1.5.3 Big Monkey, Little Monkey

Actors:

Big Monkey BM, and

Little Monkey LM.

Game: Warifruits grow only at the top of waritrees: a monkey must climb up and shake the branch for each fruit to fall. A warifruit provides 10 Kc (kilocalories).

- If BM climbs, shakes, runs back — her cost is 2Kc.
- If LM climbs, shakes, runs back — his cost is 0Kc.
- If both climb, etc., then BM eats 7Kc, and LM eats only 3Kc, since BM is a hog.
- If BM climbs while LM waits, then BM eats 6Kc, and LM eats 4Kc, since LM eats more before BM arrives.
- If LM climbs while BM waits, then BM eats 9Kc, and LM eats 1Kc, since BM eats most of the fruit before LM arrives.

Decision to maximise net gain?

- wait, or
- climb?

Three possibilities

- BM decides first,
- LM decides first, or
- They decide together.

1.5.3.1 BM decides first:

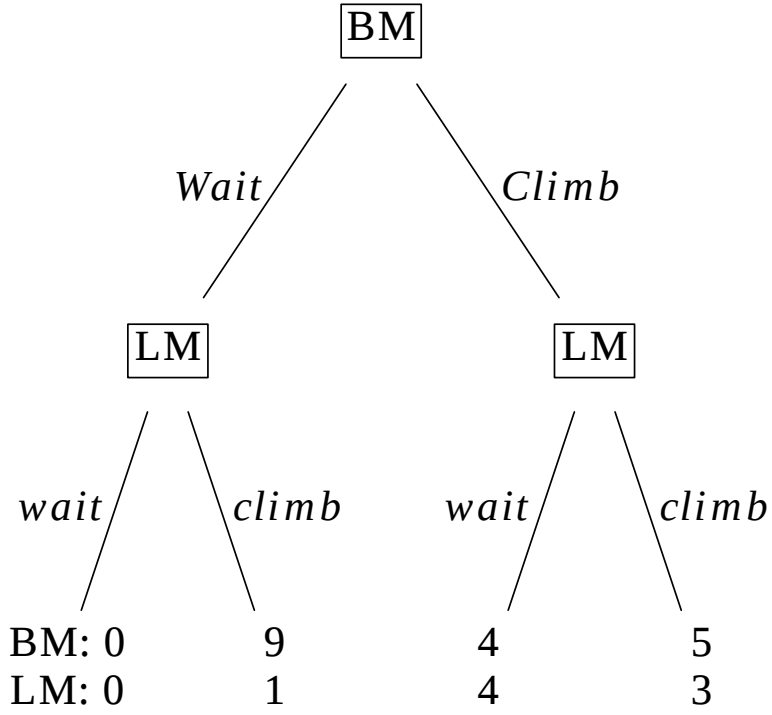

Figure 1. Two Monkeys: BM Decides First

cc: LM will climb no matter what
cw: LM will choose the opposite action of BM
wc: LM will choose the same action as BM
ww: LM will wait no matter what

BM decides first ...

BM has two strategies: *W* or *C*

LM has four strategies (each of which is 2 actions):

- climb no matter what: *cc*
- wait no matter what: *ww*
- do the same as BM: *wc*
- do the opposite of BM: *cw*

Solution:
LM opposite *cw*, and BM *W*

So the Nash equilibrium in strategies is (*W, cw*),
leading to the payoffs: (9,1)

LM delegates his decision beforehand

But what if LM has to *delegate* his decision: so LM's agent will get to move second — and will then know BM's move — but LM has to direct his agent *now*, before either of them knows BM's move.

LM can choose one of the four strategies above, and it becomes a simultaneous-move game.

Use the Normal Form or Payoff Matrix:

		Little Monkey			
		cc	*cw*	*wc*	*ww*
BM	*W*	9, 1	9, 1	0, 0	0, 0
	C	5, 3	4, 4	5, 3	4, 4

TABLE 1. The payoff matrix when BM moves first (BM, LM)

From the payoff matrix …

See that there is a N.E. at (*W,cw*) with a payoff of (9,1), and
a N.E. at (*W,cc*), with a payoff of (9,1).

Note that (*W,cc*) is weakly dominated by (*W, cw*), because 4 > 3.

There is also a N.E. at (*C,ww*), payoff = (4,4),

but this requires a non-credible threat:
LM says, "No matter what you do, I'm waiting."

Non-credible because BM knows that if she plays *W*,
then LM will play not *w*, but *c*, because 1 > 0. (Remember, BM plays first.)

So (*C,ww*) relies on a non-credible threat and is not subgame perfect.

1.5.3.2 LM decides first

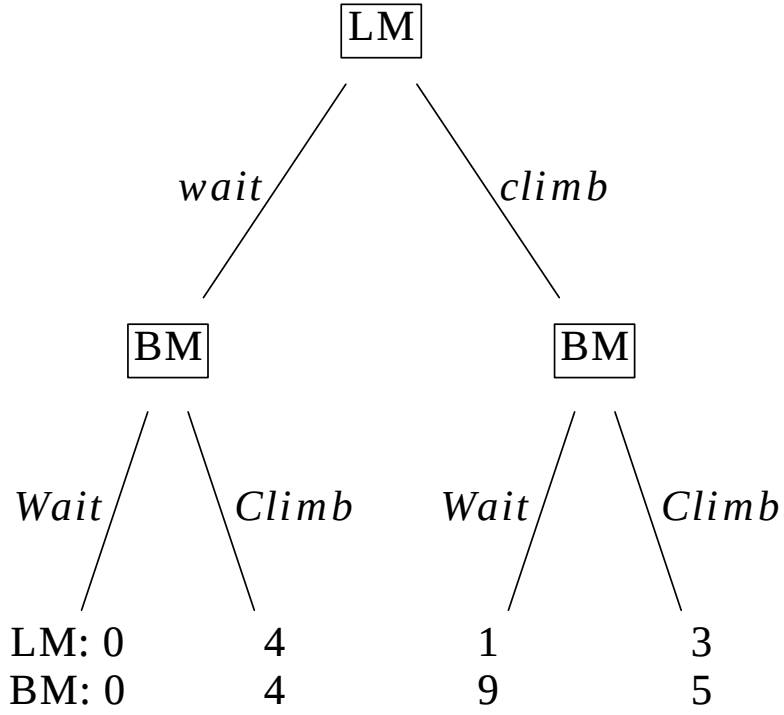

Figure 2. Two Monkeys: LM Decides First

The Normal Form:

		Big Monkey			
		CC	*CW*	*WC*	*WW*
LM	*w*	4, 4	4, 4	0, 0	0, 0
	c	3, 5	1, 9	3, 5	1, 9

TABLE 2. The payoff matrix when LM moves first (LM, BM)

Again, two N.E.: (*w*,*CC*) and (*w*,*CW*),

and also a N.E. not evident from the game tree:

BM's non-credible threat of playing *WW*, to which LM's best response is *c*.

1.5.3.3 The Monkeys decide simultaneously:

Or in ignorance of the other's choice.

Each monkey has two options: climb the tree, or wait.

The tree:

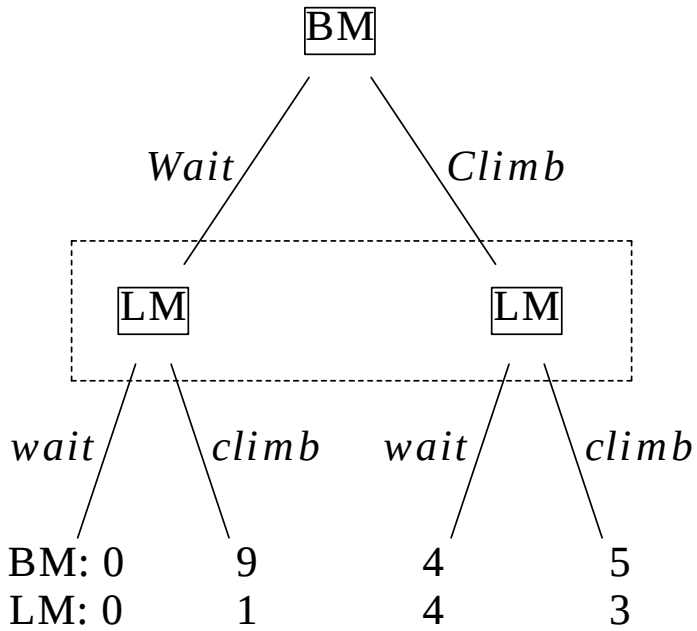

Figure 3. Two Monkeys: Simultaneous Moves

There is an *information set* because LM doesn't know what BM chooses.

Normal Form version:

		Little Monkey	
		c	*w*
BM	*C*	5, 3	4, 4
	W	9, 1	0, 0

TABLE 3. The payoff matrix (Big Monkey, Little Monkey)

See two clear N.E.: at (*W,c*) and (*C,w*) — the first favours BM and the second favours LM.

Plus a third N.E.:
BM randomises and chooses *C* and *W* with probability ½, and LM does the same — a *mixed strategy Nash equilibrium*: results in *expected* payoffs of 4½ for BM and 2 for LM.

(With probability ¼ both wait and get zero reward , and sometimes both climb the tree.)

Market Analogy

e.g. Consider OPEC as an effective cartel:

Saudi Arabia was the “swing” producer, it would unilaterally act to keep oil prices high by reducing its production when one of the smaller member cheated and increased its production of oil.

Not through altruism, but—as with Big Monkey—through the logic of the situation: the smaller producers took advantage of the common knowledge that the cartel would collapse unless the Saudis limited their production.

Saudi captured for itself a sufficiently large share of the benefits of the high prices that it was rationally willing to bear a disproportionate share of the cost of maintaining the cartel.

1.5.4 A Grade Game

- Choose your grade for this assignment.
- Write your grade & your name on the paper.
- Fold the paper and return it.
- But your grade depends on whom you're matched with:

		The other student	
		HD	PS
You	HD	48%	90%
	PS	48%	63%

TABLE 4. Your payoff matrix

1.5.5 The Gift of the Magi

(from O. Henry's short story)

The players and the game:

Della and Jim were deeply in love and eager to make any sacrifice to get a really worthy Christmas present for the other.

- Della would sell her hair (to wig-makers) to get Jim a chain for his heirloom watch, and
- Jim would sell the watch to buy a comb for Della's beautiful hair.

The Gift of the Magi

		Jim	
		Sell watch	Keep watch
Della	Keep hair	**1, 2**	0, 0
	Sell hair	−2, −2	**2, 1**

TABLE 5. The payoff matrix (Della, Jim)

A non-cooperative, positive-sum game,

with two Nash equilibria.

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