3. Strategic Moves

3.1 Game Trees and Subgame Perfection

3.1.1 Subgame Perfect Equilibria

Nash Equilibria from non-credible threats are poor predictors of behaviour.

A subgame: is a smaller game within a larger game with two special properties:

1. once players begin playing the subgame, they do so for the rest of the game;
2. the players all know when they are playing the subgame.

The subgame's *subroot* node: the initial node: the subgame consists of the subroot and all its successors — property 1.

If every information set that contains a decision node of the subgame *does not* contain decision nodes that are not part of the subgame — property 2.

The subgame preserves the original game's:

- set of players,
- order of play,
- set of possible actions, and
- information sets.

Rational behaviour in the full game should be rational in the subgame.

Defn: A strategy profile is a **subgame-perfect equilibrium**[1] (SGPE) of a game *G* if this strategy profile is also a N.E. for *every* subgame of *G*.

With perfect information (singleton information sets), the SGPE = those from backwards induction (B.I.).

B.I. eliminates non-credible threats, so a N.E. ⇔ a SGPE, with perfect information.

The real power of SGPE occurs when there is *not* perfect information — when players are not always aware of what their opponent did (modelled by multi-node information sets).

So long as there is perfect information, backwards induction results in Nash equilibria that are S.G.P.

S.G.P.E. don't rely on non-credible threats/promises.

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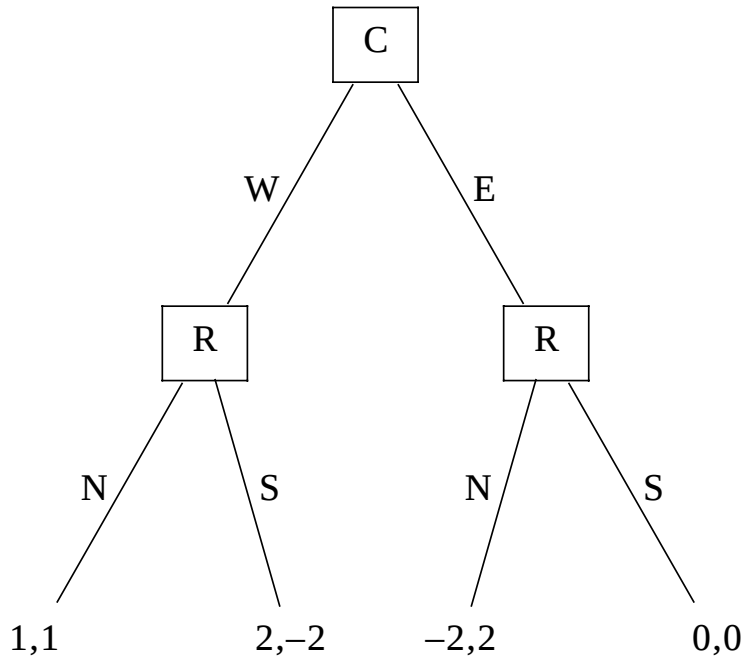
1. Reinhart Selten received the 1994 Nobel Prize in Economics for his development of this concept.

3.1.2 An example: side payments

A sequential game, in which there may be "side payments" from one player to another:



Side Payments 1 (R,C)

The second mover, R, has the following option:

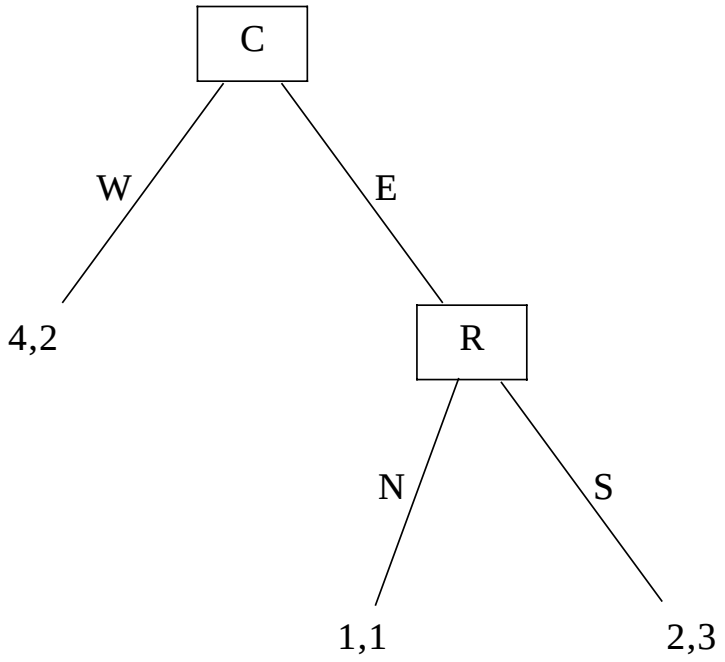
- Before his move, he can promise or threaten to reduce his payoff to any positive number or zero by adding the same amount to C's payoff.
- This can only happen at one final outcome (only one of NE, NW, SE, or SW).
- This is a promise (or threat) of a *side payment*

What would happen without the promise of a side payment?
What would happen with such a promise?

Side payments in general can only occur when there are binding contracts: cooperative games.

3.1.3 Another example: side payments

A second game tree with side payments possible:



Side Payments 2 (R,C)

We distinguish:

- compellent promises (threats) from
- deterrent promises (threats).

Two separate possibilities:

- R: "If W, then payoffs 2½, 3½"
  a compellent promise
- R: "If SE, then payoffs of ½, 4½"
  a deterrent promise

Which is *credible* or *a self-enforcing contract*?