1.5 More Strategic Interactions

1.5.1 Battle of the Bismark Sea

It's 1943: *Actors*:

- Admiral Imamura: ordered to transport Japanese troops across the Bismark Sea to New Guinea, and
- Admiral Kenney: wishes to bomb Imamura's troop transports.

Decisions/Actions:

- Imamura:
  - — a shorter Northern route or
  - — a longer Southern route
- Kenney: where to send his planes to look for Imamura's ships; he can recall his planes if the first decision was wrong, but then the number of days of bombing is reduced.

Some ships are bombed in all four combinations. Kenney and Imamura each have the same action set — {*North*, *South*} — but their payoffs are never the same. Imamura's losses are Kenney's gains: a **zero-sum game**.

Market analogue ?
Two companies, K and I, trying to maximise their shares of a market of constant size by choosing between two product designs *N* and *S*. K has a marketing advantage, and would like to compete head-to-head, while I would rather carve out its

own niche.

The Battle of the Bismark Sea

(See Rasmusen pp.19)

		Imamura North	*Imamura* South
Kenney	North	**2, –2**	2, –2
Kenney	South	1, –1	3, –3

TABLE 1. The payoff matrix (Kenney, Imamura)

A non-cooperative, zero-sum game,
with an iterated dominant strategy equilibrium.

There is no other equilibrium combination: with all other combinations, at least one of the players stands to gain by changing his action, given the other's action.

Neither player has a **dominant strategy**:

- Kenney would choose
  - — *North* if he thought Imamura would choose *North*, but
  - — *South* if he thought Imamura would choose *South*.
  - — So Kenney's best response is a function of what Imamura does.
- Imamura would choose
  - — *North* if he thought Kenney would choose *South*, but
  - — *either* if he thought Kenney would choose *North*.
  - — For Imamura, *North* is **weakly dominant.**

And Kenney knows it and chooses *North* too.

The strategy combination (*North*, *North*) is an **iterated dominant strategy equilibrium.** (It was the outcome in 1943.)

(*North*, *North*) is a (Nash) equilibrium, because:

- Kenney has no incentive to alter his action from *North* to *South* so long as Imamura chooses *North*, and
- Imamura gains nothing by changing his action from *North* to *South* so long as Kenney chooses *North*.

1.5.2 Boxed Pigs

Actors: Two pigs are put in a box:

- Big Pig, dominant
- Piglet, subordinate.

Game:

- a lever at one end of the box dispenses food at the other end.
- So the pig that presses the lever must run to the other end to eat;
- but by the time it gets there, the other pig has eaten most, but not all, of the food.
- Big Pig is able to prevent Piglet from getting any of the food when both are at the food.

Assuming the pigs can reason like game theorists, which pig will press the lever?

Six units of food are delivered:

- If Piglet presses the lever, then BP eats all 6 units; but
- if BP pushes the lever, then Piglet eats 5 of the 6 units before BP brushes him aside.
- If both press together, then Piglet, who runs faster, gets 2 units before BP arrives;
- running costs half a unit.

Decision:

- wait for the food, or
- press the lever & run for the food

The Boxed Pigs

(See Rasmusen pp.22)

		Piglet	
		Press	Wait
Big Pig	Press	3½, 1½	**½, 5**
	Wait	6, –½	0, 0

TABLE 2. The payoff matrix (Big Pig, Piglet)

A non-cooperative, positive-sum game, with a Nash equilibrium.

What is best for Piglet?

- He is better off to wait.

What is best for Big Pig?

- If Piglet presses, then BP gets:
  3½ if she presses or
  6 if she waits.
- If Piglet waits, then BP gets
  ½ if she presses or
  0 if she waits.
- So BP's *best response* differs depending on what she conjectures her rival will do.

How to resolve this dilemma?

If BP puts herself in the shoes of her rival, then BP realises that Piglet's best action is unambiguous: Wait.

If BP presumes Piglet is rational, then she knows she should use her best response to her rival's waiting: thus she presses.

Rational behaviour, therefore, indicates a surprising conclusion:

Big Pig presses the lever and Piglet gets most of the food.

Weakness, in this case, is strength!

Unlike the Prisoner's Dilemma, Boxed Pigs generates no conflict between individual rationality and collective rationality.

$\because$ The Nash Equilibrium is Pareto efficient in this game.

The outcome cannot be changed without making one of the players (Piglet) worse off.

The outcome may not be fair—the pig that does all the work gets the smaller share—but there is no alternative that the players unanimously prefer.

e.g. Consider OPEC as an effective cartel:

Saudi Arabia was the "swing" producer, it would unilaterally act to keep oil prices high by reducing its production when one of the smaller member cheated and increased its production of oil.

Not through altruism, but—as with Big Pig—through the logic of the situation: the smaller producers took advantage of the common knowledge that the cartel would collapse unless the Saudis limited their production.

Saudi captured for itself a sufficiently large share of the benefits of the high prices that it was rationally willing to bear a disproportionate share of the cost of maintaining the cartel.

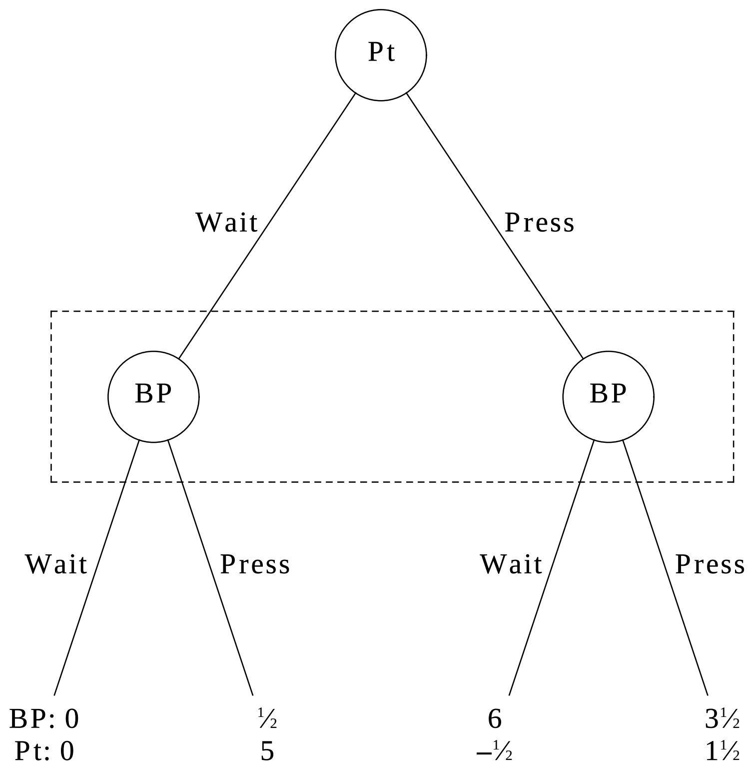

Figure 1. Boxed Pigs, Extensive Form

NB: dashed information set.
(or use a dashed line between nodes in the set)

Consider Piglet's choice; consider BP's choice.

Introduce: extensive form, game tree, information set.

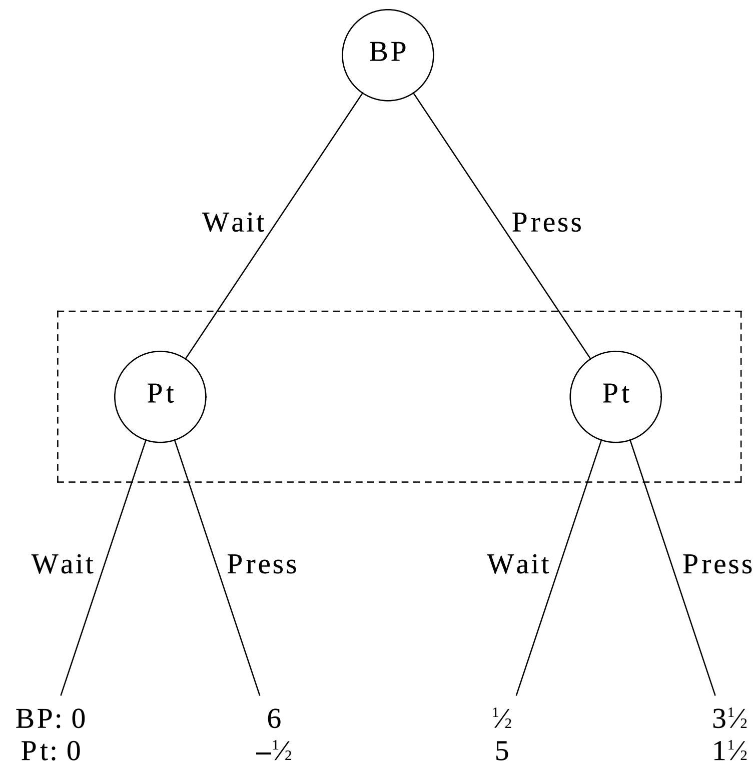

Figure 2. Boxed Pigs, Extensive Form

NB: dashed information set.

Piglet doesn't know which node it's at: BP-W or BP-P.
Whichever, Wait > Press (why?)

1.5.3 A Grade Game

- Choose your grade for this assignment.
- Write your grade & your name on the paper.
- Fold the paper and return it.
- But your grade depends on whom you're matched with:

		The other student HD	*The other student* PS
You	HD	48%	90%
You	PS	48%	63%

TABLE 3. Your payoff matrix

1.5.4 The Battle of the Sexes

The Players & Actions:

- a man (Hal) who wants to go to the Theatre and
- a woman (Shirl) who wants to go to a Concert.

While selfish, they are deeply in love, and would, if necessary, sacrifice their preferences to be with each other.

No iterated dominant strategy equilibrium.
Two *Nash equilibria*:

- (*Theatre*, *Theatre*): given that Hal chooses *Theatre*, so does Shirl.
- (*Concert*, *Concert*), by the same reasoning.

How do the players know which to choose?

If they do not talk beforehand, Hal might go to the Concert and Shirl to the Theatre, each mistaken about the other's beliefs.

Focal points?

Repetition?

Each of the Nash equilibria is collectively rational (Pareto-efficient): no other strategy combination increases the payoff of one player without reducing that of the other.

There is a **first-mover advantage** in this sequential-move game.

Market analogue ?

- An industry-wide standard when two dominant firms have different preferences but both want a common standard.
- The choice of language used in a contract when two firms want to formalise a sales agreement but prefer different terms.

The Battle of the Sexes

		Shirl	
		Theatre	Concert
Hal	Theatre	**2, 1**	–1, –1
	Concert	–1, –1	**1, 2**

TABLE 4. The payoff matrix (Hal, Shirl)

A non-cooperative, positive-sum game, with two Nash equilibria.

Q: which are Pareto Optimal or efficient?

1.5.5 The Gift of the Magi

(from O. Henry's short story)

The players and the game:

Della and Jim were deeply in love and eager to make any sacrifice to get a really worthy Christmas present for the other.

- Della would sell her hair to get Jim a chain for his heirloom watch, and
- Jim would sell the watch to buy a comb for Della's beautiful hair.

The Gift of the Magi

		Jim	
		Sell watch	Keep watch
Della	Keep hair	**1, 2**	0, 0
	Sell hair	–2, –2	**2, 1**

TABLE 5. The payoff matrix (Della, Jim)

A non-cooperative, positive-sum game,

with two Nash equilibria.