STRATEGIC GAME THEORY FOR MANAGERS

Problem Set 1

Note: Make any economic assumptions you think necessary, but make them explicitly. You may talk to fellow students about this, but ***do not copy*** *others' work.*

1. A debtor owes $15,000 to each of two creditors, but he only has $25,000. If he defaults on the debt, he will lose the whole amount, and the legal costs of filing for bankruptcy and litigating the liquidation of his assets will be $15,000, so each of the debtors will collect $5,000. The debtor has his solicitor draw up the following letter, which he sends to each of the creditors: "I hereby offer you $5,001 if both and my other creditor agree to cancel my debt. If either or both of you decline this offer, I will be legally in default."

   Write a game tree for this situation. and show that it is a dominant strategy for each creditor to accept the offer, allowing the debtor to eradicate his debt and retain the amount $14,998 for himself. (What does this tell you about bankruptcy law?)

2. Bob and Mike both sell DVD playback machines, and both have per-unit costs of $250. They compete on price: the low-price seller gets all the market, and they split the market if they have equal prices. Each prices without knowing the other's price.

   a. Explain why the only Nash equilibrium has both firms charging $250, splitting the market, and making zero profit.

   Suppose the monopoly price for DVD players (the price that maximises the sum of the profits of both firms) is $300. Now suppose Bob advertises that if a customer buys a DVD player from him for $300 and discovers he or she can buy it cheaper at Mike's, then Bob will sell the customer the DVD player with a rebate equal to twice the price difference between the two stores (e.g., if Mike charges $275, then Bob will give the customer a rebate of ($300 − $275) × 2 = $50). Suppose Mike does the same thing.

b. Show that it is now Nash for both stores to charge $300. (Conclusion: pricing strategies that seem to be super competitive can in fact be anticompetitive!)

3. In the Cold War days, the USA and the Soviet Union had both conventional ground and nuclear forces. The Soviets had superior conventional forces. If the Soviets launched a ground attack on NATO countries in Europe, the USA could decide to use either conventional ground forces or nuclear to retaliate. A conventional retaliation would leave the Soviet Union better off and the United States worse off by an equal amount. If the USA retaliated with nuclear force, a nuclear war would ensue and the USA would be, say, one hundred times worse off than in the conventional case. The Soviet Union would suffer just as much as the USA in the nuclear case.

   a. Draw a game tree for this problem, show the payoffs (set the status quo—in which the Soviet Union does not attack—to a payoff of zero for both sides), and find an equilibrium for the game.

   b. Repeat the above, supposing the USA can precommit to using nuclear deterrence.

4. A group of ten students in game theory class play the following game. Each student is given $1 and is handed the following structions:

   > You may anonymously deposit any portion of your $1 in a "public account." Whatever you do not deposit in the public account you may keep for yourself. The money in the public account will be multiplied by five and shared among the ten of you.

   Thus, each student gets $5 if all cooperate (contribute their entire dollars to the public good), and each gets only $1 if all defect (contribute nothing to the public account).

   a. Show that this is a ten-player Prisoner's Dilemma,

   b. What are the Nash equilibria to this game if it is repeated a finite number of times?

   c. What are the equilibria if the game is repeated indefinitely with probability $p = 0.99$ each time, assuming the players have discount rate zero?

d. What do you think really happens when this game is played?

5. Consider Figure 5 in Rothschild's paper (Reading 6, p.28, in the Package).

   a. Show that if his High$-X$ > Low$-Y$, then there is no Nash equilibrium in pure strategies (that is, without probabilistic mixing).

   b. How, if at all, does this affect his conclusion that commitment can make a threat credible?

6. What is the definition of a strategic interaction? Consider a strategic situation that you are familiar with from work, uni, or through social contacts.

   a. Describe it, briefly. Who are the players?

   b. What are the possible actions of each of them? Does one (or more) move first (and be seen to move first)? Who?

   c. Plot an outcomes matrix (if the number of players is not too many, and the number of possible actions is not too many) with the outcomes for each. If the matrix is a cube or worse, discuss a few of the possible combinations of actions and the payoffs for each player.

   d. Can you reduce the numbers of possible actions? If so, do so.

   e. Are there one or more players who are peripheral (whose actions have only a marginal impact on the other players)? If so, remove them.

   f. Can the outcomes be easily ranked for each remaining player? If so, do so.

   g. Can you solve for the equilibrium of the interaction? Do so. If not, why not? (What additional information would allow solution?)