Question 1

In each case an investment grows from an initial value $P$ to a final value $S$ over $T$ years. Give both the annual ordinary compound growth rate and the annual continuously compounded growth rate

(a) $P=100, S=17000, T=34$
(b) $P=3.84, S=7.38, T=3$
(c) $P=13.35, S=425.6, T=45$

<table>
<thead>
<tr>
<th>$P$</th>
<th>$S$</th>
<th>$F=S/P$</th>
<th>$T$</th>
<th>$r_o=F^{(1/T)}-1$</th>
<th>$r_c=\ln(1+ro)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>17000</td>
<td>170</td>
<td>34</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>3.84</td>
<td>7.38</td>
<td>1.921875</td>
<td>3</td>
<td>0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>13.35</td>
<td>425.6</td>
<td>31.88015</td>
<td>45</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Question 2

Consider the following two equations

\[ S = P(1 + r)^T \] (1)

and

\[ S = Pe^{rT} \] (2)

(a)

a. If a contract specifies a continuously compounding rate of 5% p.a. how long until the investment increases by 80%?

Increases by 80% means that the growth factor $S/P=1.8$

\[ \frac{S}{P} = e^{rT} \]

\[ = 1.8 \]

\[ r_cT = \ln(1.8) \]

\[ T = \frac{\ln(1.8)}{r_c} \]

\[ = \frac{\ln(1.8)}{0.05} \]

\[ = 11.75 \text{ years} \]
b. If the contract specifies an ordinary compounding rate of 5% p.a. how long until the investment increases by 80%?

\[
\frac{S}{P} = (1 + r_o)^T
\]

\[= 1.8\]

\[(1 + r_o)^T = 1.8\]

\[T \ln(1 + r_o) = \ln(1.8)\]

\[= \frac{\ln(1.8)}{\ln(1.05)}\]

\[= 12.05 \text{ years}\]

(b)

a. If contract specifies a continuously compounding rate and you wish your investment to double within 10 years, what annual rate do you require?

\[
\frac{S}{P} = e^{r_c T}
\]

\[= 2\]

\[r_c T = \ln(2)\]

\[r_c = \frac{\ln(2)}{T}\]

\[= \frac{\ln(2)}{10}\]

\[= 0.0693\]

\[= 6.93\%\]

b. Do the same for a contract which specifies ordinary compounding.

\[
\frac{S}{P} = (1 + r_o)^T
\]

\[= 2\]

\[1 + r_o = 2^{\frac{1}{T}}\]

\[r_o = 2^{\frac{1}{T}} - 1\]

\[= 2^{\frac{1}{10}} - 1\]

\[= 0.0718\]

\[= 7.18\%\]
You have the choice of the following three investments
a. 1.5% per quarter, continuously compounding
b. 0.55% per month ordinary compounding
c. 3.2% semi annually ordinary compounding

Which investment would you choose to maximize your return?

Need to choose a common interest rate, say we choose $r_c$ per quarter then
(a) $r_c=1.5\%$ per quarter
(b) $r_c=\ln(1+r_o)=\ln(1.0055)=0.00548$ per month. Therefore quarterly
    $r_c=3 \times 0.00548=0.0165=1.65\%$
(c) $r_c=\ln(1+r_o)=\ln(1.032)=0.0315$ per 6 months. Therefore quarterly
    $r_c=0.0315/2=0.0157=1.57\%$
So investment (b) maximizes return.

Question 3

Give an equation of the form $y=mx+b$.

(a) When $x=0$, $y=2$. For each increase in $x$, $y$ decreases by 2.

    We are told the slope $m=-2$. The intercept is the point at which the line crosses the
    y-axis. This is the value of $y$ when $x=0$, which are also told is 2, so $b=2$. Therefore
    equation of the straight line is
    \[ y = 2 - 2x \]

(b) When $x=3$, $y=0$ while when $x=0$, $y=4$

\[ \text{Intercept } b=4 \]
\[ \text{Slope } m = \frac{\Delta y}{\Delta x} \]
\[ \Delta y = 0 - 4 \]
\[ = -4 \]
\[ \Delta x = 3 - 0 \]
\[ = 3 \]
\[ m = \frac{-4}{3} \]
So equation of line is
\[ y = 4 - \frac{4}{3}x \]
(c) When \( x=2 \ y=5 \) and when \( x=9 \ y=5 \)

\[
\text{Slope } = m = \frac{\Delta y}{\Delta x}
\]
\[
\Delta y = 5 - 5 = 0
\]
\[
\Delta x = 2 - 9 = -7
\]
\[
m = \frac{0}{-7} = 0
\]
\[
b = y - mx
\]
\[
= 5 - 0 \times 2 = 5
\]

So equation of line is \( y = 5 \)

(d) When \( y=0 \ x=4 \) and for each 2 unit increase in \( x \) \( y \) decreases by 1 unit

\[
\text{Slope } = m = \frac{\Delta y}{\Delta x}
\]
\[
\Delta y = -1
\]
\[
\Delta x = 2
\]
\[
m = -\frac{1}{2}
\]
\[
b = y - mx
\]
\[
= 0 - \left( -\frac{1}{2} \times 4 \right)
\]
\[
= 2
\]

So equation of line is \( y = 2 - \frac{x}{2} \)

(e) When price \( x \) equals the unit cost 8 the quantity \( y \) supplied is 0. For each 50 cents above this price, the quantity supplied increases by 3 units.

\[
\text{Slope } = m = \frac{\Delta y}{\Delta x}
\]
\[
\Delta y = 3
\]
\[
\Delta x = 0.5
\]
\[
m = \frac{3}{0.5} = 6
\]
$b = y - mx$
$= 0 - 6 \times 8$
$= -48$
So equation of line is
\[ y = 6x - 48 \]
where \( y \) = quantity supplied and \( x \) = price.

**Question 4**

(a) Suppose the demand function is given by \( Q = 200 - 10P \) and the supply function is \( S = 5P - 25 \). Find the equilibrium price and quantity.

Equilibrium occurs when \( S = Q \)

\[ 5P - 25 = 200 - 10P \]
\[ 15P = 225 \]
\[ P = 15 \]
When \( P = 15 \), \( Q = 200 - 15 \times 10 = 50 \)

(b) Suppose that demand decreases by 5 units for every increase in price of $2, and that when price equals $60 the demand is zero. Suppose further that the quantity supplied is zero for the price is less than or equal to $20 and that for each $1 increase in price the quantity supplied increases by 4 units. Find the equilibrium quantity and price.

**The demand Function**

Slope \( m = \frac{\Delta Q}{\Delta P} \)

\( \Delta P = 2 \); \( \Delta Q = -5 \)

\[ m = \frac{-5}{2} \]
\[ = -2.5 \]

\[ Q = mP + b \]; \( 0 = -2.5 \times 60 + b \);
\[ b = 150 \]

So equation of line is

\[ Q = 150 - 2.5P \]
The Supply Function

\[ \text{Slope } m = \frac{\Delta S}{\Delta P} \]

\[ \Delta P = 1 \]
\[ \Delta S = 4 \]

\[ m = \frac{4}{1} = 4 \]

\[ b = S - mP \]
\[ = 0 - 4 \times 20 \]
\[ = -80 \]

So equation of line is

\[ S = 4P - 80 \]

If \( S = Q \) then \( 150 - 2.5P = 4P - 80 \);
\[ 230 = 6.5P \]

\[ P = \frac{230}{6.5} = \$35.38 \]

\[ Q = 150 - 2.5 \times 230/6.5 \text{ OR } 4 \times 230/6.5 - 80 = 61.5 \text{ units} \]