LOUIS YEUNG

- LOCATION : ROOM 3122, QUADRANGLE BUILDING

CONTACT : TEL : 9385 3286

FAX : 9313 6337

EMAIL : L.Yeung@unsw.edu.au

OR l.yeung@unsw.edu.au

WELCOME

- FIRST, PLEASE ALLOW ME TO WELCOME YOU ALL WARMLY TO THE MBA MATHS COURSE DEALING WITH MATHEMATICAL CONCEPTS AND METHODS

PROGRAM OBJECTIVE

- To achieve sufficient mathematical confidence, proficiency and skills so as to be able to cope well with other quantitative subjects such as data analysis, finance subjects or any other course requiring such mathematical knowledge

A FEW HINTS ON HOW TO STUDY MATHS

- Initiative: do not be afraid to try, to make mistakes
- Learning from mistakes breeds understanding
- Perseverance: try, try, try… until you get it
- Practise, practise,……for
- PRACTICE MAKES PERFECT

- Understanding what you are doing and why you are doing it is crucial, essential for mastering mathematics
- Never carry out the mathematical operations or procedures in a mechanical way without understanding what is going on. It is the surest way to disaster.

Program/Curriculum/Road Map

- Algebra M1
- Powers and Logs M2
- Functions M3
- **Calculus M4**

PROGRAM TIMETABLE

- MODULES M1,M2,M3 IN THE NEXT 3 WEEKS
- MODULE 4 (CALCULUS) IN JUNE 2009 FOR 2 WEEKS

BIBLIOGRAPHY

- ESSENTIAL TEXT:

- WATSON, J. Managing mathematics
- UNSW Printing Unit

OTHER REFERENCES

- SPERLING, A. & STUART, M. :
  Mathematics Made Simple. Doubleday

HAEUSSLER,E.F.,PAUL, R.S. & WOOD, R.
Introductory Mathematical Analysis.
Pearson Prentice Hall (Specially the
Algebra Refresher chapter)

COURSE WEBSITE

- http://www2.agsm.edu.au/Maths5231/index.html

ROAD MAP

- MODULE 1 : ALGEBRA
- MODULE 2 : POWERS AND LOGARITHMS
- MODULE 3 : FUNCTIONS
- MODULE 4 : CALCULUS

MODULE 1
Algebra

- Rules for + - × ÷
- Negative Numbers
- Rules for fractions
- Factorizing Examples
- Solving equations
- Powers

MODULE 2
Powers and Logs

- Powers Logs
- Compound Interest and Growth Factors
- Continuously compounded Interest

MODULE 3
Functions

- Linear Functions
- Quadratic Functions
- Exponential Functions
- Power Functions
- Hyperbolic Functions

<u>**Module 4** Calculus</u>
(2 weeks in JUNE 2009)

- Slope of a quadratic function
- Formal definition of a derivative
- Derivatives of power functions
- Derivatives of sums of power functions
- Derivatives of exponential functions and Logarithmic functions
- Product Rule, Quotient Rule and Chain Rule
- Applications

Module M1
Algebra

- PREREQUISITES

- Knowledge of basic arithmetic operations and their priority rules

INTENDED LEARNING OUTCOMES

- Develop the ability to manipulate and interpret algebraic expressions
- Develop the ability to express simple and practical problems algebraically
- Develop the ability to solve these algebraically expressed problems

Module M1
Algebra

- Rules for + - × ÷
- Negative Numbers
- Rules for fractions
- Factorising Examples
- Solving equations
- Powers

THE NUMBER LINE

SOME PRELIMINARIES

- $27 - 23 = ?$; $23 - 27 = ?$
- $0.24 + 3.791 = ?$; $4.6 - 8.92 = ?$
- $16.71 \times 0.05 = ?$; $173.42/0.12 = ?$

Evaluate the following

- 16% of 800 = ?
- 27 as a percentage of 60 = ?

Algebra
Rules for + - × ÷

• Rule	• Example
$a+b=b+a$	$3+2=2+3$
$a-b\neq b-a$	$3-2 \neq 2-3$
$a-b= -b+a= -(b-a)$	$3-2 = -2+3= -(2-3)$
$a\times b=b \times a$	$3 \times 4=4 \times 3$
$a/b \neq b/a$	$8/2 \neq 2/8$
$a/b =1/b \times a$	$8/2=1/2 \times 8$
$a \times(b+c)=a \times b+a \times c$	$2 \times(3+4)=2 \times 3+2 \times 4$
$-a \times -b=a \times b$	$-2 \times -3=2 \times 3$
$(a+b)/c=a/c+b/c$	$(6+8)/2=6/2+8/2$
$a/c+b/d=(ad+bc)/cd$	$6/3+8/2=(12+24)/6)$

ADDITION AND SUBTRACTION

- 1. $2a + b - a + 3b$
- 2. $a + c - a + 2c$
- 3. $8 + 2x - 5 + 3x$
- 4. $5ab + 9 - 3ba + 6$
- 5. $3x - 2y - 2x + 5y$
- 6. $2x + 3y + x + 5y$

MULTIPLICATION AND DIVISION

- 1. $3a \times 5$ 2. $6a \times 2b$
- 3. $5ab \times 2b$ 4. $5ab \times ac$
- 5. $8p \times 4q$ 6. $-5a \times -2b$

- 7. $12a \div 3$ 8. $18b \div 6b$
- 9. $15a \div 10b$ 10. $48ab \div 16b$
- 11. $28abc \div 7ac$ 12. $14a \div (-7a)$

Algebra
Rules for + - × ÷

- Rules for order of operations
  - GODMAS
  - G&O=grouping and ordering symbols
    (Parentheses and exponents)
  - D&M=division & multiplication
  - A&S=addition and subtraction

Examples

$3\times4+2=14; \neq 18$; $3\times(4+2)=18$

$2 + 12/4 = ?$; $(10 + 6)/4 = ?$

$6x5-11 = ?$; $5 \text{ x } 6/15 = ?$

Algebra
Negative Numbers

- Negative numbers represent value subtracted e.g. debt, loss
- Negative numbers are used in any context where measurement has 2 possible directions e.g. a longitude of 270° is the same as -90°.
- Rules are

  positive ×positive=positive

  positive ×negative=negative

  negative ×positive=negative

  negative ×negative=positive

MORE GENERALLY

- Multiplying or dividing two numbers with the same sign yields a positive answer
- Multiplying or dividing two numbers with different signs yields a negative answer

Evaluate

- $-4 \times 7 = ?$
- $(-4) \times (-7) = ?$
- $50/(-10) = ?$

Algebra
Negative Numbers

- Examples

$x+(-y)=x-y$

$x\times(-1)=-x$

$x\times(-y)=-xy$

$-6(-3-7)=-6\times-10=60$

$-5xy\times-6\times-z/(-3x)=10yz$

ABSOLUTE VALUE

- Absolute value of $x = |x|$
- $|x| = x$ if $x > 0$
- $|x| = -x$ if $x < 0$

Algebra
Rules for Fractions

- Rules

  $Q/m=FQ/Fm$

  $Q_1/m+Q_2/n=(nQ_1+mQ_2)/nm$

- Examples

  $700/200=7/2$

  $15xy/3y=5x$

  $2xz^2/4xy=z^2/2y$

  $7/2+5/3=(21+10)/6=31/6$

  $3+R/100=(300+R)/100$

  $(1+r)/(1+i)-1= (1+r)/(1+i)-(1+i)/(1+i)=(1+r-1-i)/(1+i)=(r-i)/(1+i)$

SUBTRACTION OF FRACTIONS

- $Q_1/m - Q_2/n=(nQ_1 - mQ_2)/nm$

- *Examples*

  $7/2 - 5/3=(21-10)/6=11/6$

  $3 - R/100=(300 - R)/100$

MULTIPLICATION OF FRACTIONS

- $Q_1/m \times Q_2/n = Q_1Q_2/mn$

- *Example*

- $11/12 \times 3/7 = 11 \times 3/12 \times 7 = 33/84 = 11/28$

DIVISION OF FRACTIONS

- $Q_1/m \div Q_2/n = Q_1/m \times n/Q_2 = Q_1\, n/mQ_2$

- *Examples*
- $1 \div 1/2 = 1 \times 2/1 = 2$
- $1 \div \frac{1}{4} = 4$
- $1/15 \div 5/6 = 1/15 \times 6/5 = 6/75 = 2/25$

More numerical examples

- $2/3 + 2/5 = ?$; $4/9 - 1/6 = ?$
- $8/9 + 2/3 = ?$; $1\ 1/3 + 2\ ¼ = ?$
- $3\ 1/4 \times 4\ ½ = ?$; $16 \div ¼ = ?$

Algebraic fractions: Examples

- Watson p. 10
- Simplify

$$\text{(i)}\ \frac{2}{3a}+\frac{5}{2a} \quad \text{(ii)}\ \frac{2a}{b}+\frac{3c}{4d}$$

$$\text{(iii)}\ \frac{5a}{2b}-\frac{c}{3b} \quad \text{(iv)}\ \frac{3a}{4d}-\frac{2c}{5b}$$

Examples

(i) $\frac{2}{3a} \times \frac{5}{2a}$ (ii) $\frac{2a}{b} \times \frac{3c}{4d}$

(iii) $\frac{5a}{2b} \div \frac{c}{3b}$ (iv) $\frac{3a}{4d} \div \frac{2c}{5b}$

Algebra

- Common Expansions

$(x+y)^2 = (x+y)(x+y) = x^2+2xy+y^2$

$(x-y)^2 = (x-y)(x-y) = x^2-2xy+y^2$

$(x-y)(x+y)=x^2-y^2$

$(x+a)(x+b)=x^2+(a+b)x+ab$

$(cx+a)(dx+b)=cdx^2+(ad+bc)x+ab$

$(x+y)^3=x^3+3x^2y+3xy^2+y^3$

$(x-y)^3=x^3-3x^2y+3xy^2-y^3$

Algebra

- Examples

$105^2-95^2=$

$(x+5)^2-(x-5)^2=$

$255^2-245^2=$

Algebra

- Example: The value of an investment of \$$P$ after 2 years compounding at 4% interest is

  $P(1+0.04)^2$

  What percentage of the total interest is simple interest?

Factorisation

- Definition of a factor or factors. Examples?
- What does factorisation mean?
- Factorisation as the reverse of expansion
- Breaking up an algebraic expression into its several factors, if possible, i.e., where they exist
- Possibility always exists where it might not be possible to do so

Rules for factoring
See Haeussler et al. p.21, 11th ed.

- 1. $xy + xz = x(y + z)$
- 2. $x^2 + (a + b)\,x + ab = (x + a)(x + b)$
- 3. $abx^2 + (ad + cb)x + cd = (ax + c)(bx + d)$
- 4. $x^2 + 2ax + a^2 = (x + a)^2$
- 5. $x^2 - 2ax + a^2 = (x - a)^2$
- 6. $x^2 – a^2 = (x + a)(x – a)$

- 7. $x^3 + a^3 = (x + a)(x^2 – ax + a^2)$
- 8. $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

FACTORISING QUADRATIC EXPRESSIONS

- SEE WATSON pages 19 - 20

Simple cases of quadratic expressions

- In the following, b,c,d,e are all positive whole numbers and $d > e$
- 1. $x^2 + bx + c = (x + d)(x + e)$
- 2. $x^2 - bx + c = (x - d)(x - e)$
- 3. $x^2 + bx - c = (x + d)(x - e)$
- 4. $x^2 - bx - c = (x - d)(x + e)$

The general case of an algebraic, quadratic expression

- $a \neq 1$;
- $ax^2 + bx + c = (dx + e)(fx + g)$
- $ax^2 + bx + c = (df)x^2 + + g.dx + e.fx + eg$
- $ax^2 + bx + c = (df)x^2 + (gd + ef)x + eg$
- Obviously, $a = df$; $b = gd + ef$; $c = eg$

Factorise the following examples

- 1. $4x^2 + 16x + 15 = ???$
- 2. $6x^2 + 7x - 5 = ???$
- 3. $6x^2 - 5x - 6 = ???$
- 4. $3x^2 - 8x + 4 = ???$

Algebra
Factorising; examples

- factorise examples
- Haeussler p.23 examples (passim)
- 1. $10xy + 5xz =$??
- 2. $x^2 + 4x + 3 =$??
- 3. $x^2 + 3x - 4 =$??

Factorisation examples

- 4. $5x^2 + 25x + 30 =$??
- 5. $x^2 + 8x + 16 =$??
- 6. $x^2 - 6x + 9 =$??
- 7. $x^2 - 25 =$??
- 8. $x^3 + 125 =$??
- 9. $x^3 - 64 =$??

Challenging examples

- 10. $x^6 - y^6 =$??
- 11. $x^2y^2 - 4xy + 4 =$??
- 12. $x^3y^2 - 14x^2y + 49x =$??
- 13. $x^3y - 4xy + z^2x^2 - 4z^2 =$??

SOLVING AN EQUATION BY FACTORISATION

- Solve the following equations

- 1. $(x - 2)(x + 5) = 0$
- 2. $3x (x + 7) = 0$
- 3. $(3x - 5)(2x + 1) = 0$
- 4. $2x^2 = 5x$
- 5. $x^2 = 64$

- 6. $x^2 + 6x + 5 = 0$
- 7. $x^2 + 3x - 18 = 0$
- 8. $x^2 - 5x + 6 = 0$
- 9. $x^2 - x - 12 = 0$
- 10. $3x^2 + 8x + 4 = 0$
- 11. $2x^2 + 3x - 5 = 0$
- 12. $4x^2 - 14x + 6 = 0$

- 13. $5x^2 - x - 4 = 0$
- 14. $3x^2 + 5x = 12$
- 15. $6x^2 = x + 15$

SOLVING LINEAR EQUATIONS

(a) $$\frac{5x - 2}{x + 1} = 0$$

(b) $$\frac{4x}{7 - x} = 1$$

SOLVING QUADRATIC EQUATIONS

- Using the factorisation method
- (a) $9x^2 + 30x + 25 = 0$
- (b) $6x^3 - x^2 - 2x = 0$

USING THE FORMULA

For a general quadratic equation,

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE

$$2x^2 + 3x - 2 = 0 \Rightarrow$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4.2.(-2)}}{2.2}$$

$$= \frac{-3 \pm \sqrt{9 + 16}}{4} = \frac{-3 \pm \sqrt{25}}{4}$$

$$= \frac{-3 \pm 5}{4} = \frac{2}{4} \text{ or } \frac{-8}{4} \Rightarrow$$

$$x = \frac{1}{2} \text{ or } -2$$

FURTHER EXAMPLES

- WATSON p.23, Set 2.8, nos.1-6.

Powers and Logs

- Rules for Powers
- Power and Fraction Examples
- Rules for Logs
- Compound Interest and Growth Factors
- Continuously compounded interest

Rules for Powers

- In addition we have
- $3+3+3+3+3+3+3 = 3 \times 7$

- In multiplication we use *powers* (also known as i*ndices* or *exponents*) for repeated multiplication. We use the notation

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$$

We say that 7 is the *index* or *exponent* and 3 is the *base*.

- In general, we have

- 3+3+3+3+….n times = 3 x n

- In multiplication we use *powers* (also known as i*ndices* or *exponents*) for repeated multiplication. We use the notation

3x3x3x3x….n times = 3^n

We say that n is the *index* or *exponent* and 3 is the *base*.

Rules for Powers/Exponents

- Example

  $3^2 \times 3^3 = (3x3)x(3x3x3) = 3^5 = 243 = 9 \times 27$

  $x \times x^3 = x^1 \times x^3 = x^4$

- Rule1. MULTIPLICATION
- $a^m \times a^n = a^{m+n}$

  A further example $(1.05)^{10} \times (1.05)^{12} = 1.05^{22}$
-

RULE 2. DIVISION

$$\frac{3^3}{3^2} = \frac{3x3x3}{3x3} = 3^{3-2} = 3^1 = 3$$

$$\text{N.B.} \frac{1}{3^2} = 3^{-2}$$

$$\text{More generally, } \frac{1}{a^n} = a^{-n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

RULE 3.
Raising a power to another power

- $(3^2)^3 = (3^2)x(3^2)x(3^2) = (3^{2x3}) = 3^6$

- More generally, $(a^m)^n = a^{mn}$

Further examples : $(2^2)^3 = 2^6 = 4^3 = 64$;

$(y^2)^4 = y^8$; $(1.07^2)^3 = 1.07^6$

Rule 4.

- $(ab)^m = a^m b^m$
- Examples

  $(xy)^3 = x^3y^3$

  $(1.05 \times 1.07)^3 = 1.05^3 \times 1.07^3 = 1.1235^3$
- $(4x5)^2 = (4^2 \times 5^2) = 16 \times 25 = 400$

Alternatively, $(4x5)^2 = 20^2 = 400$

- What about $(a^p b^q)^r$???

Rule 5

- $(a/b)^m = a^m/b^m = a^m\ b^{-m}$
- Example

$$(1.05 \div 1.07)^3 = 1.05^3 \div 1.07^3 = 0.981^3$$

Rule 6. Negative Powers

$$\frac{5x5x5}{5x5x5x5x5} = \frac{1}{5x5} = \frac{1}{5^2}$$

Using powers or exponents,

$$\frac{5^3}{5^5} = 5^{3-5} = 5^{-2} \text{ using division rule}$$

Thus, $5^{-2} = \frac{1}{5^2}$, that is,

5^{-2} is the reciprocal of 5^2

More Examples

$4^{-2}=1/4^{2}=1/16$

$4^{-0.5}=1/4^{0.5}=1/2$

More generally,

- In general $a^{-m}=1/a^{m}$
- If $a^{2}=a^{3}/a=a^{3} \times a^{-1}$, then a^{-1} must be $1/a$
- Similarly $a^{-2}= 1/a^{2}$
- Further examples
- $a^{2}/a^{-1}=a^{3}$; $(a^{2}b^{-3}c^{-5})/(a^{-1}bc^{2})=a^{3}/b^{4}c^{7}$

- What about $1/a^{-m}$???

THE POWER ZERO

$$\frac{5x5x5}{5x5x5} = 5^{3-3} = 5^{0} = 1$$

Thus, $\frac{a^{n}}{a^{n}} = a^{0} = 1$

N.B. $a \neq 0$

Fractional Powers

- If $a=a^1=a^{1/2} \times a^{1/2}$
- Then $a^{1/2}$ must be that number which, when multiplied by itself, gives a. i.e $a^{1/2}=\sqrt{a}$

- Examples

  $9^{0.5}=\sqrt{9}=3$; also $(9^{0.5})^2= (\sqrt{9})^2=9$

- Similarly $a^{1/3} = \sqrt[3]{a}$ which we call the cube root of a

- Example

- $8^{4/3} = (\sqrt[3]{8})^4 = 2^4 = 16$

- In general $a^{1/m} = \sqrt[m]{a}$
- and
- $a^{n/m} = (\sqrt[m]{a})^n$

Examples

- $x^{4/5}x^{2/5}=x^{6/5}=(\sqrt[5]{x})^6$
- $\sqrt{(x^4\times 25)}= \sqrt{x^4}\times\sqrt{25} =5x^2$

Further results

$$\sqrt{a^2} = a \quad ; \quad \sqrt{a}\,\sqrt{b} = \sqrt{ab} \quad ;$$

$$a\sqrt{b} = \sqrt{a^2 b} \quad ; \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Power and Fraction Examples

- Power and Fraction examples
- More Problems.doc

POWERS EXAMPLES

- Express as positive integers

$$\text{(i)}\ \frac{125x^6y^7}{80x^2y^9} \quad \text{(ii)}\ \frac{48x^2y^5z^3}{27x^4y^3z}$$

$$\text{(iii)}\ \left(\frac{12x^2}{15y^3z^4} \div \frac{3x^3}{5yz^2}\right) \div \left(\frac{9y^4z^3}{2x^2}\right)$$

FRACTIONAL POWERS EXAMPLES

(i) $(\sqrt[3]{x}.\,y^2)^6.(x^{-2}y^{-5})$ (ii) $\dfrac{(\sqrt{xy})^5}{x^2}$

(iii) $\dfrac{(\sqrt{8x^6})(\sqrt{6x^5})}{\sqrt{12x^2}}$

PRACTICE QUIZ 1

-