

AGSM 5306  
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2 August, 2006  
due 4 pm: 16 August, 2006

## ***STRATEGIC GAME THEORY FOR MANAGERS***

### *Problem Set 2*

*Note: Make any economic assumptions you think necessary, but make them explicitly. You may talk to fellow students about this, but do not copy others' work.*

1. Imagine your friend is being tested for HIV. You have read and believe that this test is 97.5% accurate: if he or she has HIV, then it will show positive 97.5% of the time, and if he or she does not have HIV, then it will show negative 97.5% of the time. You have also read that 0.9%, or 9 in 1000, of the population actually have HIV. Now your doctor tells you that your friend has tested positive. What is the probability that your friend is HIV-positive, given the positive test result?
2. Your company has recently developed Florix, a new sugar-free chewing gum that contains fluoride. Not only does it taste good, but it's also good for your teeth. You're faced with the decision of whether or not to put Florix on the market, (i.e. whether to launch it).

Total sales of all brands of chewing gum are expected to be about \$20 million over the next ten years (in present-value terms).

Your marketing people believe that with their best efforts and with a front-end marketing expenditure of \$400,000, your company could capture from 2% to 10% of the chewing gum market with Florix. They have given you the following probabilities:

<i>Market share</i>	<i>Probability</i>
High (10%)	0.30
Medium (6%)	0.50
Low (2%)	0.20

Your financial advisors point out that the profit margin on Florix is quite uncertain because of unusual manufacturing requirements: they say that there is a 40% chance that the profit

margin will be only 25% of sales revenue, and a 60% chance that it will be 50% of sales revenue.

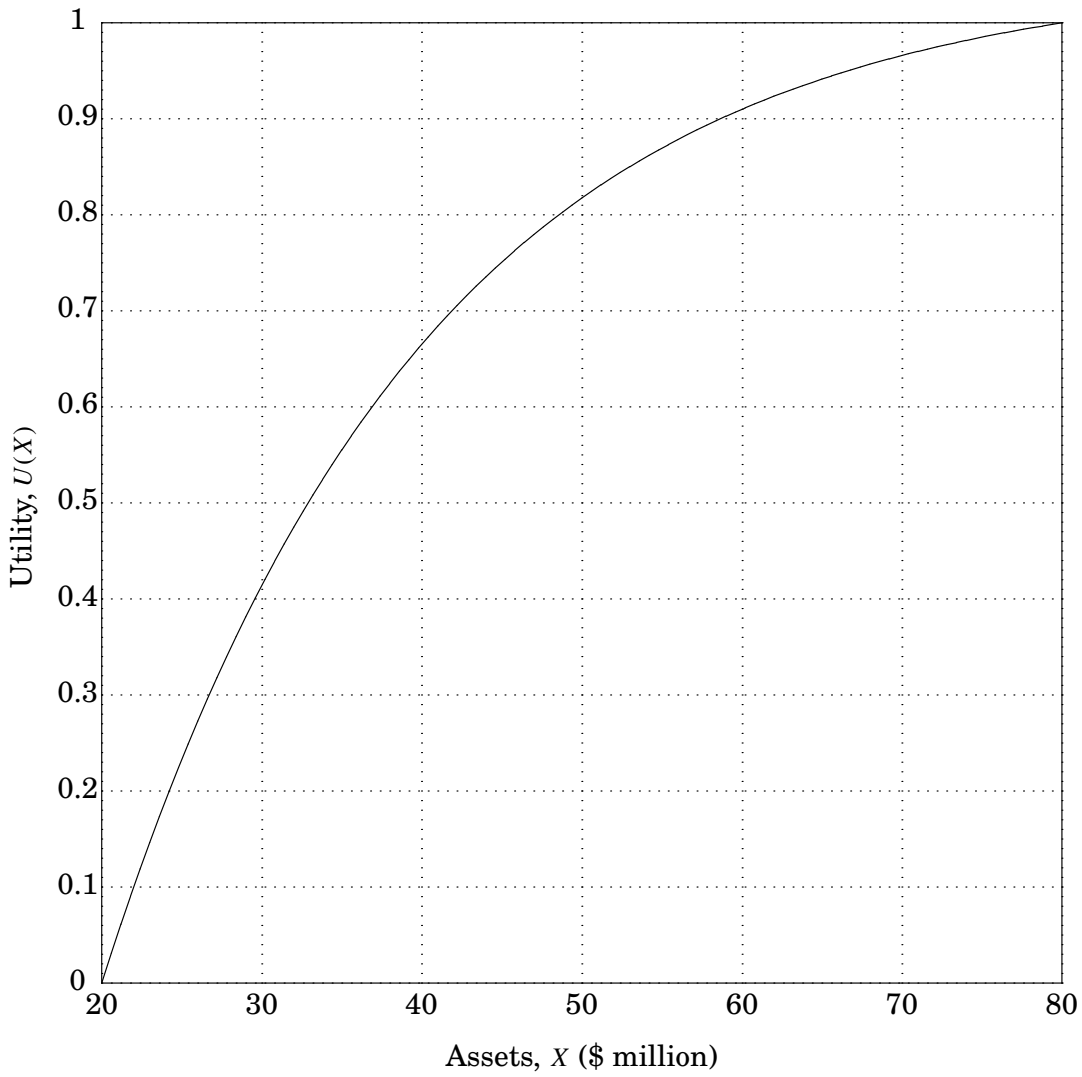
- a. Plot a decision tree for Florix. (Chance nodes, decision nodes, payoffs, and probabilities.)
  - b. What is the preferred decision and the expected value?
  - c. The engineers say that if you postpone the launch decision for six months, they will be able to tell you the manufacturing requirements (and hence the profit margin), but there will be extra costs to be borne for waiting. You decide to wait.  
How high would these extra costs have to be to eliminate any advantage from waiting to find out? (Ignore any time-value-of-money effects.)
  - d. From part (c) you have derived the value of perfect information on the profit margin. What is the value of perfect information on market share? Assume here that you have no additional information on profit margin, unlike part (c).
  - e. What is the value of perfect information on both uncertainties together? Compare your answer here with the previous answers in parts (c) and (d).
3. The Arrow Insurance Company has been approached to insure a building against destruction by fire. The building is valued at \$20,000,000, and Arrow estimates that there is a chance of approximately one in twenty-five that such a fire will take place in any calendar year. Arrow's current assets amount to \$40,000,000. You may ignore any administrative charges.
- a. Assume that Arrow is a risk-neutral decision maker. What is the minimum annual premium that Arrow will require to take on the fire policy? (Hint: What is the lottery faced by Arrow? What is its certain equivalent?)
  - b. Assume now that Arrow is an enlightened organisation which has been persuaded to base the premiums it charges on values from a utility curve. The relationship between its money assets and its utility is represented in the attached curve. What now is the minimum premium that Arrow will require to take on the fire policy? (Hint: if

the utility of a lottery is its expected utility, what is the certain equivalent of the lottery facing Arrow?)

- c. With items which could involve payments greater than 25% of current assets, Arrow usually has an understanding with the Comstock Insurance Company to share the risk on a 50–50 basis. If Comstock happens to work from the same utility curve and currently also has assets of \$40,000,000, what now would the combined premium be? Comment: why are the answers to b. and c. different?
  - d. What would the effect in the answer in part c. be if Comstock's assets were \$80,000,000? Is this what you'd have expected? Explain.
4. Ronaldo, a striker, has the ball near the goal but off to one side. Barthez, the goalie, is poised in front of the goal, somewhat towards the side the ball is on. Ronaldo can aim for the near side or the far side of the goal, i.e., to one side or the other of Barthez. Barthez, at the moment the ball is kicked, can lunge towards one side or the other, but will be too late if he waits to see the direction of the ball. If he lunges to the side Ronaldo aims for, then he has a 90% chance of blocking the goal; that is, there is a 10% chance of a goal. If he lunges to the wrong side, there is a greater likelihood of a goal, but the chances depend on whether the unguarded side is the near side or the far side. If Ronaldo has aimed the ball at the near side *and* Barthez lunges the other way, there is 40% chance of a goal. If Ronaldo has aimed the ball at the far side and Barthez lunges the wrong way, there is a 20% chance of a goal.

These estimates are common knowledge to Barthez and Ronaldo. No coordination of teammates is needed.

- a. Your favourite team has asked your advice on the “correct” way to play this situation. Both Barthez, the goalie, and the Ronaldo, the striker, want to know. What do you tell them and why?
- b. What if Ronaldo's success rate at the far side with Barthez going the wrong way improves from 20% to 25%? Discuss.



**Arrow's Utility Curve,  $U(X) = \frac{1 - e^{-\gamma Y}}{1 - e^{-\gamma}}$ ,  $\gamma = 3$ ,  $Y = \frac{X - 20}{60}$**

$X$	$U(X)$
20	0.0
30	0.4141
38.670	0.6386
39.200	0.6494
39.324	0.6519
39.488	0.6552
39.600	0.6574
40	0.6652
70	0.9660
79.488	0.9986
79.600	0.9989
80	1.0