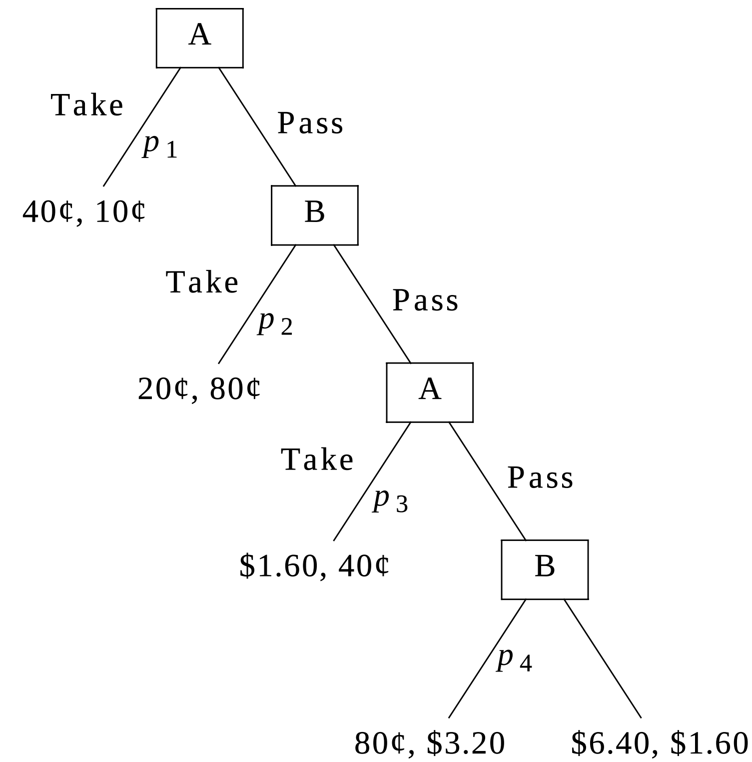
1.10 GameTrees: Extensive Form

To completely specify a game, we need to know:

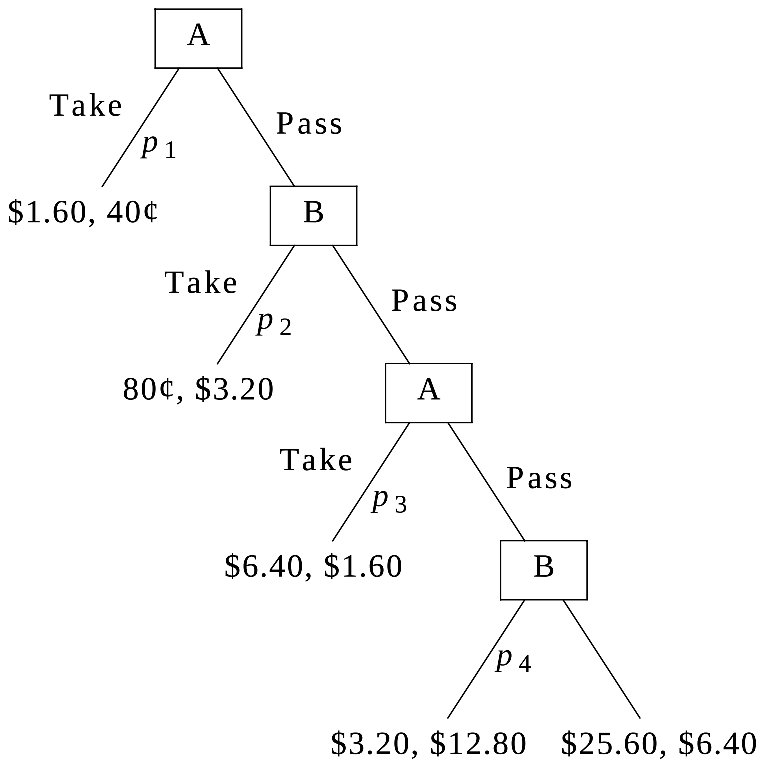
- *the set of players* (who)
- the set of situations (nodes) and the sequence of their appearance, i.e., *the game tree*
- who moves at each decision node, i.e., *the player partition* (order of play , i.e., sequential)
- which decision nodes each player can distinguish i.e., *the information partition* (who knows what when)
- what choices each player has at each information set (choice of actions)
- what the payoffs to the players are whenever the game ends, i.e., *the payoff function* (who gets what when)

Games in which each player knows exactly what has happened in previous moves are called *games with perfect information*. Otherwise they are *games with imperfect information*. This is a structural property of the game.

1.10.1 CentipedeGame 1



1.10.2 Centipede Game 2



1.11 Common Knowledge

Common knowledge = complete information

An example from Halpern:

- Two divisions camped on two hilltops overlooking a common valley.
- where awaits the enemy.
- if both divisions attack the enemy simultaneously they will win a battle, whereas
- if only one division attacks it will be defeated.
- no plans for launching an attack on the enemy
- but General Able of the first division wishes to coordinate with General Baker of the second a simultaneous attack (at some time the next day).
- Neither will attack unless he is sure that the other will attack with him.
- can only communicate by means of a messenger.
- one hour to get between Able and Baker
- possible that he will get lost in the dark or be captured by the enemy.
- Question: How long will it take them to coordinate an attack?

Suppose the messenger sent by General Able makes it to General Baker with a message saying, "Let's attack at dawn." Will Baker attack?

Of course not, since Able does not know that Baker has received the message, and thus may not attack.

So Baker sends the messenger back with an acknowledgement. Suppose the messenger makes it. Will Able attack?

No, because now Baker does not know that Able got the message, so Baker thinks that Able may think that Baker didn't get the original message, and thus not attack.

So Able sends the messenger back with an acknowledgement. But of course this is not enough either.

How many acknowledgements, suitable acknowledged, are necessary for Able and Baker to know with absolute certainty that the other division will attack at dawn?

No number of acknowledgements sent back and forth ever guarantees agreement.

There is a lack of *common knowledge* about the schedule. This is a case of *incomplete information*.

In practice, most generals would act without perfect information, perhaps after Able had received Baker's first acknowledgement?

1.12 Definitions— Rasmusen

- The **players** are the individuals who make decisions. Each *player's goal* is to maximise his utility by choice of actions.
- **Nature** is a non-player who takes random actions at specified points in the game with specified probabilities.
- An **action** or *move* by player i is a choice he can make.
- Player i's **action set** is the entire set of actions available to him.
- An **action combination** is an ordered set of one action for each of the n players in the game.
- The **players' order of play** must be specified.
- Player i's **strategy** s_i is a rule that tells him which action to choose at each instant of the game, given his information set.
- Player i's **strategy set** or *strategy space* is the set of strategies or rules available to him.
- A **strategy combination** is an ordered set consisting of one strategy for each of the n players in the game.
- By player i's **payoff** we mean either:
  1. The utility he receives after all players and Nature have picked their strategies and the game has been played out; or

2. The expected utility he receives as a function of the strategies chosen by himself and the other players.

- The **outcome** of the game is a set of interesting elements that the modeller picks out from the values of actions, payoffs, and other variables after game is played out.
- An **equilibrium** is a strategy combination consisting of a best strategy for each of the *n* players in the game.
- An **equilibrium concept** or *solution concept* is a rule that defines an equilibrium based on the possible strategy combinations and the payoff functions.
- Player *i*'s **best response** or *best reply* to the strategies chosen by the other players is the strategy s_i^* that yields him the greatest payoff.
- The strategy s_i^* is a **dominant strategy** if it is a player's strictly best response to **any** strategies the other players might pick, in the sense that whatever strategies they pick, his payoff is highest with s_i^*.
- His inferior strategies are **dominated strategies**.
- A **dominant strategy equilibrium** is a strategy combination consisting of each player's dominant strategy.

- A **cooperative game** is a game in which the players can make binding commitments, as opposed to a **non-cooperative game**, in which they cannot.
- Cooperative games often allow players to split the gains from cooperation by making **side-payments**—transfers among themselves that change the prescribed payoffs.
- Strategy s_i^* is a **weakly dominant strategy** if it is a player's best response to any strategies the other player might pick, in the sense that whatever strategies they pick, his payoff is no smaller with s_i than with any other strategy, and is greater in some strategy combination.
- An **iterated dominant strategy equilibrium** is a strategy combination found by deleting a weakly dominated strategy from the strategy set of one of the players, recalculating to find which remaining strategies are weakly dominated, deleting one of them, and continuing the process until only one strategy remains for each player.
- A **zero-sum game** (or constant-sum game) is a game in which the sum of the payoffs of all the players is zero (or constant) whatever they choose.
- The strategy combination s^* is a **Nash equilibrium** if no player has incentive to deviate from his strategy given that the other players do not deviate.

- A **subgame** is a game consisting of a node which is a singleton in every player's information partition, that node's successors, and the payoffs at the associated end nodes.
- A strategy profile is a **subgame perfect Nash equilibrium** if (*a*) it is a Nash equilibrium for the entire game; and (*b*) its relevant action rules are a Nash equilibrium for every subgame.

The term **sequential rationality**: the idea that a player should maximise his payoffs at each point in the game, re-optimising his decisions at each point and taking into account the fact that he will re-optimise in the future.

A blend of the economic ideas of rational expectations and ignoring sunk costs.

Sequential rationality has great power.

(See Chapter 4 of Rasmusen: 4.3 Nuisance Suits.)

1.13 Oligopolies:Continuous Strategies

e.g., duopoly — two sellers — **von Stackelberg** model describes a dominant firm (once IBM, now Microsoft, or OPEC, etc.)

Model:

☐ Leader (Spring) produces quantity Q_S of bottled water, and

☐ Follower (Crystal) produces Q_C of identical water.

- market demand → price P (equal for both)

$$P = 10 - (Q_S + Q_C), \quad Q_S + Q_C \leq 10.$$

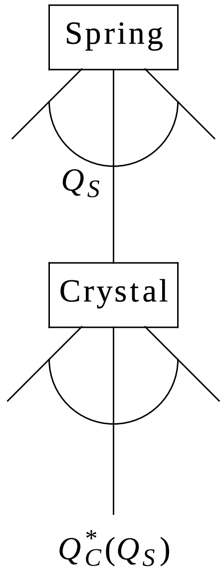
- common knowledge: Spring sets its production level before Crystal does
- each firm's production level: common knowledge
- order of play & output choices: common knowledge
- no capacity constraints
- each firm has identical costs ($i = C, S$):

$$MC_i = AC_i = \$3/\text{unit}$$

- ∴ payoffs for either firm i are ($i = C, S$):

$$\pi_i(Q_S, Q_C) = (10 - Q_S - Q_C)Q_i - 3Q_i$$

- zero output → zero profit



Cournot Rivalry Game Tree

(Quant ➩ Cournot,
Price ➩ Bertrand.)

- What should the Leader (Spring) do? Depends on how the Leader thinks the Follower (Crystal) will react.

  Spring should: *Look forward and reason back*.

- for every fixed level of output for Spring, what is the best (profit-maximising) level that Crystal can do? What is the Follower's *reaction function*?

- Crystal (the Follower): choose Q_C^* to max his profit:

$$\pi_C = P\,(Q_S + Q_C)\,Q_C - TC_C\,(Q_C)$$

$$\pi_C = (10 - (Q_S + Q_C))\,Q_C - 3\,Q_C$$

- From Crystal's viewpoint, Spring's (the Leader's) output is predetermined — a constant Q_S, because Spring will have moved first,

  so Crystal sets $MR_C\,(Q_S, Q_C) = MC\,(Q_C) = 3$ to get Q_C^*:

$$MR_C\,(Q_S, Q_C^*) = (10 - Q_S - Q_C^*) + \frac{\partial P}{\partial Q_C}\,Q_C^* = 3$$

$$\rightarrow Q_C^* = f_C\,(Q_S),$$

  the profit-maximising output Q_C^* of Follower (Crystal) is a function of the Leader's choice Q_S

- This function is known as the *reaction function*, since it tells us how the Follower will react to the Leader's choice (of output in this case).

- It is the locus of points where the family of Crystal's isoprofit curves have zero slope, w.r.t output Q_C. Why?

- Solving for Q_C^*:

$$Q_C^* = \frac{7 - Q_S}{2}$$

  Crystal's *reaction function*, contingent on Spring's action Q_S

- Looking forward & reasoning back, Spring knows this, so Spring's profit:

$$\pi_S = (10 - (Q_S + \frac{7 - Q_S}{2}))\,Q_S - 3Q_S$$

  Solving for maximum π_S ($d\pi / dQ_S = 0$) gives:

  π_S max when $Q_S^* = 3.5$,

  with $\pi_S^* =$ \$6.125 and $P =$\$4.75

  and $Q_C^* = 1.75$ and $\pi_C^* =$ \$3.06

- So Spring's *first-mover advantage* is

  \$6.125 –\$3.06 = \$3.06

- leadership — first mover
- leadership — innovator, monopolist, faced with threat of entry
- incumbent erects barriers to entry by newcomer
- long-term contracts reduce incumbent's flexibility and increase the credibility of defence
- The monopolist quantity in this case is $Q_M = 3.5$, with $P_M =$ \$6.5 and profit $\pi_M = 6.5 \times 3.5 - 3 \times 3.5 =$ \$12.25

  This means that the Leader could pay the Follower not to enter the market by offering him his (Follower's) profit of \$3.06 not to, and still be ahead by \$12.25 – 3.06 – 6.125 = \$3.06.

1.14 Benchmarking Oligopoly Behaviour

Two companies produce homogeneous output.

Linear industry demand curve of $P = 10 - Q$, where Q is the sum of the two companies' outputs, $Q = y_1 + y_2$.

Both companies have identical costs, $AC = MC =$ \$1/unit.

Five possibilities for the equilibrium levels of prices, outputs, and profits.

1.14.1 Competitive Price Taking

Each setting price equal to marginal cost.

Price $P_{PC} =$ \$1/unit, the total quantity $Q = 9$ units between them, and each produces output $y_1 = y_2 = 4.5$ units.

Since $P_{PC} = AC$, their profits are zero: $\pi_1 = \pi_2 = 0$.

1.14.2 Monopolistic Cartel

Collude and act as a monopolistic *cartel*. Each produces half of the monopolist's output and receives half the monopolist's profit.

Output Q_M such that $MR(Q_M) = MC =$ \$1/unit.

MR curve is given by $MR = 10 - 2Q$, which results in $Q_M = 4.5$ units, $P_M =$ \$5.5/unit, and $\pi_M = (5.5 - 1) \times 4.5 =$ \$20.25.

Each produces output $y_1 = y_2 =$ 2.25 units, and earns $\pi_1 = \pi_2 =$ \$10.125 profit.

1.14.3 Cournot Oligopolists

Cournot oligopolists, each choosing an amount of output to maximise its profit, each assuming that the other is doing likewise: not colluding, but competing. They choose simultaneously.

Cournot equilibrium occurs where their *reaction curves* intersect and the expectations of each of what the other is doing are fulfilled.

Firm 1 determines Firm 2's reaction function: "If I were Firm 2, I'd choose my output y_2^* to maximise my Firm 2 profit conditional on the expectation that Firm 1 produced output of y_1^e."

$$\max_{y_2} \pi_2 = (10 - y_2 - y_1^e) \times y_2 - y_2$$

Thus $y_2 = ½(9 - y_1^e)$, which is Firm 2's reaction function.

Since the two firms are identical, Cournot equilibrium occurs where the two reaction curves intersect, at $y_1^* = y_1^e = y_2^* = y_2^e = 3$ units.

So $Q_{Co} = 6$ units, price P_{Co} is then \$4/unit, and the profit of each firm is \$9.

1.14.4 Stackelberg Quantity Leadership

What if one firm, Firm 1, gets to choose its output level y_1 first?

It realises that Firm 2 will know what Firm 1's output level is when Firm 2 chooses its level: this is given by Firm 2's reaction function from above, but with the actual, not the expected, level of Firm 1's output, y_1.

So Firm 1's problem is to choose y_1^* to maximise its profit:

$$\max_{y_1} \pi_1 = (10 - y_2 - y_1) \times y_1 - y_1,$$

where Firm 2's output y_2 is given by Firm 2's reaction function: $y_2 = ½(9 - y_1)$.

Substituting this into Firm 1's maximisation problem: $y_1^* = 4.5$ units, and so $y_2^* = 2.25$ units, so that $Q_{St} = 6.75$ units and $P_{St} =$ \$3.25/unit.

Profits are $\pi_1 =$ \$10.125 (the same as in the cartel case above) and $\pi_2 =$ \$5.063 (half the cartel profit).

1.14.5 Bertrand Simultaneous Price Setting.

Remember: Equilibrium means there is no incentive for either firm to undercut the other.

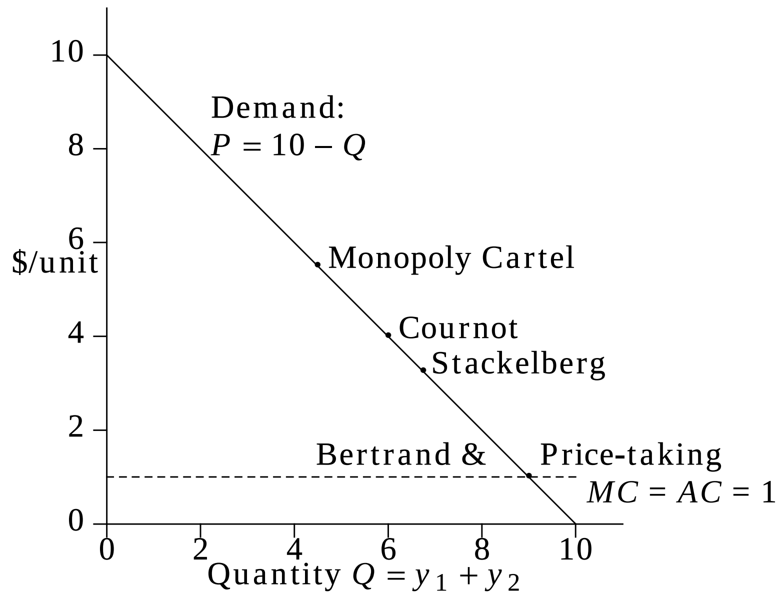
The only equilibrium when they compete using price is where each is selling at $P_1 = P_2 = MC_1 = MC_2 = \$1$/unit.

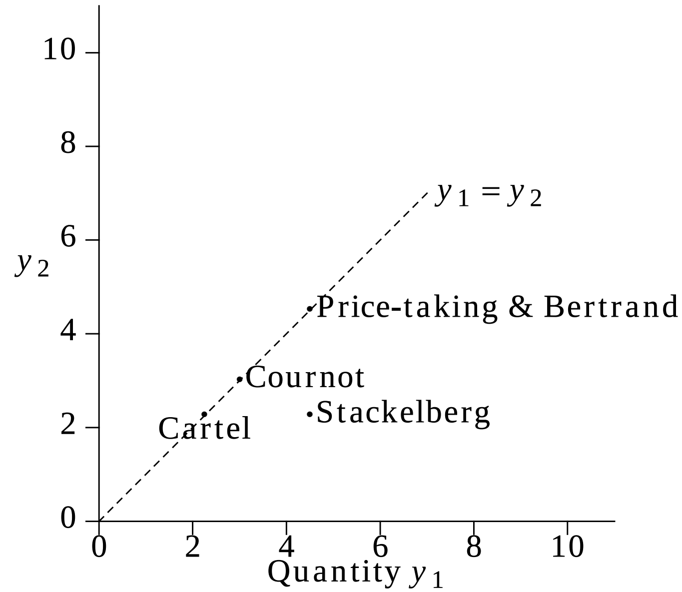
Identical to the price-taking case above.

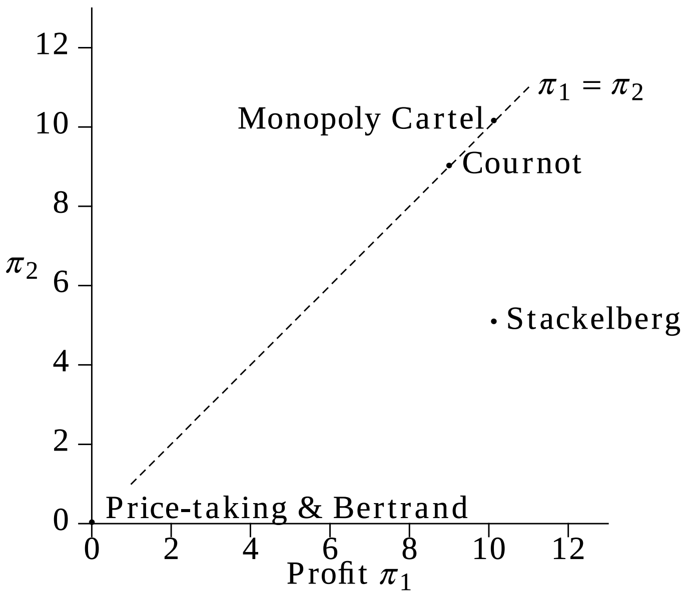
If MC_1 is greater than MC_2, then Firm 2 will capture the whole market at a price just below MC_1, and will make a positive profit; $y_1 = 0$.

	y_1	π_1	y_2	π_2	P	Q
Price-taking	4.5	0	4.5	0	1	9
Cartel	2.25	10.125	2.25	10.125	5.5	4.5
Cournot	3	9	3	9	4	6
Stackelberg	4.5	10.125	2.25	5.063	3.25	6.75
Bertrand	4.5	0	4.5	0	1	9

Summary of Outcomes.






12
10
8
6
4
2
0
π_2
$\pi_1 = \pi_2$
Monopoly Cartel
Cournot
Stackelberg
Price-taking & Bertrand
0 2 4 6 8 10 12
Profit π_1