Quiz 2 Solutions 2009

Question 1

Evaluate

(a) $\text{Log}_7 49=2$ because $7^2 = 49$

(b) $\log_{10}0.0001= - 4$ because $10^{-4}=1/10000=0.0001$

(c) $\log_3 3^{1/2}+\log_3 3^{3/2}=2$ because $\log_3 3^{1/2}+\log_3 3^{3/2}= 1/2\log_3 3+3/2\log_3 3=1/2+3/2=2$

(d) $\log_{25}0.2=-0.5$ because $25^{-1/2}=1/5=0.2$

(e) $\log_4 32+ \log_4 2=2$ because $\log_4 32+ \log_4 2= \log_4 64=3$

(f) $\log_6 \sqrt[4]{72} - \log_6 \sqrt[3]{2}$

This was a typo. This question should have read $\log_6 \sqrt[4]{72} - \log_6 \sqrt[4]{2}$, in other words $\sqrt[3]{2}$ should have been $\sqrt[4]{2}$. In which case the answer would have been ½ because

$$\begin{aligned}\log_6 \sqrt[4]{72} - \log_6 \sqrt[4]{2} &= \frac{1}{4}\left(\log_6 72 - \log_6 2\right)\\ &= \frac{1}{4}\log_6 36\\ &= \frac{2}{4} = \frac{1}{2}\end{aligned}$$

Question 2

(a) If an initial investment of 1000 is worth 1250 at the end of 30 months, what was the interest rate expressed as

i. A monthly compound rate?

Using growth factor $F = S/P = 1250/1000 = 1.25 = (1+r)^T = (1+r)^{30}$

$1 + r = (1.25)^{1/30}$; $r = (1.25)^{1/30} - 1 = 0.0075 = 0.75\%$ per month

ii. An annual compound rate? $T = 2.5$ (30 months = 2.5 years)

Using growth factor $F = S/P = 1250/1000 = 1.25 = (1+r)^T = (1+r)^{30/12}$

$1 + r = (1.25)^{12/30} = (1.25)^{2/5}$; $r = (1.25)^{2/5} - 1 = 0.0934 = 9.34\%$ per year

(b) If the growth factor for 1 year is 1.15 what is the interest rate expressed as

i. An annual compound rate?

$F = 1.15 = (1 + r)^1$; $r = 1.15-1 = 0.15 = 15\%$ per year

ii. A quarterly compound rate?

$1 + r = (1.15)^{1/4}$; $r = (1.15)^{1/4} - 1 = 0.03556 = 3.556\%$ per quarter

(c) Referring to part (b) how much will the investment be worth at the end of 18 months?

You could use either the annual or the quarterly interest rates to calcuate this, the answer would be the same.

If you used the annual interest rate then T = 18months = 1.5 years,
the growth factor, F, would be $(1.15)^{1.5} = 1.233$.

Note that if you used quarterly interest rates then $F=(1.03556)^6 = 1.233$

(d) If you invest $1000 at a quarterly compound interest rate of 3% how long will it take for your investment to double?

$F = S/P = 2 = (1+r)^T = (1+ 0.03)^T$; $T = \ln 2/ \ln 1.03 = 23.445$ quarters or 5.86 years