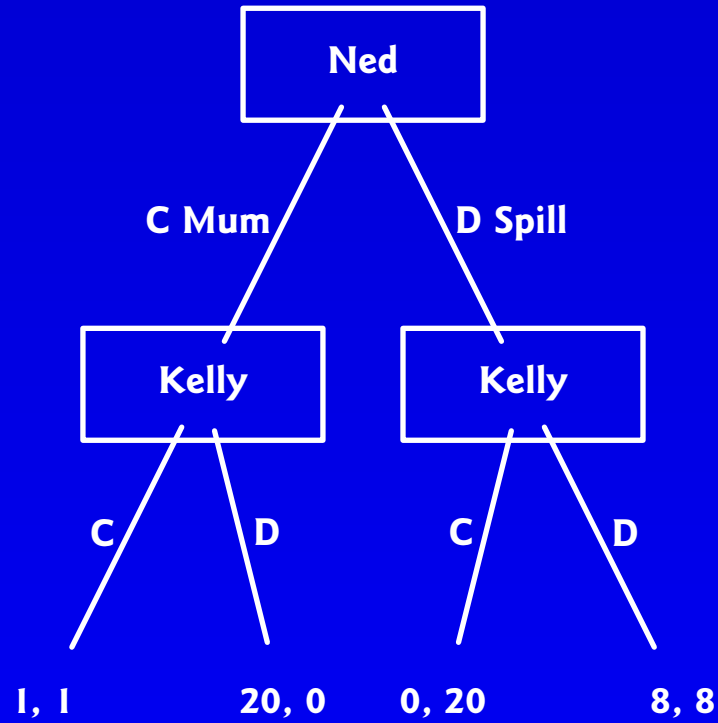
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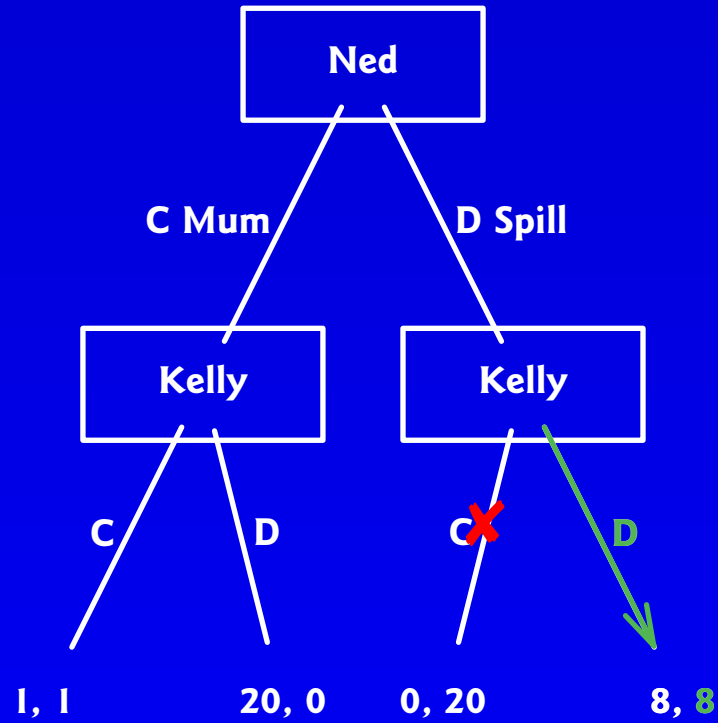
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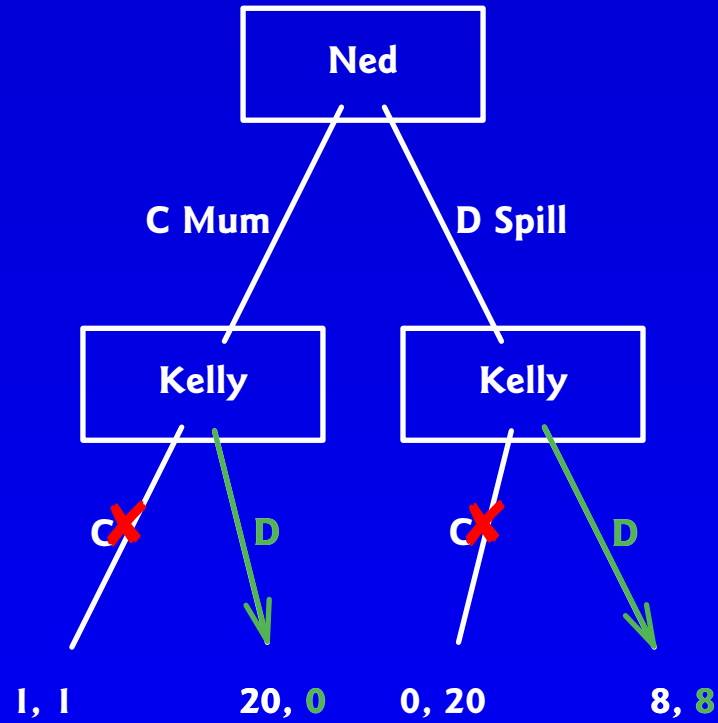
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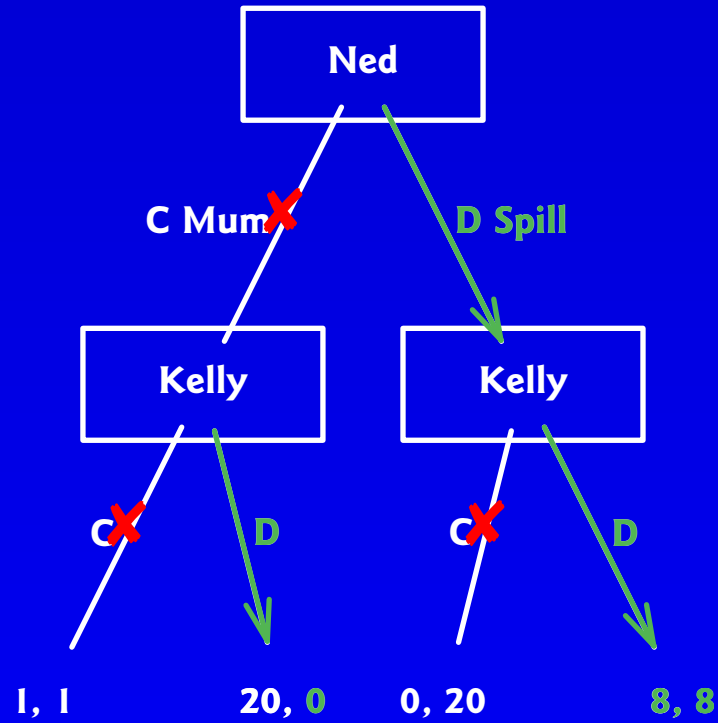
Fewer years in prison are better: D,D is the N.E. (8,8).

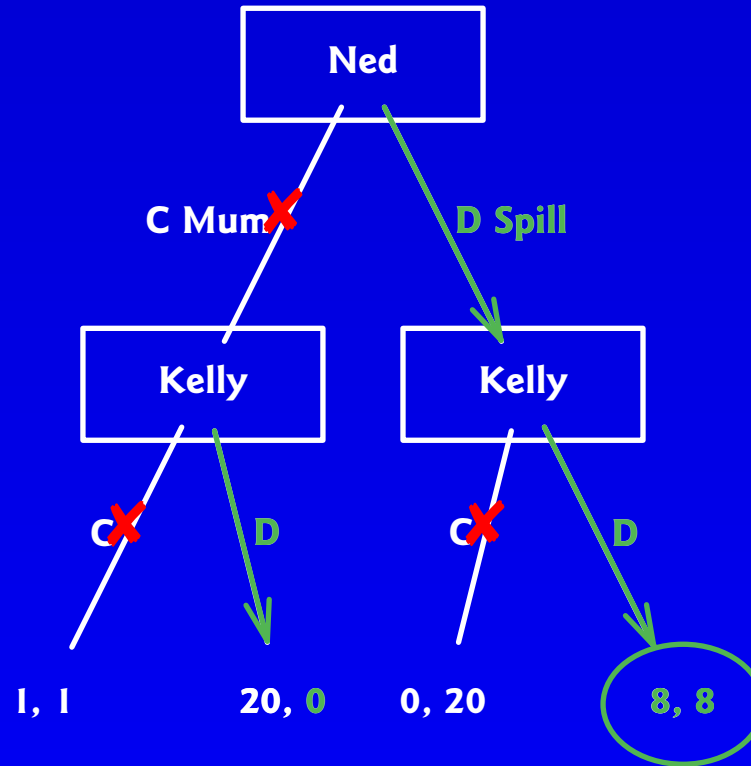


**PD: Ned moves first:**

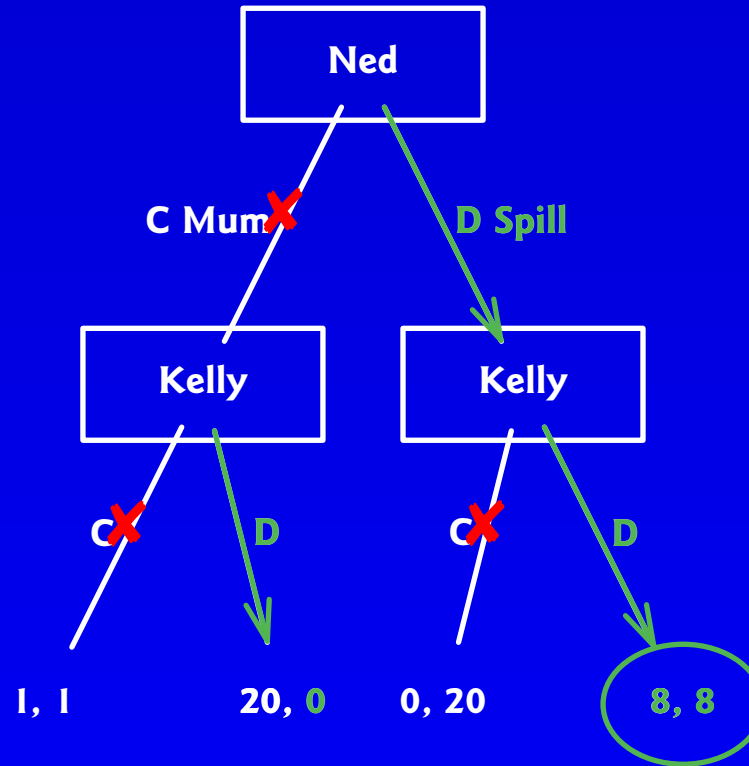
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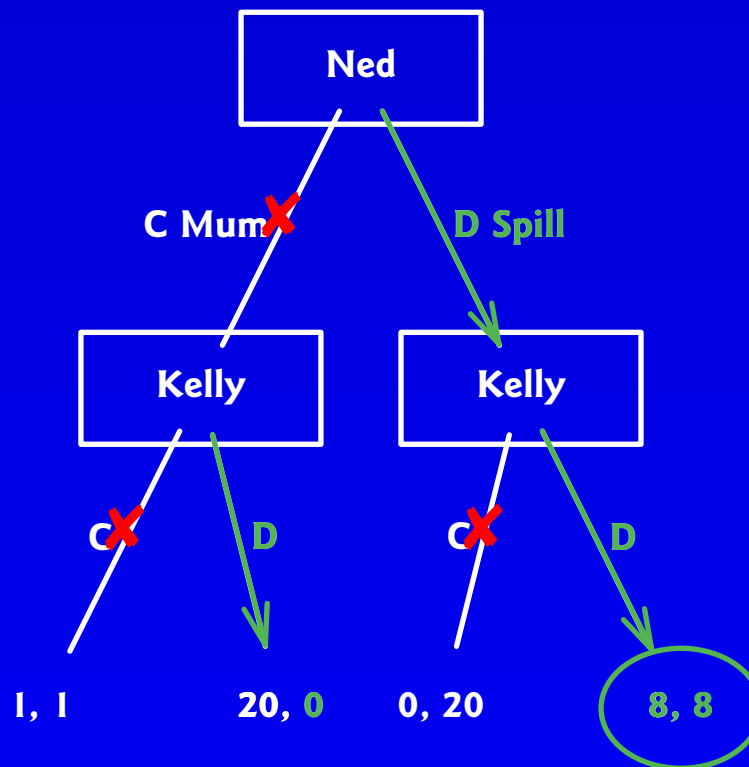
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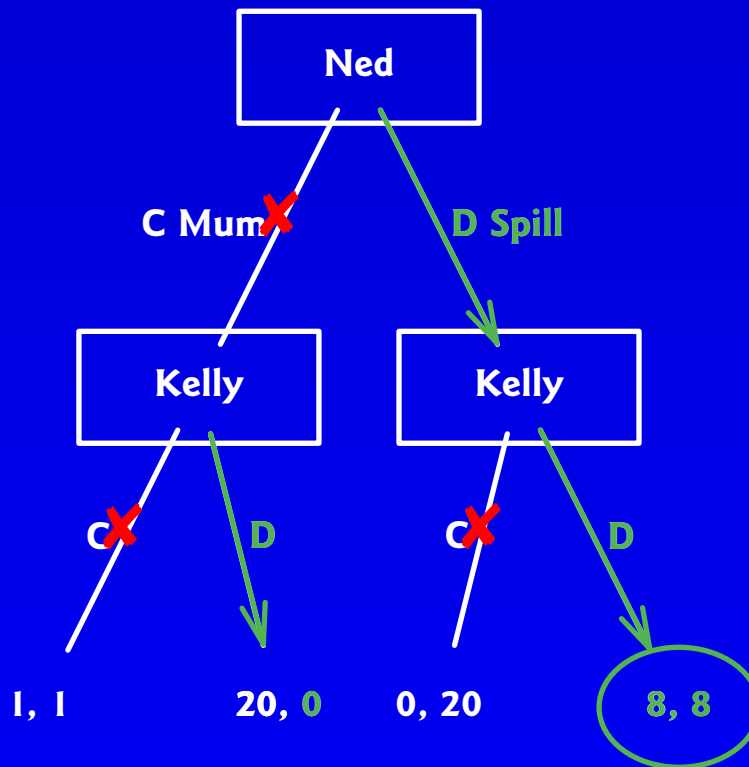
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∴ No first- or second-mover advantage in a PD.



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	Straight	Winner, Chicken!	Death? Death?

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Diagram illustrating the Chicken! game matrix. The matrix shows the outcomes for the Alien (rows) and Bomber (columns) based on their choices (Veer or Straight). Red arrows indicate the path of play: Bomber chooses Straight, Alien chooses Veer, resulting in the outcome "Chicken!, Winner". A green circle highlights the "Chicken!, Winner" outcome.

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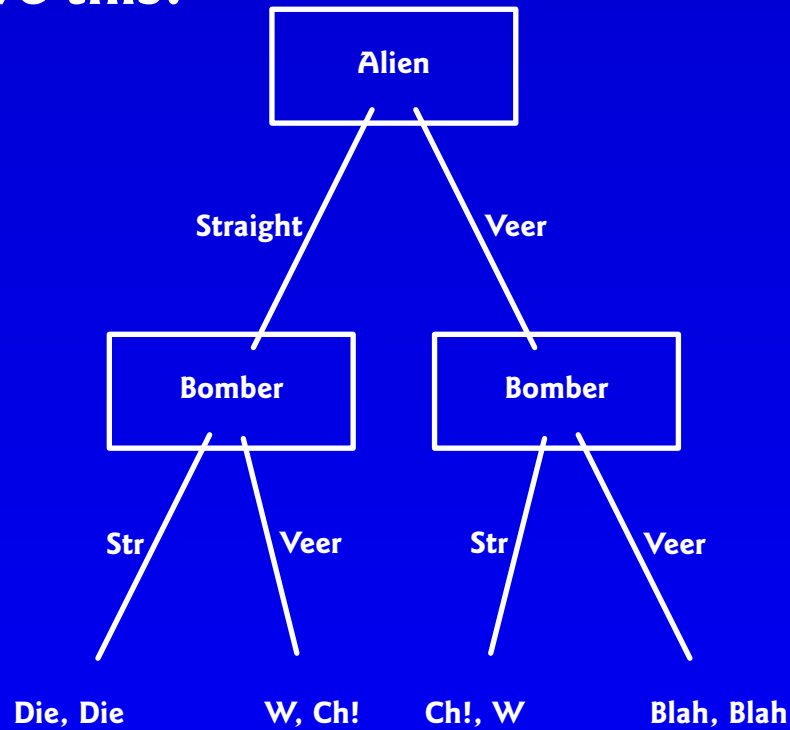
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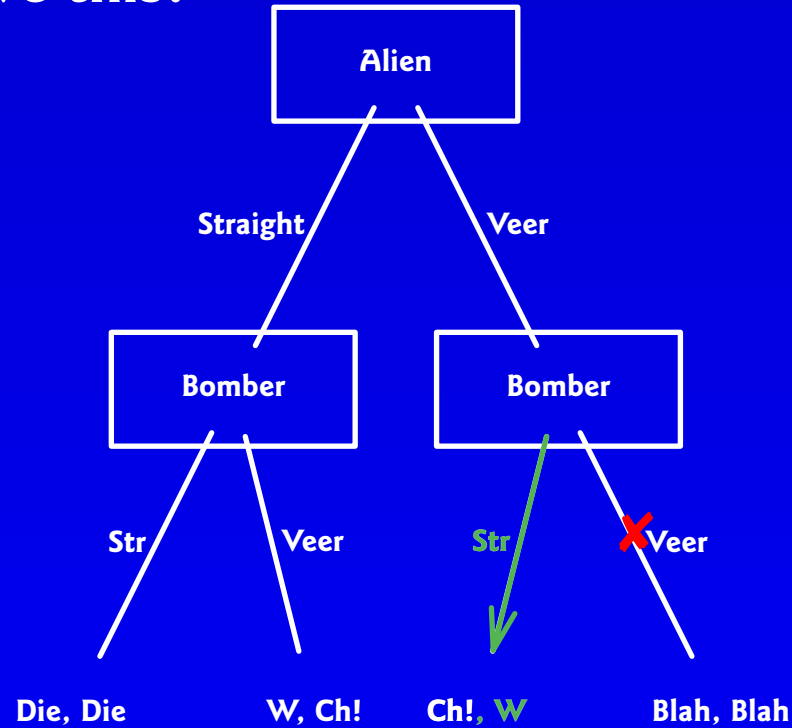
The table includes red arrows indicating best responses: a downward arrow from the top-left cell to the bottom-left cell, an upward arrow from the bottom-right cell to the top-right cell, a horizontal arrow from the top-right cell to the top-left cell, and a horizontal arrow from the bottom-right cell to the bottom-left cell. The top-right and bottom-left cells are circled in green.

Two N.E., but no easy way to coordinate on one or the other.

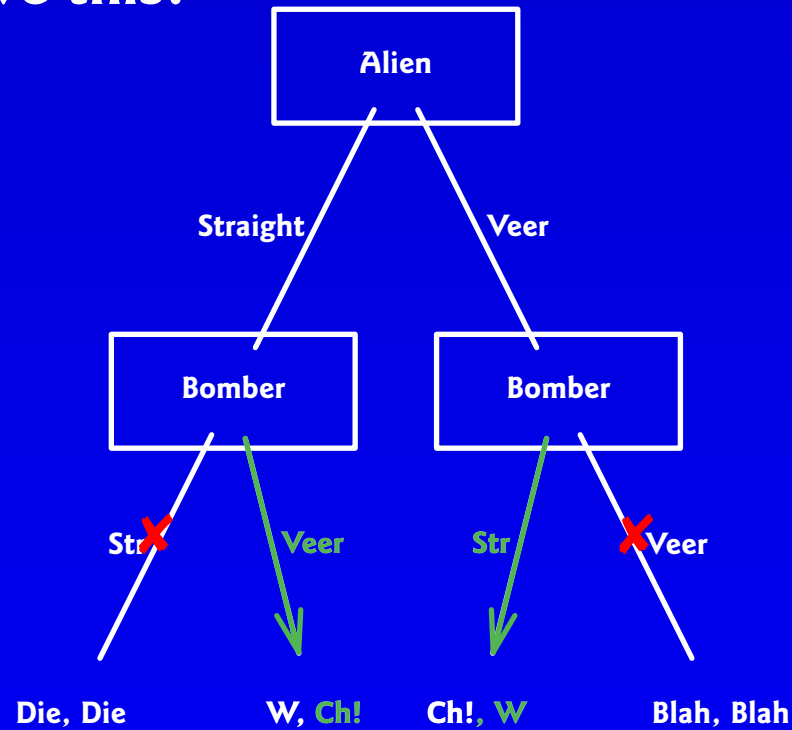
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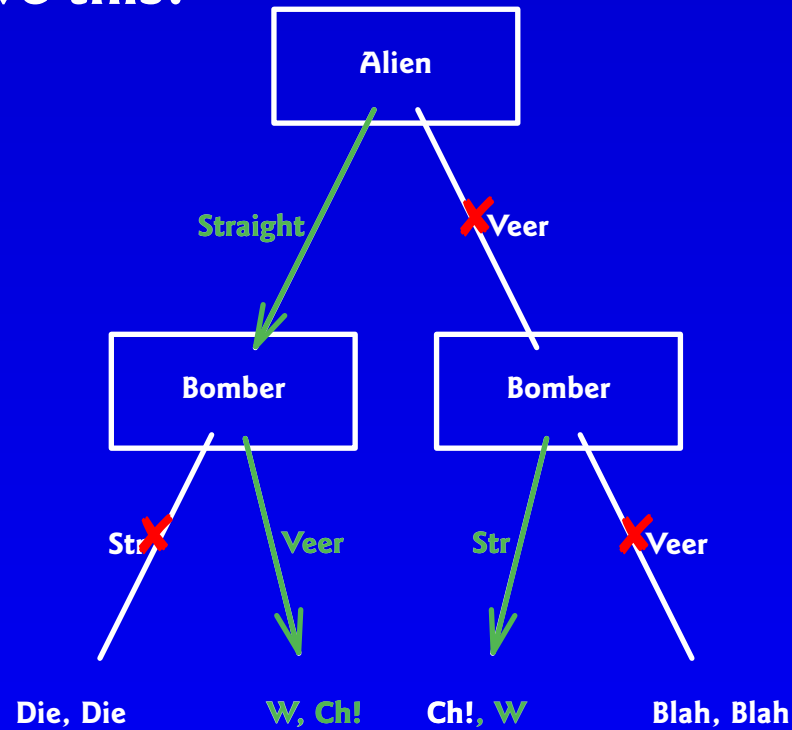


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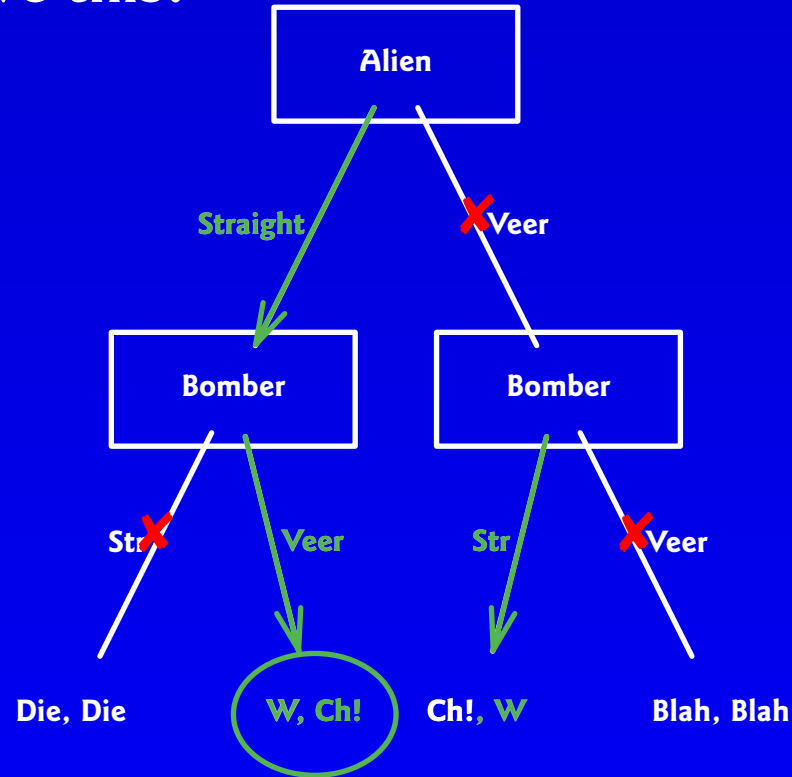




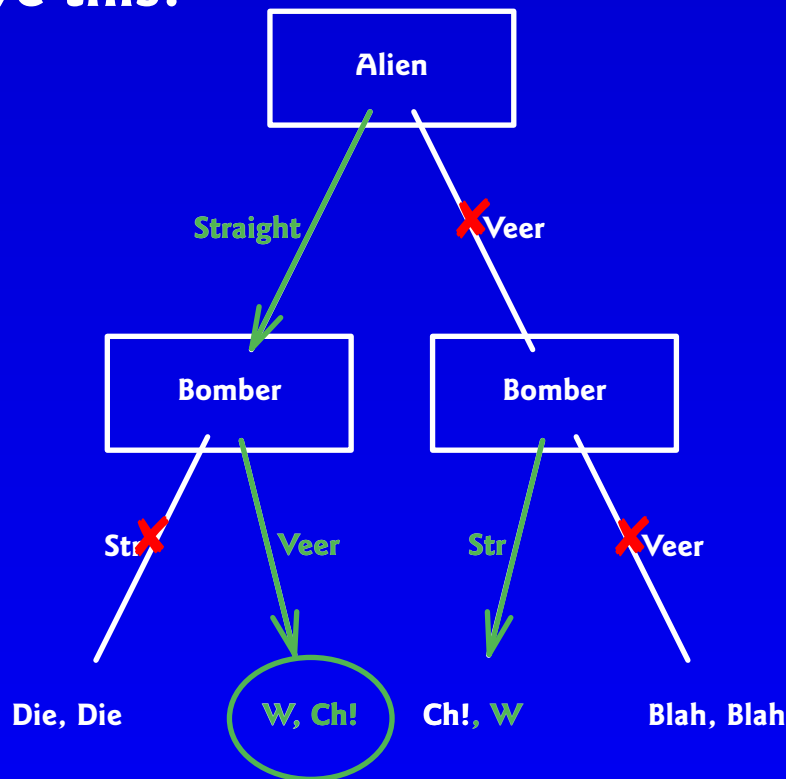
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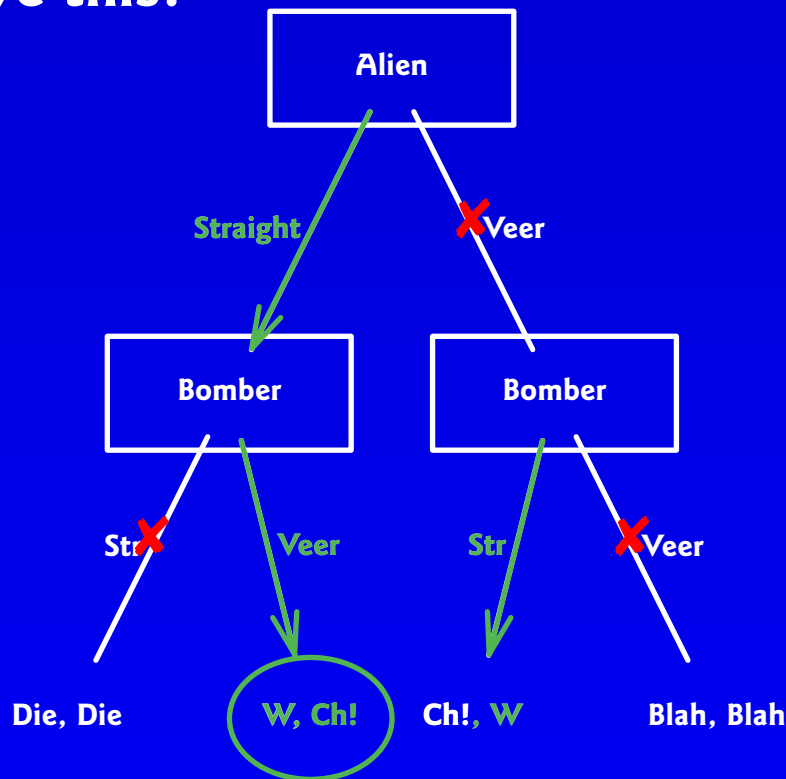


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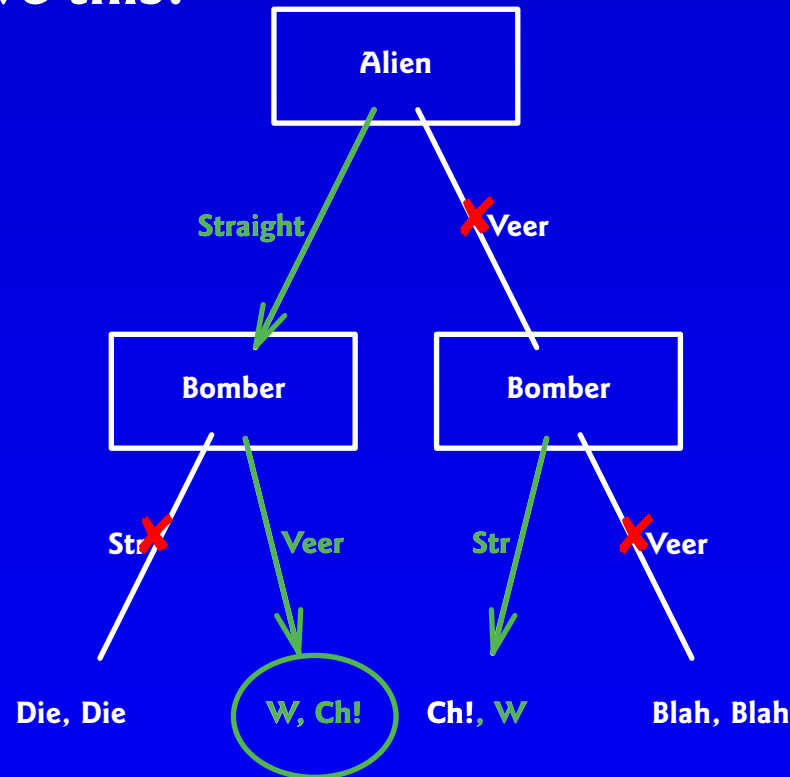
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(So: how to commit to Straight credibly?)

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 $\therefore$  no N.E. (in pure strategies).

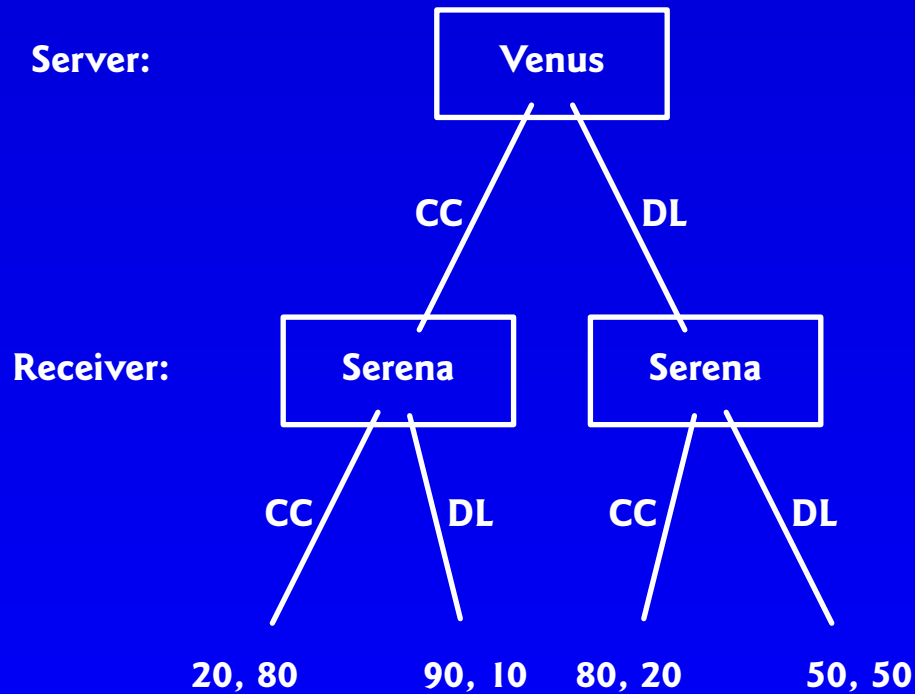
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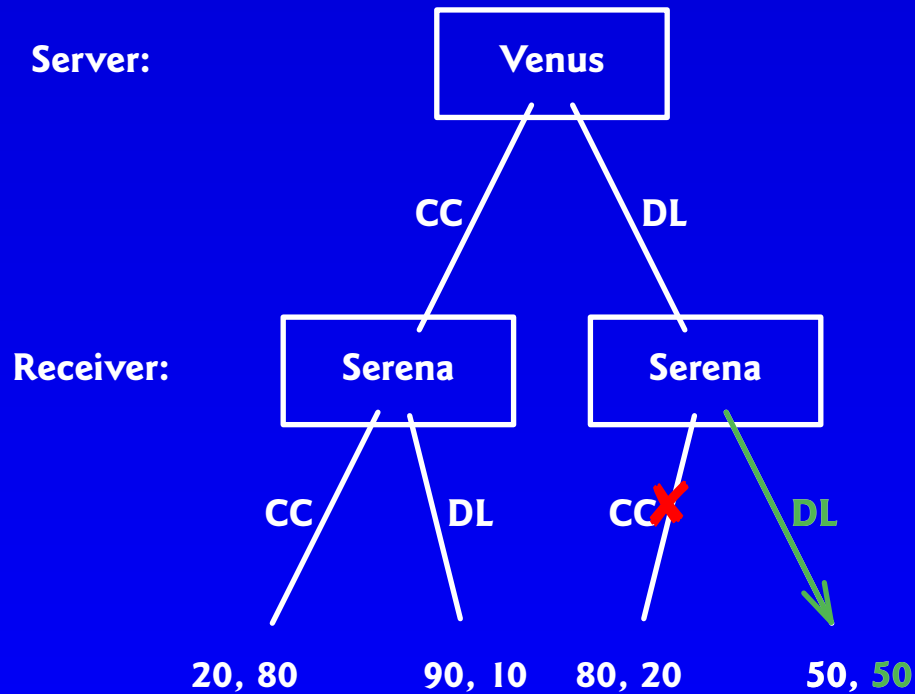
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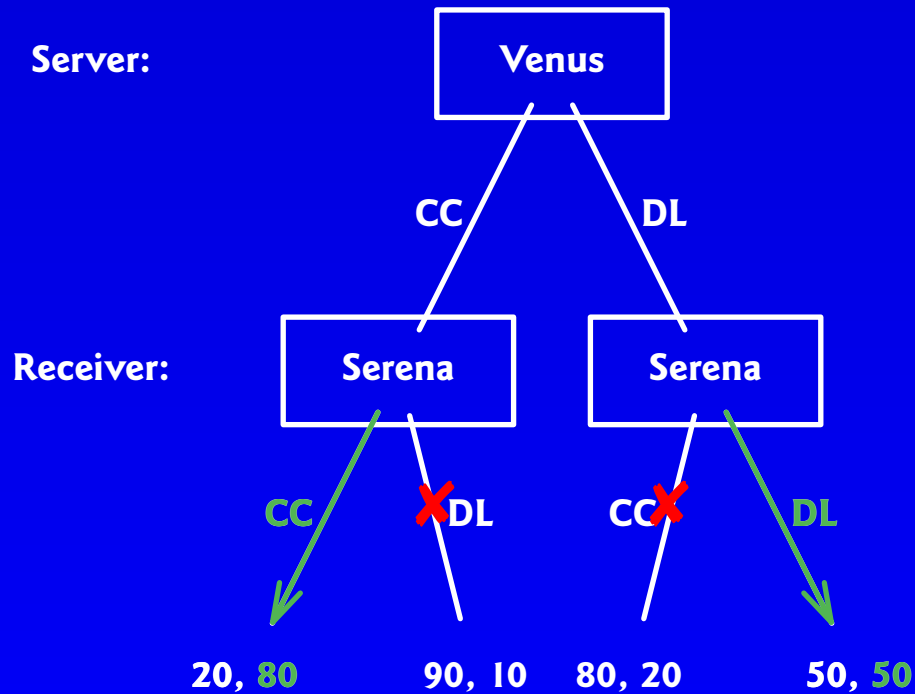
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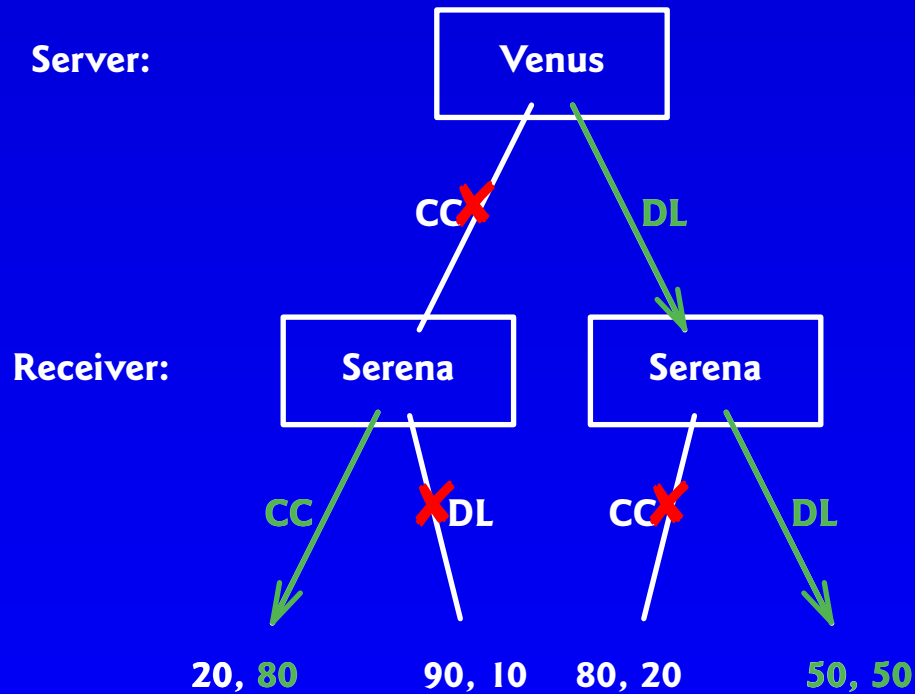
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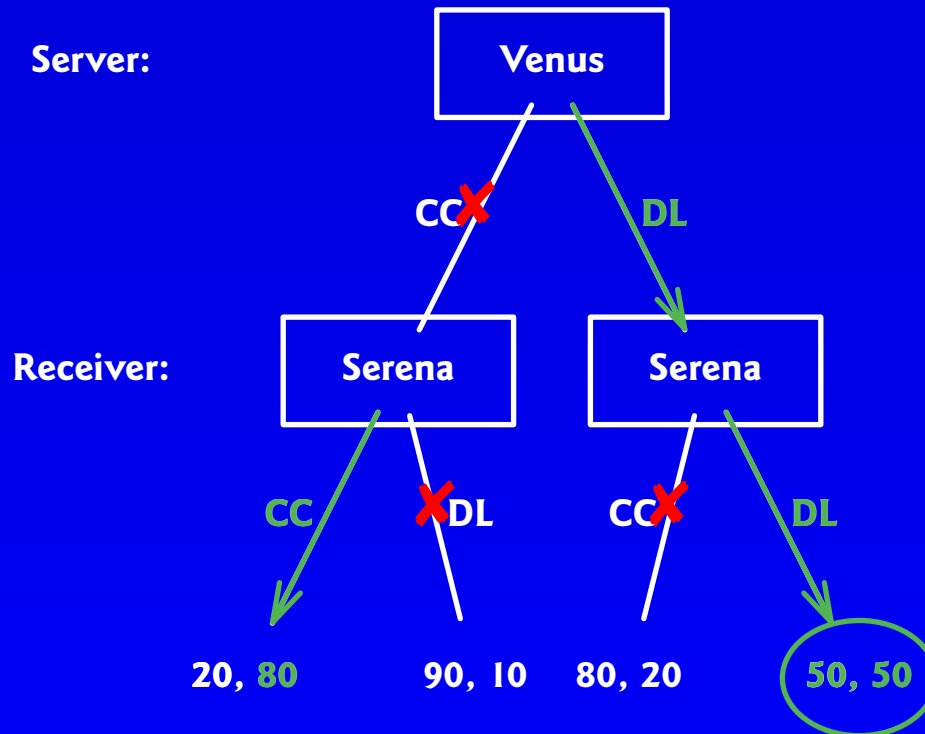
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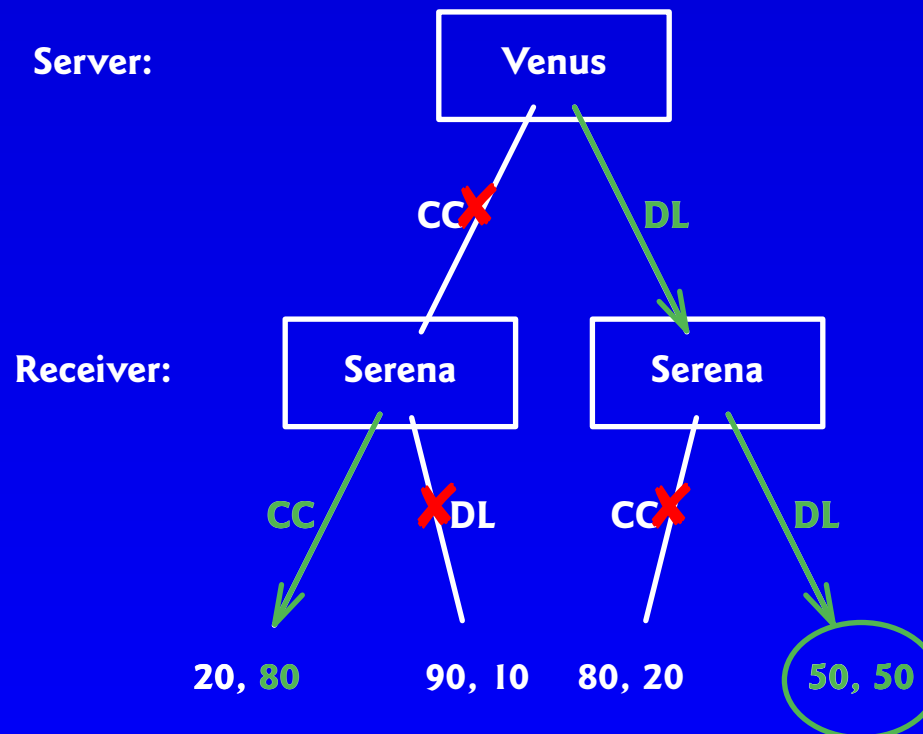
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**Serena has the second-mover advantage, and wins the shot half (50%) the time. Venus wins 50% too.**

## **Serena the receiver moves first**

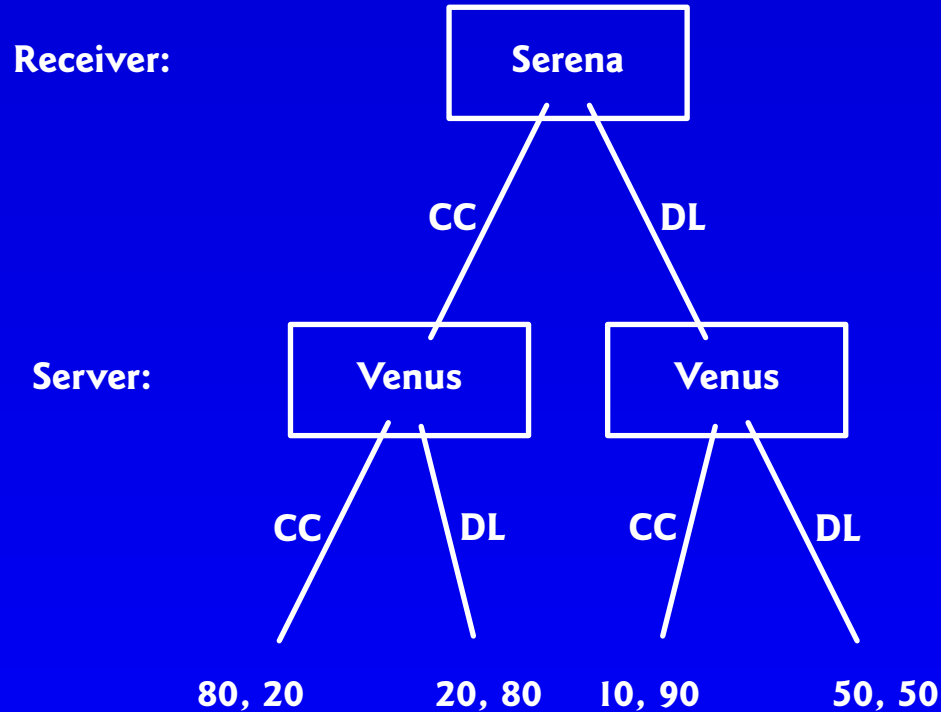
---

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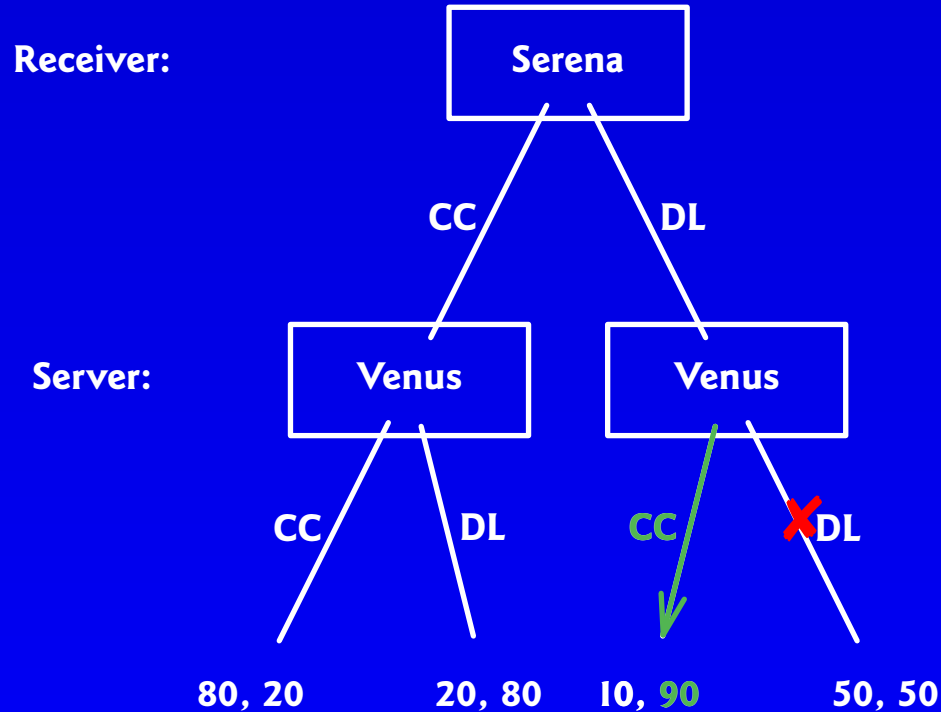
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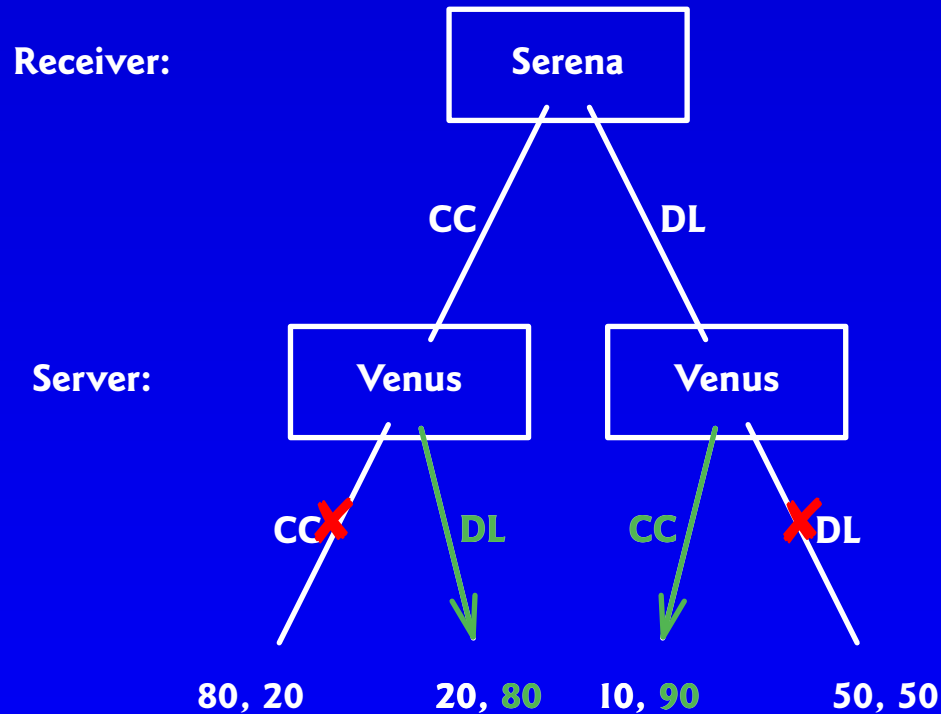
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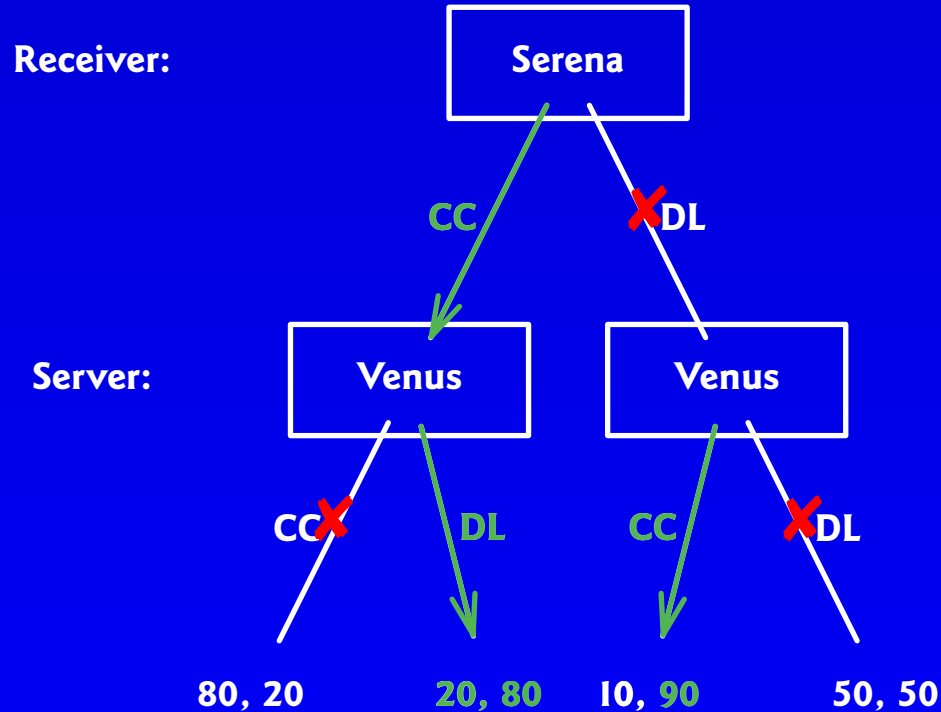
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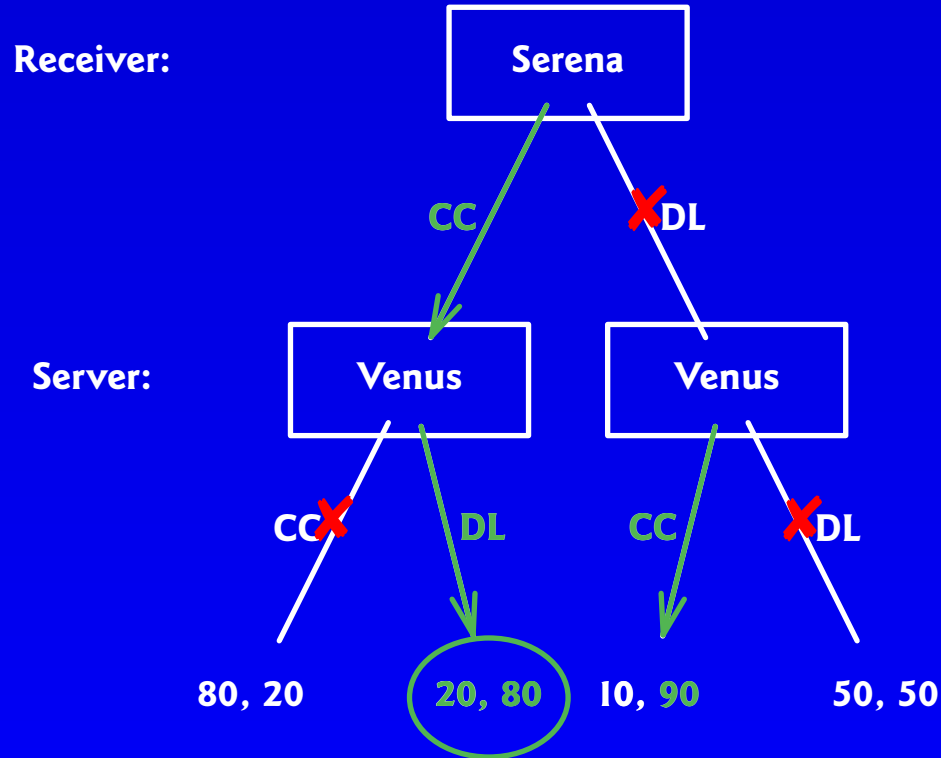
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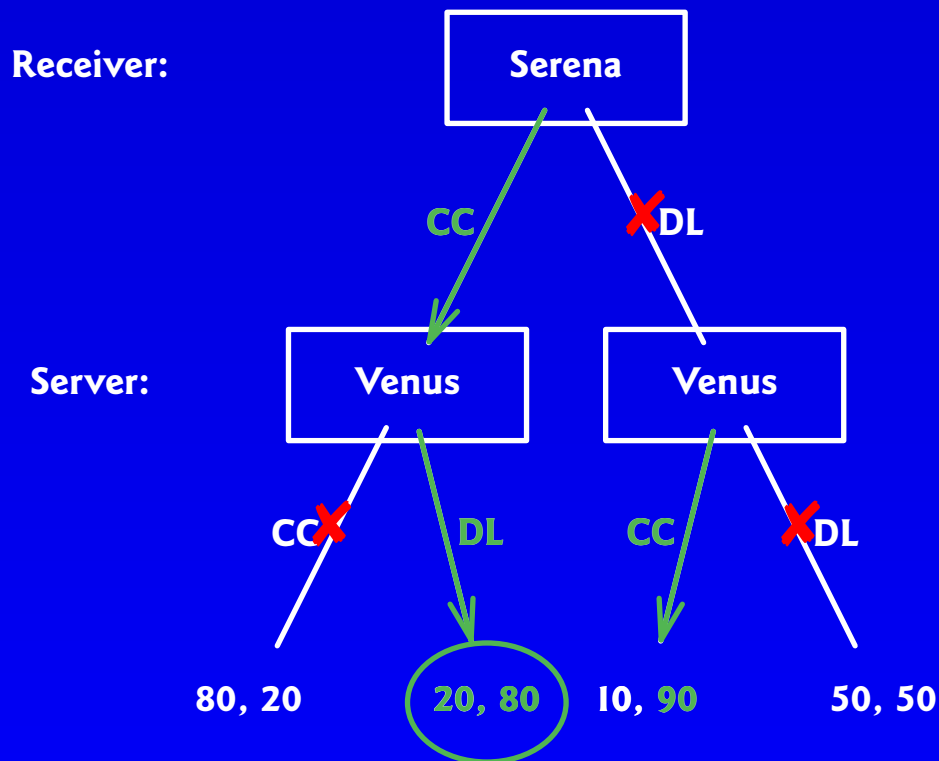
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## Serena the receiver moves first

Or if Venus serving can pick Serena's choice in time:



Venus has the second-mover advantage, and wins the shot most (80%) of the time.

---

## **2c. *Both Players May Do Better***

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The Macro game from Lecture 2 has the POM:

		<i>RBA</i>	
		Low	High
<i>Gov't</i>	Balanced	3, 4	1, 3
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Red arrows indicate the following best responses: Gov't chooses Deficit if RBA chooses Low, and Balanced if RBA chooses High. RBA chooses Low if Gov't chooses Balanced, and High if Gov't chooses Deficit.

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Diagram annotations: Red arrows point from (3,4) to (4,1) and from (1,3) to (2,2). A green circle highlights the (2,2) cell.

The Gov't has a dominant strategy of Deficit, which the RBA knows, and  $\therefore$  chooses High interest rates.

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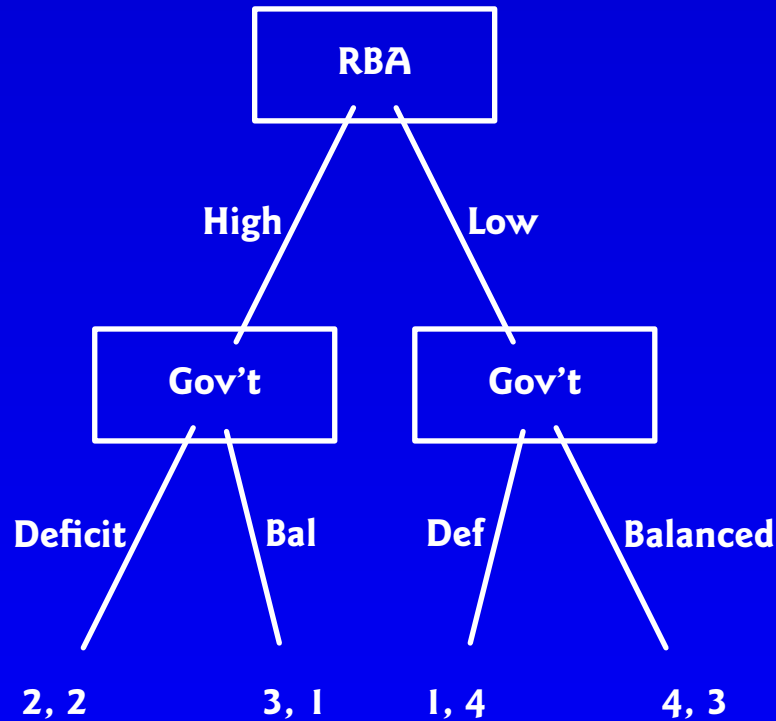
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Yields payoffs of 2 (the second-worst outcome) for each.

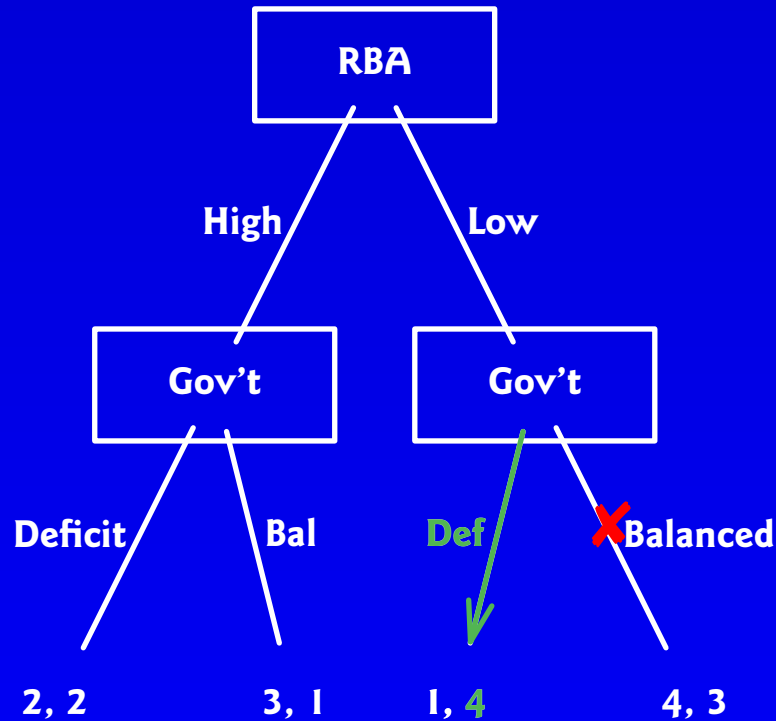
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The game tree (4 = best, 1 = worst):



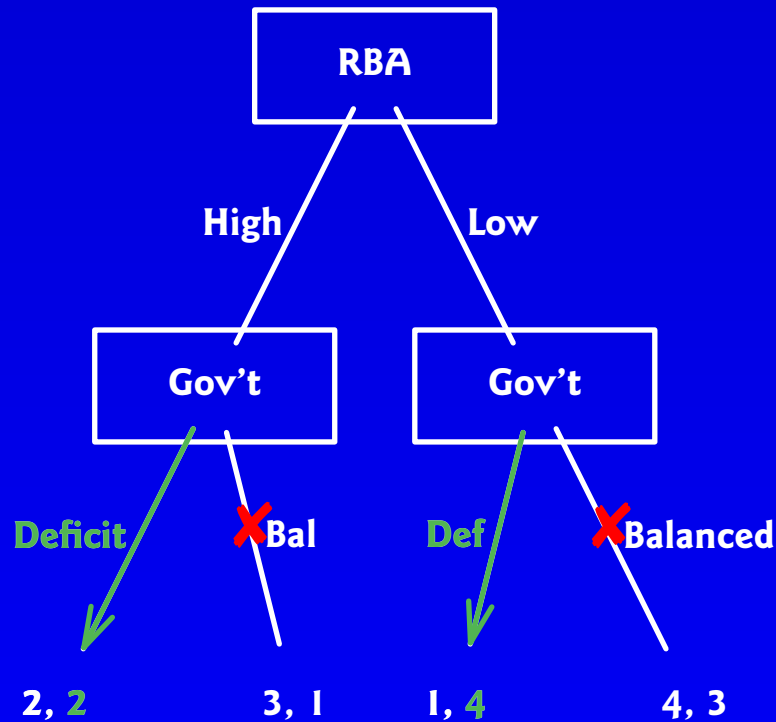
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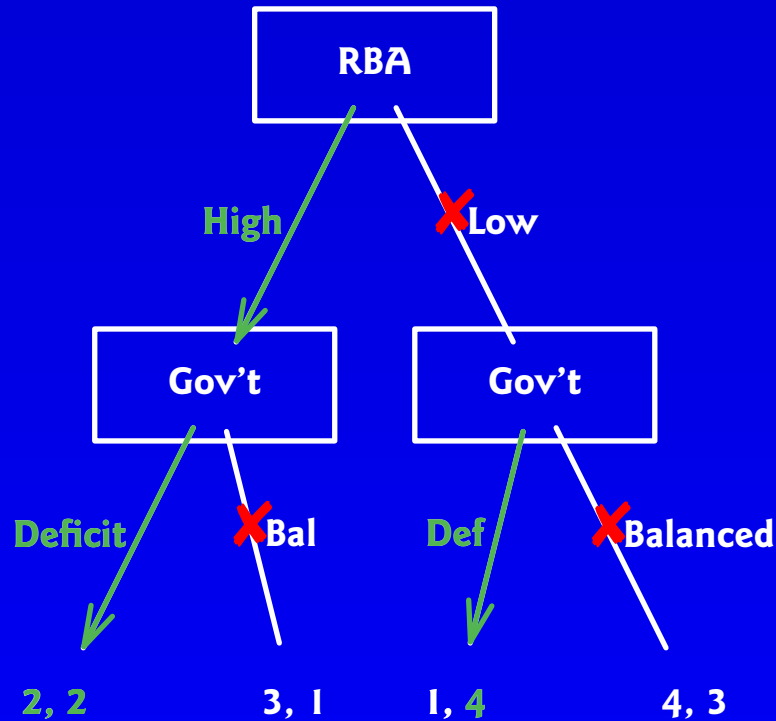
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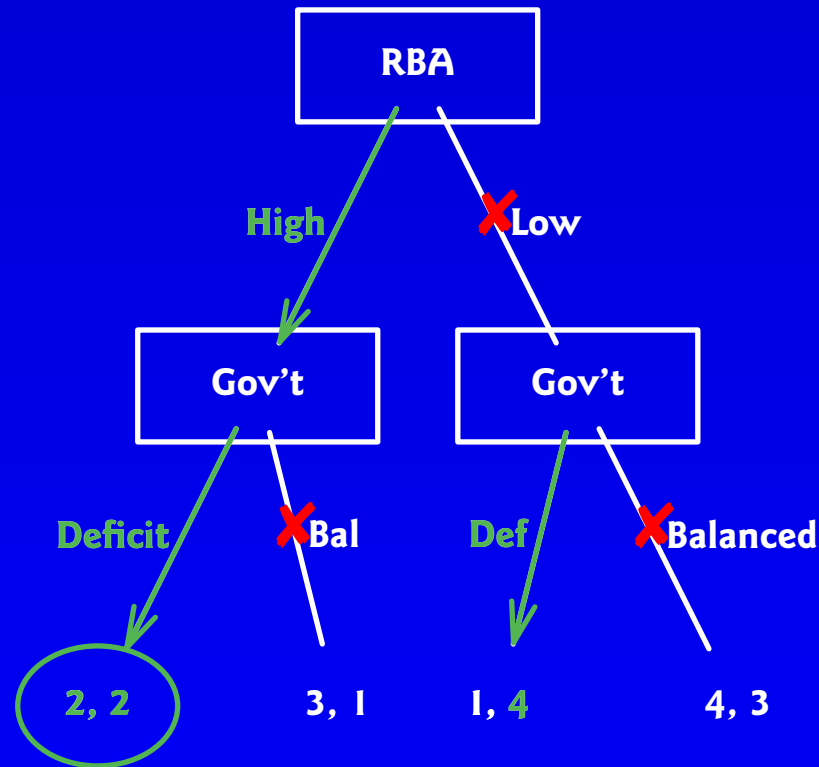
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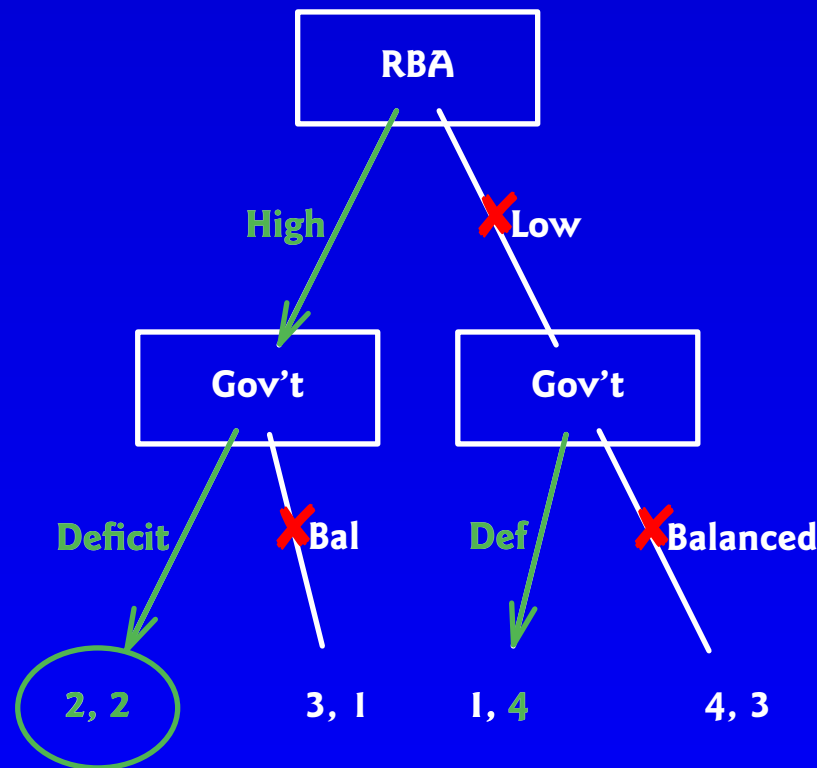
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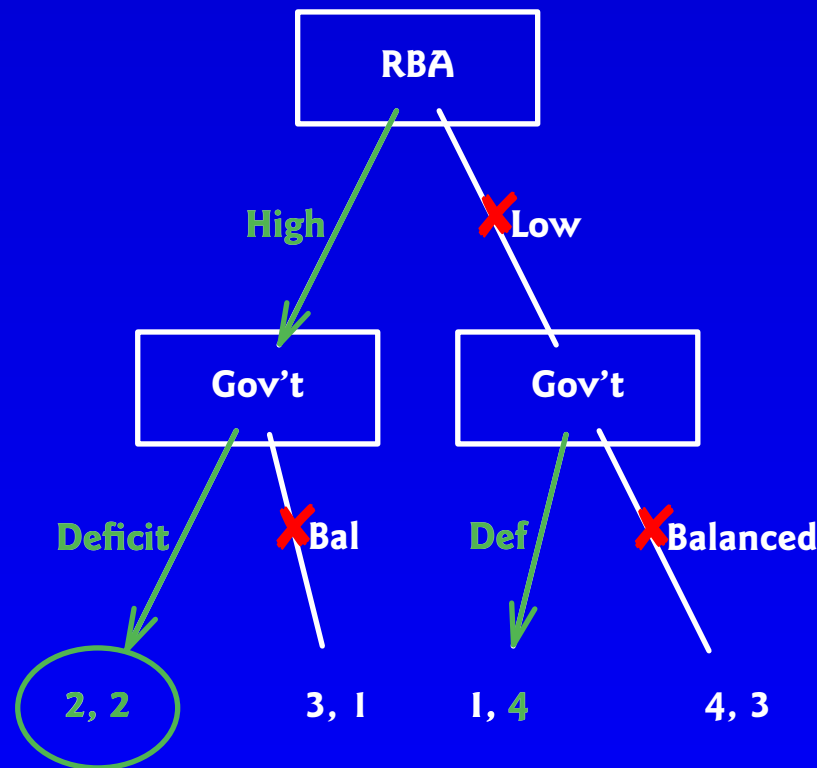
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The RBA knows that the Gov't will go into Deficit, come what may, and so chooses High interest rates, yielding the RBA 2 instead of 1.

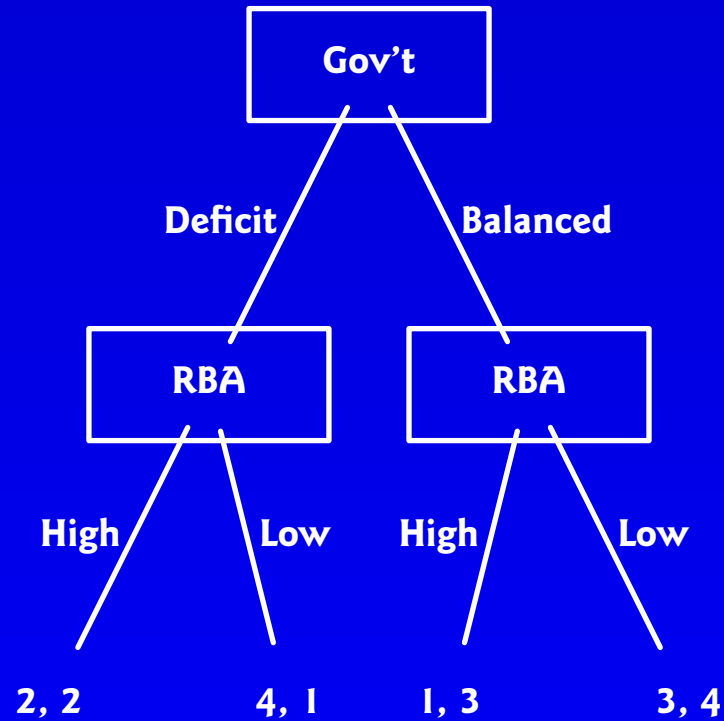
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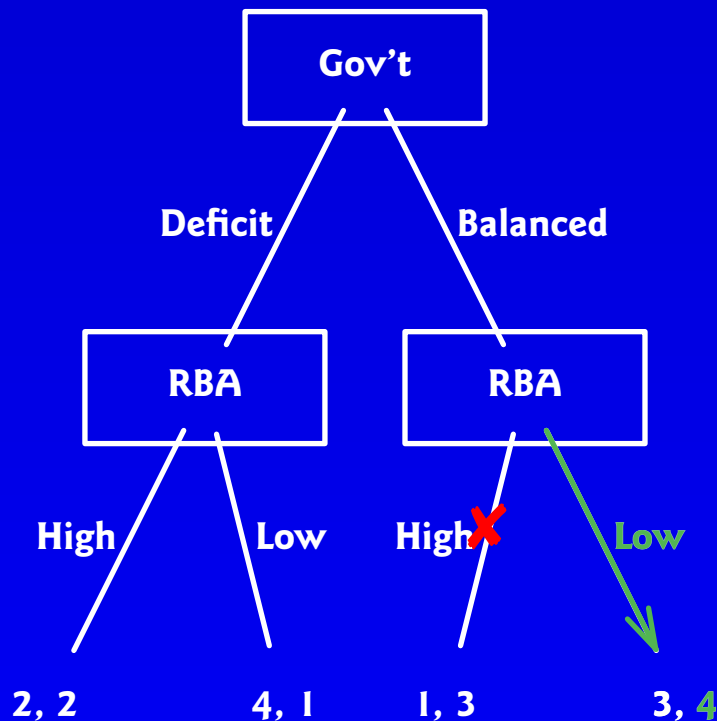
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**But if the Gov't moves first:**

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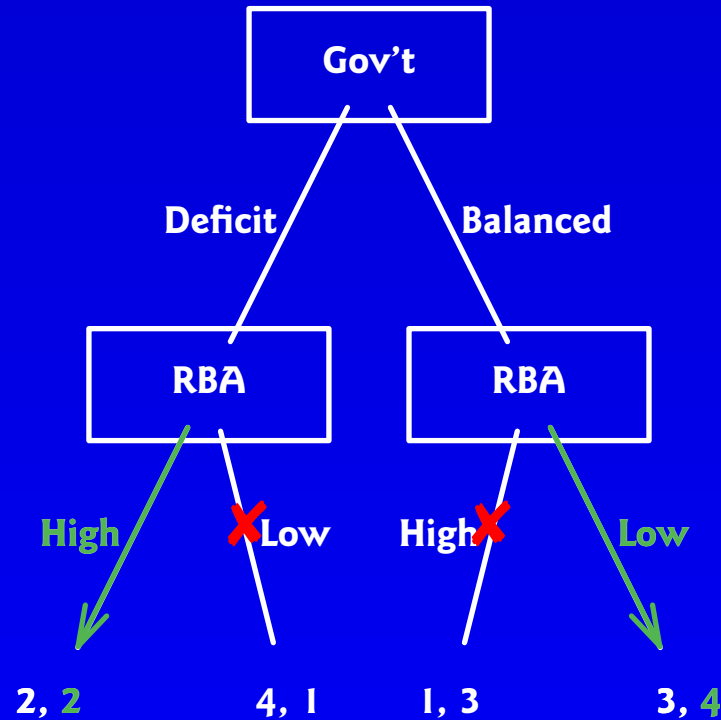
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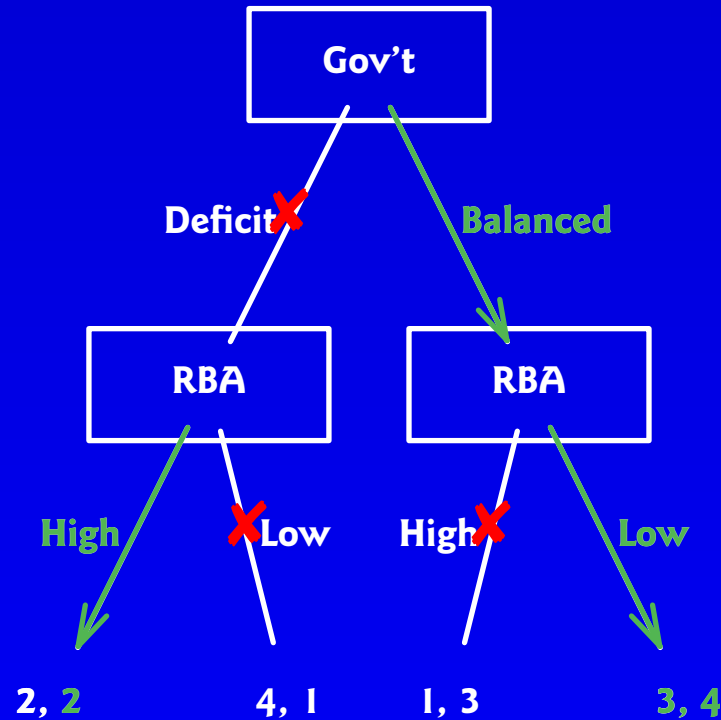
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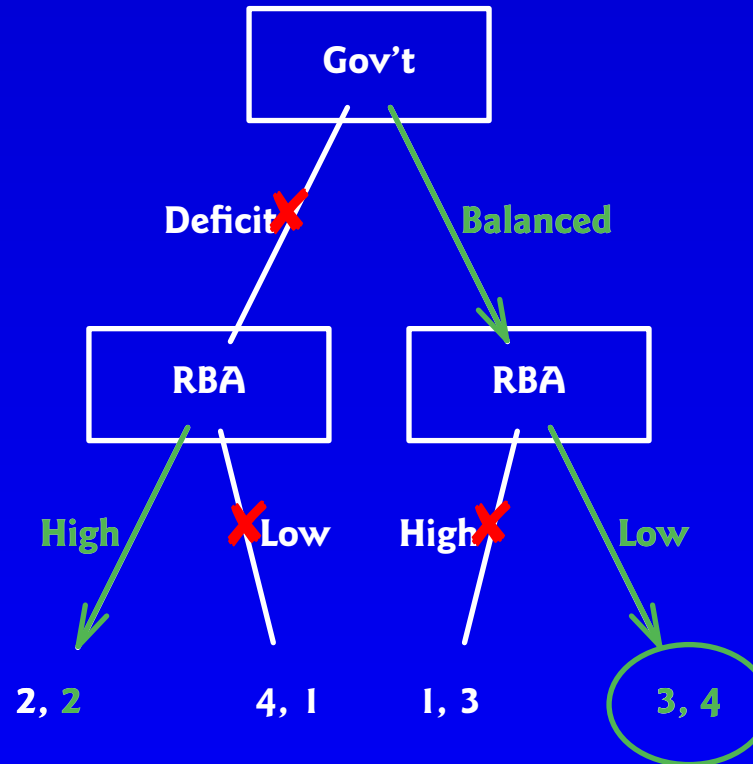
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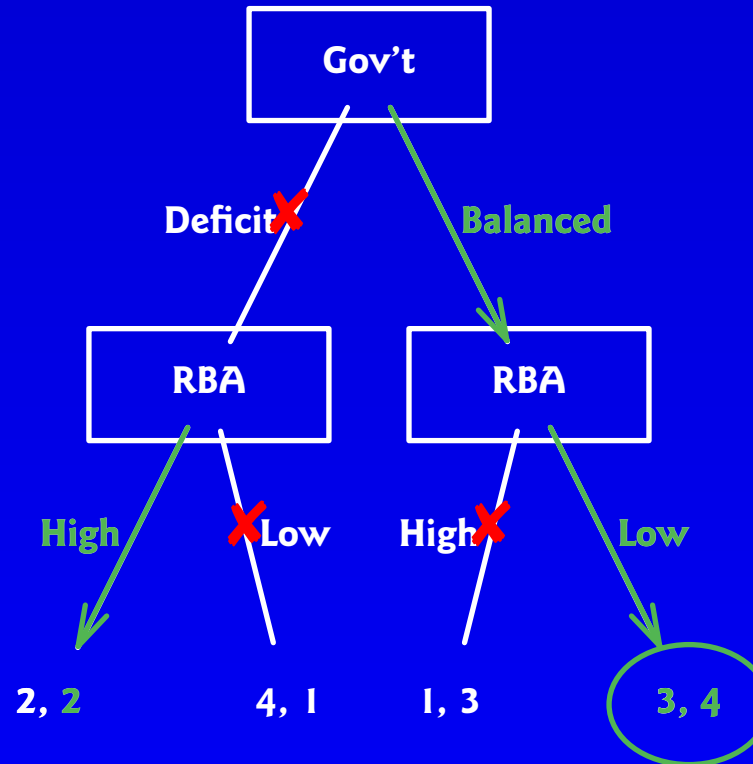
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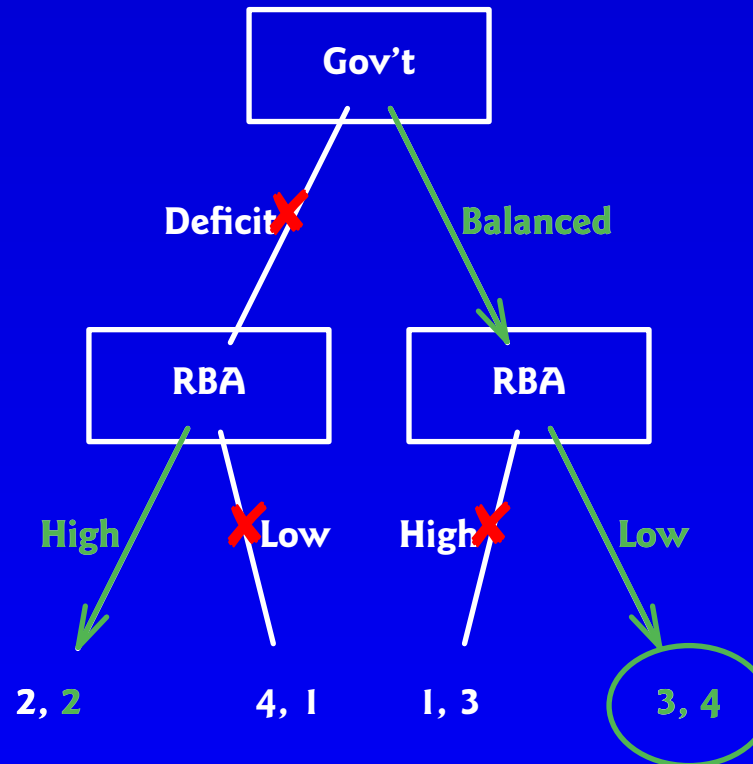
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**The chosen combination of strategies is {Balanced, Low}:**

## But if the Gov't moves first:

The game tree is:



The chosen combination of strategies is {Balanced, Low}: this is the **Rollback Equilibrium** (R.E.), and, surprisingly, yields a better outcome for *both* players than does {Deficit, High}.

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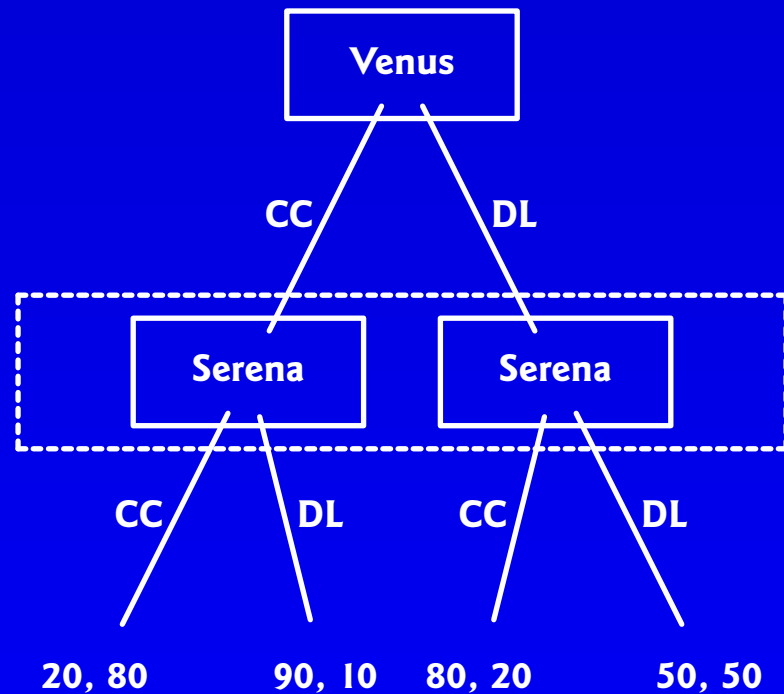
**(See also the Capacity Game Revisited, Lecture 4, pp. 6–8, where a dominated move — Large — is also chosen in R.E.)**

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## 3. Trees for Simultaneous Games

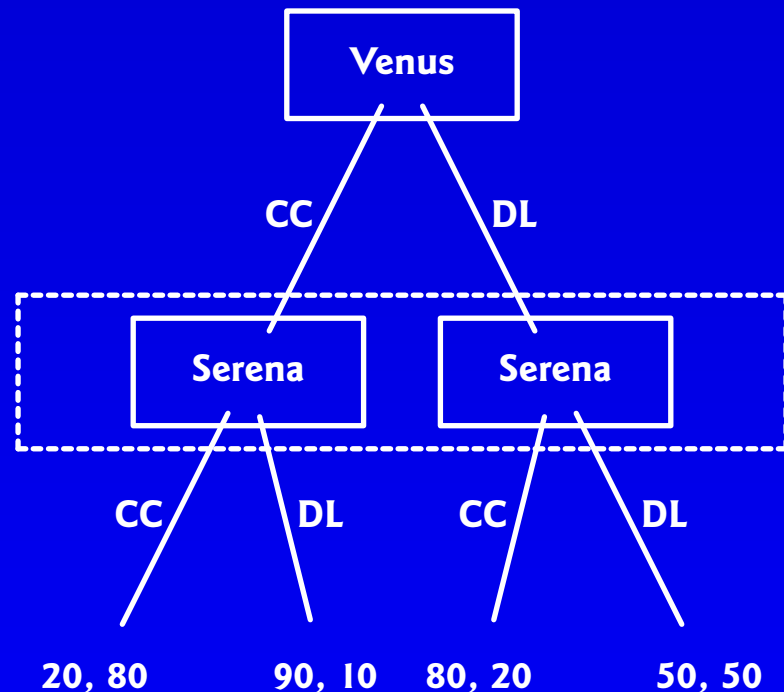
### 3. Trees for Simultaneous Games

The simultaneous Tennis game as a tree:



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The simultaneous Tennis game as a tree:



The dotted box is an **Information Set**: Serena can't tell which of the two decision nodes she's at since she doesn't (yet) know how Venus will serve (CC or DL) and so she cannot do CC at one and DL at the other — *there can only be 1 action per Info Set.* (DSkR p.194)

## Information Sets

**Serena must choose without knowing what Venus has picked: Serena doesn't know which decision node she's at.**

**Use a dotted box around the relevant decision nodes to indicate her *lack of specific information*.**



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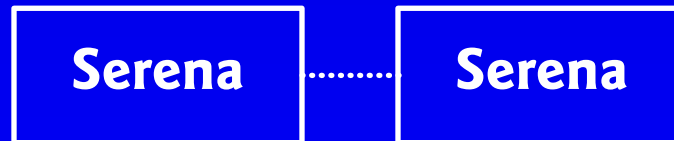
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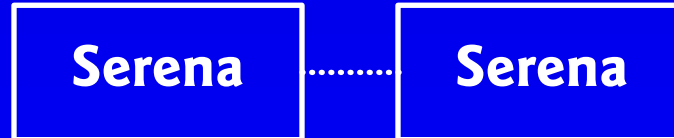


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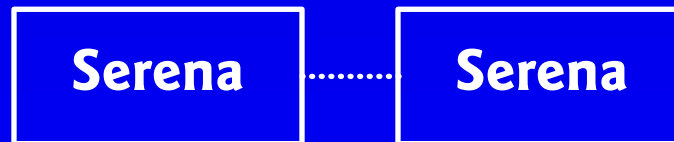


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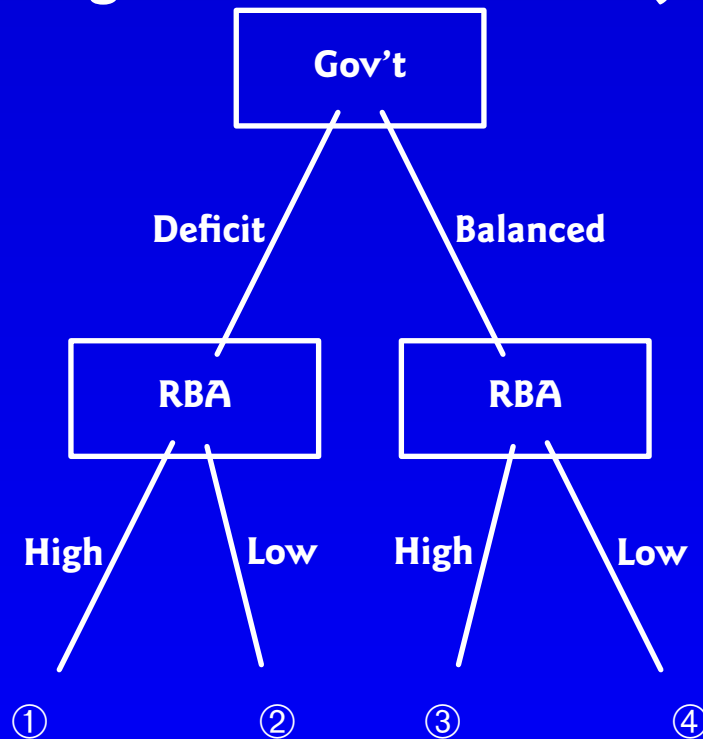
**So a *strategy*: a complete plan of action, specifying the move a player would make at each Information Set (instead of each decision node) when the rules of the game specify that it is her turn to move.**

---

## 4. Matrices for Sequential Games

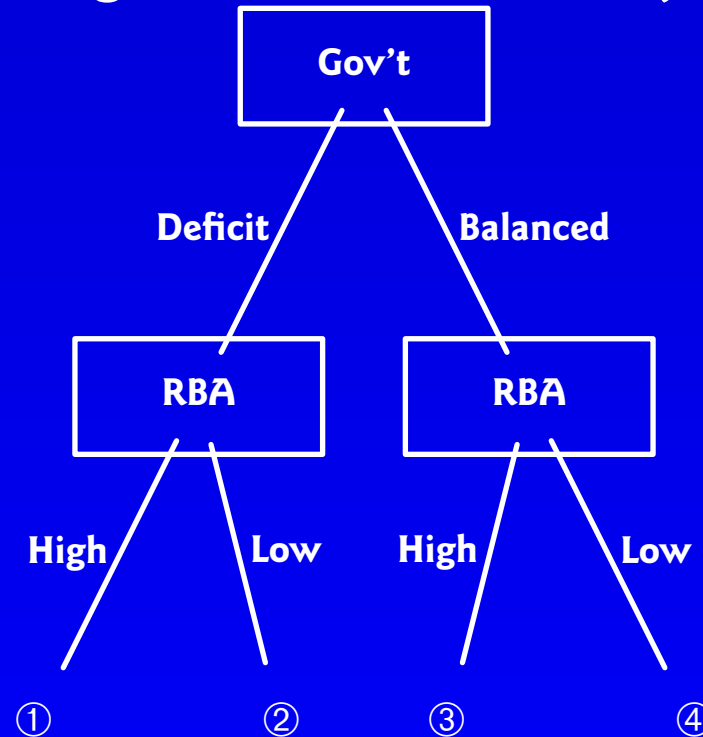
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Use the Macro game tree, where the Gov't moves first.  
(Instructions for *delegation* of RBA action.)



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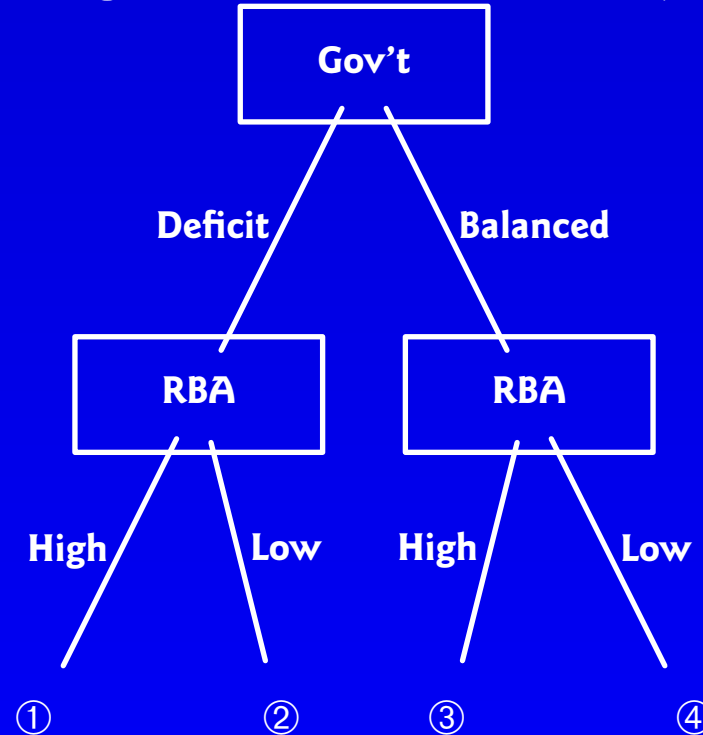
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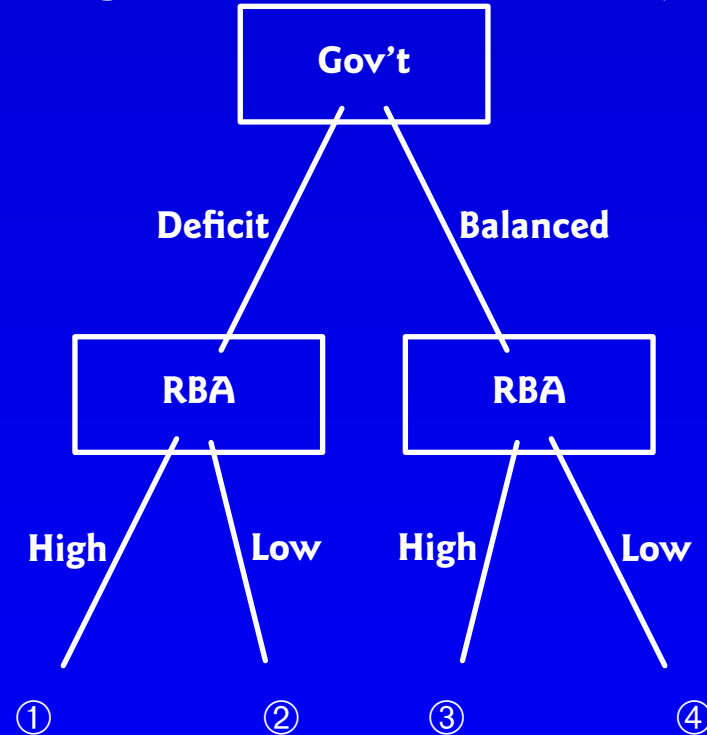


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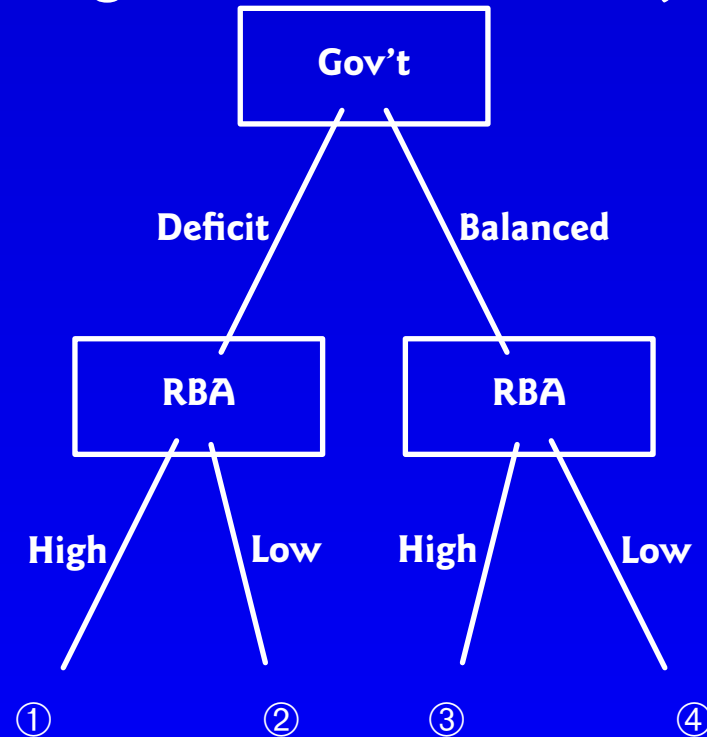


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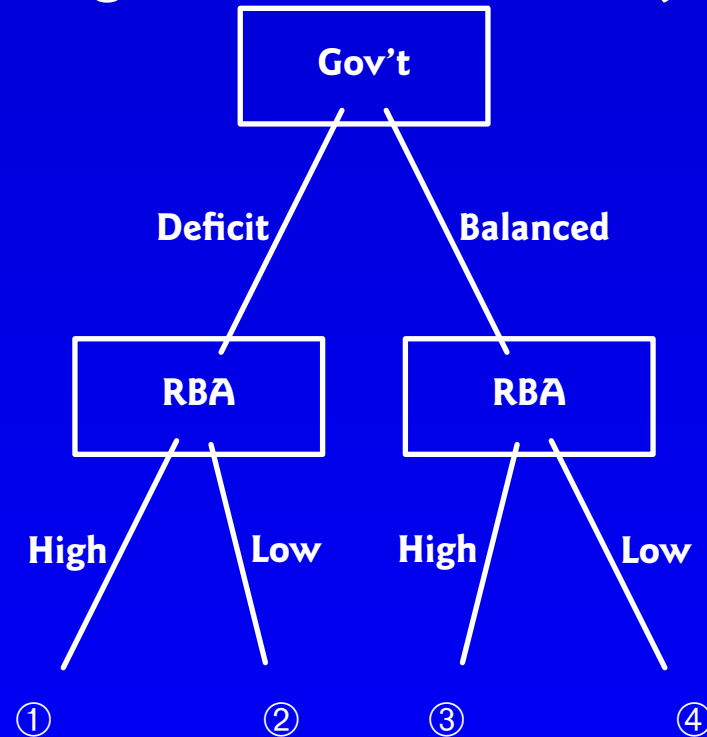


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**first, ① & ③: always H; second, ② & ④: always L; third, ④ & ①: L if B & H if D; and fourth, ③ & ②: H if B & L if D.**

---

**As a payoff matrix:**

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*R B A*

*Gov't*    **Bal**  
          **Def**

## As a payoff matrix:

		<i>R B A</i>			
		L if B & H if D	H if B & L if D	L always	H always
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		<i>RBA</i>			
		L if B & H if D	H if B & L if D	L always	H always
<i>Gov't</i>	Bal	3, 4	1, 3	3, 4	1, 3
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Red arrows in the original image indicate best responses: Gov't chooses Def if B chooses L or H, and Bal chooses L if Gov't chooses Def.



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**Gov't's possible strategies: Balanced or Deficit.**

**RBA has four possible strategies: always High; always Low; Low if Balanced and High if Deficit (L if B & H if D); High if Balanced and Low if Deficit (H if B & L if D).**

## As a payoff matrix:

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**RBA has four possible strategies: always High; always Low; Low if Balanced and High if Deficit (L if B & H if D); High if Balanced and Low if Deficit (H if B & L if D).**

**The last two columns are as if the game were simultaneous, but in the first two columns RBA's decision depends on Gov't's.**

---

**Using arrow or cell-by-cell, find two NE:**

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**“H always” is not optimal at one decision node: if Gov't chose Balanced, then RBA chooses Low (4 preferred to 2). So R.E. can't include “H always”.**

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Why are there two N.E. in this analysis of the sequential game but only one using rollback (R.E.) (p.16)?

N.E. when: neither player gains from moving, given the other's strategy. But R.E. asks: what would the player do, at each decision node (or Info Set)?

“H always” is not optimal at one decision node: if Gov't chose Balanced, then RBA chooses Low (4 preferred to 2). So R.E. can't include “H always”.

But the N.E. doesn't do the R.E. test: “H always” and Deficit are mutual best responses (“H always” equal to “L if B & H if D”).

## Using arrow or cell-by-cell, find two NE:

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SPE, however, excludes non-credible strategies.

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∴ "H always" lacks *credibility*: it is not in the R.E. and ∴ is not in SPE.