Question 1

In each case an investment grows from an initial value $P$ to a final value $S$ over a time period $T$. Give both the annual ordinary compound growth rate and the annual continuously compounded growth rate.

(a) $P=4.5$, $S=7.2$, $T=4$ years.

$$ F = \frac{S}{P} = \frac{7.2}{4.5} = 1.6 $$

$T = 4$ years

**Continuously Compounded**

$$ F = e^{r_c T} $$

$$ r_c = \frac{\ln(F)}{T} $$

$$ r_c = 0.1175 $$

$$ r_c = 11.75\% $$

**Ordinary Compounding**

$$ F = (1 + r_0)^T $$

$$ r_0 = F^{1/T} - 1 $$

$$ r_0 = 0.1247 $$

$$ r_0 = 12.47\% $$

(b) $P=100$, $S=140$, $T=32$ months.

**Continuous compounding**

$$ F = S/P = 140/100 = 1.4 \ ; \ T = 32 \text{ months} = 32/12 \text{ years} $$

$$ r_c = \frac{\ln(F)}{T} = \frac{(\ln 1.4)(12/32)}{32/12} = 0.1262 = 12.62\% $$

**Ordinary compounding**

$$ F = (1 + r_0)^T \ ; $$

$$ r_0 = F^{1/T} - 1 $$

$$ r_0 = (1.4)^{1/32} - 1 = 0.1344 = 13.44\% $$
(c) If contract specifies a continuously compounding rate and you wish your investment to triple within 20 years, what annual continuously compounding rate do you require?

**Solution**

Triplet; \( F = 3 \)

\( T = 20 \) years

Continuously Compounded

\[
F = e^{rcT}
\]

\[
r_c = \frac{\ln(F)}{T}
\]

\[
= \frac{\ln(3)}{20}
\]

\[
= 0.0549
\]

\[
= 5.49\%
\]

(d) If contract specifies a ordinary compounding rate and you wish your investment to increase by 50% within 5 years, what annual ordinary compounding rate do you require?

**Solution**

\( F = 1.5 \)

\( T = 5 \) years

Ordinary Compounding

\[
F = (1 + r_0)^T
\]

\[
r_0 = F^{1/T} - 1
\]

\[
r_0 = 0.0845
\]

\[
= 8.45\%
\]

(e) You have the choice of the following three investments

a. 1.6% per quarter ordinary compounding  
b. 0.52% per month continuous compounding  
c. 6.5% for 1 year ordinary compounding. 

Which investment would you choose to maximize your return?

**Solution**

Convert all interest rates into continuous compounding interest p.a.

(a) 1.6% per quarter ordinary compounding \( r_c = \ln(1.016) = 0.0159 \)  
continuously compounding per quarter = 0.0159\times4=0.0635=6.35\% \) p.a.  
(b) 0.52% per month continuously compounding = 0.0052\times12=0.0624=6.24 \% \) p.a.  
(c) 6.5% p.a. ordinary; \( r_c = \ln(1.065) = 0.063 = 6.3\% \) p.a.  
So investment (a) maximizes your return.
Question 2

Give an equation of the form $y=mx+b$.

(a) When $y=0$ $x=15$ and for each 5 unit increase in $x$, $y$ decreases by 2 units.

\[
\text{Slope } m = \frac{\Delta y}{\Delta x} = \frac{-2}{5}
\]

\[
b = y - mx = 0 - \frac{2}{5} \times 15 = 6
\]

So equation of line is

\[
y = 6 - \frac{2}{5}x
\]

(b) When $x=-6$, $y=0$ while when $x=0$, $y=3$

We have two points (-6,0) and (0,3). From this we have

\[
\text{slope } m = \frac{0-3}{-6-0} = \frac{1}{2},
\]

\[
b = 3 \text{ so } y = \left(\frac{1}{2}\right)x + 3
\]

(c) Suppose that demand decreases by 1000 units for every increase in price of $1$, and that when price equals $70$ the demand is zero. Write down the quantity demanded, $Q_D$, as a function of price, $P$, i.e. write an equation of the form $Q_D = m + bP$. You may assume the relationship between quantity demanded and price is linear.

\[
\text{Slope } m = \frac{\Delta Q_D}{\Delta P} = -1000
\]

\[
\Delta P = 1
\]

\[
m = -1000
\]

\[
b = Q_D - mP = 0 - (-1000) \times 70 = 70000
\]

So equation of line is

\[
Q_D = 70000 - 1000P
\]
(d) Suppose that the quantity supplied is zero if the price is less than $10 and that for every $2 increase in price the amount supplied increases by 1000 units. Write down quantity supplied, $Q_S$ as a function of price, i.e. write an equation of the form $Q_S = m + bP$. You may assume the relationship between quantity supplied and price is linear.

Slope $= \frac{\Delta Q_S}{\Delta P}$

$\Delta Q_S = 1000$

$\Delta P = 2$

$m = 500$

$b = Q_S - mP$

$= 0 - (500) \times 10$

$= -5000$

So equation of line is

$Q_S = 500P - 5000$

(e) A market is said to be in equilibrium if $Q_D = Q_S$. Using your answers to parts (c) and (d) find the equilibrium price and quantity.

Equilibrium occurs when $Q_S = Q_D$

$500P - 5000 = 70000 - 1000P$

$1500P = 75000$

$P = 50$

Equilibrium quantity $= 70000 - 1000(50) = 20000$