MBA Maths 2009

Quiz 3 Solutions

Question 1

In each case an investment grows from an initial value P to a final value S over a time period T. Give both the annual ordinary compound growth rate and the annual continuously compounded growth rate

(a) P=4.5, S=7.2, T=4 years.

$$F = \frac{S}{P} = \frac{7.2}{4.5} = 1.6$$

$$T = 4 \text{ years}$$

Continuously Compounded

$$F = e^{r_c T}$$

$$r_c = \frac{\ln(F)}{T}$$

$$= 0.1175$$

$$= 11.75\%$$

Ordinary Compounding

$$F = (1 + r_0)^T$$

$$r_0 = F^{1/T} - 1$$

$$r_0 = 0.1247$$

$$= 12.47\%$$

(b) P=100, S=140, T=32months.

<u>Continuous compounding</u>

F = S/P = 140/100 = 1.4 ; T = 32 months = 32/12 years

r_c = ln F / T = (ln 1.4)(12/32) = 0.1262 = 12.62 %

<u>Ordinary compounding</u>

F = $(1 + r_o)^T$;

$r_o = F^{1/T} - 1 = (1.4)^{12/32} - 1 = 0.1344 = 13.44$ %

(c) If contract specifies a continuously compounding rate and you wish your investment to triple within 20 years, what annual continuously compounding rate do you require?

<u>Solution</u>

Triple; $F = 3$

$T = 20$ years

Continuously Compounded

$F = e^{r_c T}$

$r_c = \dfrac{\ln(F)}{T}$

$= 0.0549$

$= 5.49\%$

(d) If contract specifies a ordinary compounding rate and you wish your investment to increase by 50% within 5 years, what annual ordinary compounding rate do you require?

<u>Solution</u>

$F = 1.5$

$T = 5$ years

Ordinary Compounding

$F = (1 + r_0)^T$

$r_0 = F^{1/T} - 1$

$r_0 = 0.0845$

$= 8.45\%$

(e) You have the choice of the following three investments

a. 1.6% per quarter ordinary compounding
b. 0.52% per month continuous compounding
c. 6.5% for 1 year ordinary compounding.

Which investment would you choose to maximize your return?

<u>Solution</u>

Convert all interest rates into continuous compounding interest p.a.

(a) 1.6% per quarter ordinary compounding r_c=ln(1.016)=0.0159 continuously compounding per quarter= 0.0159×4=0.0635=6.35 % p.a.

(b) 0.52% per month continuously compounding = 0.0052×12=0.0624=6.24 % p.a.

(c) 6.5% p.a. ordinary; r_c=ln(1.065)=0.063=6.3%p.a.

So investment (a) maximizes your return.

Question 2

Give an equation of the form y=mx+b.

(a) When y=0 x=15 and for each 5 unit increase in x, y decreases by 2 units.

$$Slope = m = \frac{\Delta y}{\Delta x}$$

$$\Delta y = -2$$

$$\Delta x = 5$$

$$m = \frac{-2}{5}$$

$$b = y - mx$$

$$= 0 - \frac{-2}{5} \times 15$$

$$= 6$$

So equation of line is

$$y = 6 - \frac{2}{5}x$$

(b) When x=-6, y=0 while when x=0, y=3

We have two points (-6,0) and (0,3). From this we have
slope m=(0-3)/(-6-0) = 1/2,
b=3 so
y=(1/2)x+3

(c) Suppose that demand decreases by 1000 units for every increase in price of \$1, and that when price equals \$70 the demand is zero. Write down the quantity demanded, Q_D, as a function of price, P, i.e. write an equation of the form $Q_D=m+bP$. You may assume the relationship between quantity demanded and price is linear.

$$\text{Slope} = m = \frac{\Delta Q_D}{\Delta P}$$

$$\Delta Q_D = -1000$$

$$\Delta P = 1$$

$$m = -1000$$

$$b = Q_D - mP$$

$$= 0 - (-1000) \times 70$$

$$= 70000$$

So equation of line is

$$Q_D = 70000 - 1000P$$

(d) Suppose that the quantity supplied is zero if the price is less than \$10 and that for every \$2 increase in price the amount supplied increases by 1000 units. Write down quantity supplied, Q_S as a function of price, i.e. write an equation of the form $Q_S=m+bP$ You may assume the relationship between quantity supplied and price is linear.

$$\text{Slope} = m = \frac{\Delta Q_S}{\Delta P}$$

$$\Delta Q_s = 1000$$
$$\Delta P = 2$$

$$m = 500$$

$$\begin{aligned} b &= Q_S - mP \\ &= 0 - (500) \times 10 \\ &= -5000 \end{aligned}$$

So equation of line is

$$Q_S = 500P - 5000$$

(e) A market is said to be in equilibrium if $Q_D=Q_S$. Using your answers to parts (c) and (d) find the equilibrium price and quantity.

Equilibrium occurs when $Q_S=Q_D$

$$\begin{aligned} 500P - 5000 &= 70000 - 1000P \\ 1500P &= 75000 \\ P &= 50 \end{aligned}$$

Equilibrium quantity = 70000 – 1000(50) = 20000