

## ***STRATEGIC GAME THEORY FOR MANAGERS***

### *Problem Set 1*

*Note: Individual work: Make any economic assumptions you think necessary, but make them explicitly. You may talk to fellow students about this, but **do not copy** others' work.*

1. The Amherst Company must decide whether to buy all or part of its steel from the Duquesne Corporation. If Duquesne provides prompt delivery of the steel it sells Amherst, then Amherst will make \$2 million if it buys all its steel from Duquesne and \$1 million if it buys only part of its steel from Duquesne. But if Duquesne does not provide prompt delivery, then Amherst will lose \$50 million if it buys all its steel from Duquesne and lose \$1 million if it buys only part from Duquesne. If it receives an order for all of Amherst's steel requirements, then Duquesne will make \$3 million if it provides prompt delivery and \$2 million if it does not do so. If it receives an order for part of Amherst's steel requirements, then Duquesne will make \$2 million if it provides prompt delivery and \$1 million if it does not.
  - a. Amherst must decide whether to buy all or part of its steel from Duquesne, and Duquesne must decide whether or not to provide prompt delivery. What is the payoff matrix for this game?
  - b. Does each player have a dominant strategy? If so, what is it? Explain.
  - c. Does this interaction have a Nash equilibrium? If so, what is it? Explain.
  - d. Suppose that Duquesne's managers are known to be inefficient and not much interested in how much money their firm makes. Do you think that Amherst will act in accord with the Nash equilibrium? Why or why not? (Hint: think *maximin*, in which Amherst maximises its minimum payoff.)

2. Consider two firms, say, Holden and Toyota. Each of them is considering whether to expand its capacity to make cars or not. Whether or not each does depends on the payoffs, which depend on what the other player does. Assume, for simplicity's sake, that each faces three possibilities: Do Not Expand production capacity (DNE), a Small expansion, or a Large expansion.

The payoff matrix is given by:

		<i>T o y o t a</i>		
		DNE	Small	Large
<i>H o l d e n</i>	DNE	\$36m, \$36m	\$30m, \$40m	\$18m, \$36m
	Small	\$40m, \$30m	\$32m, \$32m	\$16m, \$24m
	Large	\$36m, \$18m	\$24m, \$16m	\$0m, \$0m

**TABLE 1.** The payoff matrix (Holden, Toyota)

- Because of the long lead times, assume that the two players' actions are simultaneous. What is the equilibrium strategy in this game? Explain.
  - If one player, say Holden, preempts the other by moving first, plot the resulting game tree. What is the equilibrium strategy now? Explain.
  - If Holden can move first, and this is common knowledge, how might Toyota try to change the outcome? Explain. (This is asking for possible strategic moves by Toyota not included in the POM — hint: think *cooperative* game theory.)
  - Does the equilibrium strategy change if Toyota instead moves first? Explain how.
3. In a recent obituary of Alexander Ginsburg, a Russian dissident, we read:

Ginsburg's interrogation after his arrest in 1977 was carried out by a team of six KGB officers. "Their job was to break me down," he later recalled. "Their main weapon was blackmail. They said that unless I cooperated they would arrest my friends and colleagues. But I knew that they would do that anyway, whenever they wished. Then they threatened me with the death penalty for treason. I replied that, as far as I was concerned, the death penalty would be the best possible outcome. This surprised them. Then they started making use of my illness, my duodenal ulcer. They created conditions under which I was bound to be in continual pain, through lack of medical attention."

"But at least I knew that they would not try to kill me before

the trial. This is because I was a *defended person*, someone whom the West knew about and was likely to make a fuss about. Without this form of defence, political prisoners just die.”

Before his unexpected exchange with two Soviets whom the Americans had jailed for espionage, Ginsburg served ten months of his sentence of eight years hard labour in Mordovia. He died in Paris in 2002, a French citizen since 1998.

The KGB evidently wanted a confession, including names and details. Ginsburg had been administering the Solzhenitsyn Fund, based on the royalties from *The Gulag Archipelago* after the author was exiled in 1974.

Plot the interaction between Ginsburg and the KGB as a game tree. You will have to decide the order of play and the payoffs, but ranking the alternatives for each side is sufficient. Remember to add a dotted line where you wish to represent (with an information set) that one player does not observe the other’s choice. Briefly explain your tree.

4. Balminor has two food shops, Victoire’s, which sells bread, and La Fromagerie, which sells cheese. It costs \$2 to make a loaf of bread and \$4 to make a kilo of cheese. If Victoire’s price is  $P_1$  dollars per loaf of bread, and La Fromagerie’s price is  $P_2$  dollars per kilo of cheese, their respective weekly sales,  $Q_1$  hundred loaves of bread and  $Q_2$  hundred kilos of cheese, are given by the following equations:

$$Q_1 = 20 - P_1 - \frac{1}{2} P_2,$$
$$Q_2 = 24 - \frac{1}{2} P_1 - P_2$$

- a. Following the steps of reasoning and calculation as in the lecture, find the two shops’ best-response curves, and the Nash equilibrium prices of each. They choose their prices simultaneously.
- b. If the two shops colluded and set prices jointly to maximise the sum of their profits, find their prices.
- c. Give a short intuitive explanation for the differences between the N.E. prices and the joint-profit-maximising ones. Why is joint profit maximisation not a N.E.?
- d. Bread and cheese are mutual *complements*. They are often consumed together; that’s why a drop in the price of one increases sales of the other. The pizzerias’ products in class were *substitutes* for each other; that’s why a drop in the price of one hurt the sales of the other. How does this

distinction explain your findings for the best response curves and the Nash equilibrium and joint profit-maximising prices in this question and the corresponding answers in the pizzeria example?

5. What is the definition of a strategic interaction? Consider a strategic situation that you are personally familiar with from work, uni, or through social contacts.
  - a. Describe it, briefly. Who are the players?
  - b. What are the possible actions of each of them? Does one (or more) move first (and be seen to move first)? Who?
  - c. Plot an outcomes matrix (if the number of players is not too many, and the number of possible actions is not too many) with the outcomes for each. If the matrix is a cube or worse, discuss a few of the possible combinations of actions and the payoffs for each player.
  - d. Can you reduce the numbers of possible actions? If so, do so.
  - e. Are there one or more players who are peripheral (whose actions have only a marginal impact on the other players)? If so, remove them.
  - f. Can the outcomes be easily ranked for each remaining player? If so, do so.
  - g. Can you solve for the equilibrium of the interaction? Do so. If not, why not? (What additional information would allow solution?)