Multi-Attribute Decision Making

Many decisions are based on other attributes than price. Choosing a car, for instance, although you might be looking in a particular price band. Comfort, performance, reliability, size, safety, style, image, equipment, handling, noise, running costs — these are some attributes of cars.

*Example:*
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*Example:* helping a family to buy a car

*Steps:*
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*Steps:* (1) Clarify problem;
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Example: helping a family to buy a car

Steps: (1) Clarify problem; (2) Identify objectives;
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Example: helping a family to buy a car

Steps: (1) Clarify problem; (2) Identify objectives; (3) Measurement of effectiveness.

(1) Clarify problem
keep an older car?
use public transport?
constraints? —
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*Example:* helping a family to buy a car

*Steps:* (1) Clarify problem; (2) Identify objectives; (3) Measurement of effectiveness.

---

(1) **Clarify problem**

keep an older car?
use public transport?
constraints? —

$  
manual transmission / auto?
size?
power steering?
? 1. driving kids to school
? 2. reliable & safe commuting vehicle?
? 3. status symbol
? 4. help on family holidays
Example (cont.):

Attributes: Price, handling & performance, overall safety, overall comfort, brakes, visibility, manufacturer’s reputation (AFR 17/11/04)

<table>
<thead>
<tr>
<th>Identify objectives</th>
<th>1</th>
<th>comfort 5A, or 1A + 5K</th>
<th>$S_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>safe &amp; reliable</td>
<td>$S_2$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>status</td>
<td>$S_3$</td>
</tr>
</tbody>
</table>

given the $ constraint
Example (cont.):

Attributes: Price, handling & performance, overall safety, overall comfort, brakes, visibility, manufacturer’s reputation (AFR 17/11/04)

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<thead>
<tr>
<th>Identify objectives</th>
<th>(1) comfort 5A, or 1A + 5K</th>
<th>S_1</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(2) safe &amp; reliable</td>
<td>S_2</td>
</tr>
<tr>
<td></td>
<td>(3) status</td>
<td>S_3</td>
</tr>
</tbody>
</table>

given the $ constraint

<table>
<thead>
<tr>
<th>Measurement of effectiveness</th>
<th>(1) + (3) subjective—judgement intuition experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) less subjective</td>
<td></td>
</tr>
</tbody>
</table>
Additive Valuation

1.
Additive Valuation

1. Use scales for $S_1, S_2, S_3$
   
   (1) (2) (3)

   For each of the three attributes (1), (2), and (3), score the cars on a scale from 0 to 1.

2.
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2. Subject to the $\$ constraint, now weight the three attributes: i.e.
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   — How important is the first attribute (comfort) in the total decision? $\rightarrow w_1$
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   —
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   — How important is the first attribute (comfort) in the total decision? $w_1$
   — How important the second (safety and reliability)? $w_2$
   — The third (status)? $w_3$

   The three weightings $w_1$, $w_2$, $w_3$ should be normalised: $\sum w_i = 1$.

3. 
Additive Valuation

1. Use scales for $S_1, S_2, S_3$
   (1) (2) (3)
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   The three weightings $w_1, w_2, w_3$ should be normalised: $\sum w_i = 1$.

3. From part (1), each car $j$ has a score for attribute $i$:
   $\therefore x_{ij}$ is the score of car $j$ in attribute $i$.
   $\therefore$ Each car’s total score can be calculated: $\sum_i x_{ij}w_i \rightarrow$ score for car $j$

4. 
Additive Valuation

1. Use scales for $S_1$, $S_2$, $S_3$
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4. Choose the car with the highest total score, or iterate, until you feel happy with the scores, the weightings, and the final outcome.
**Multiattribute Problem**

CBA a subset
e.g. which bank?

<table>
<thead>
<tr>
<th>quality of service</th>
<th>interest rates</th>
<th>location</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comparing specific</strong></td>
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<td>projects</td>
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There are six ways: (Perry & Dillon in the Package)

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Multiattribute Problem

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Comparing specific outcomes

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There are six ways: (Perry & Dillon in the Package)

1. Pairwise comparisons
2.
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There are six ways: (Perry & Dillon in the Package)

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2. “Satisficing”
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*Comparing specific outcomes projects*

There are six ways: (Perry & Dillon in the Package)

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5. Even swaps, or Pricing out
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quality of service | interest rates | location

Comparing specific | outcomes |
projects

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6. Additive value models
1. Pairwise comparisons

“eye-balling”:
1. Pairwise comparisons

“eye-balling”:

➢ OK for small number of attributes

➢
1. Pairwise comparisons

"eye-balling":

- OK for small number of attributes
- ? OK number of alternatives?
1. Pairwise comparisons

“eye-balling”:

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but
1. **Pairwise comparisons**

   “eye-balling”:

   - OK for small number of attributes
   - ? OK number of alternatives?
   - large number of alternatives or attributes
   - no complete preference ordering
   - but – time consuming, costly
     - continuous variables
     → no information for *delegation*
2. “Satisficing”
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- Set minimum levels ("satisfy") of all attributes but one (the "target" attribute)
2. “Satisficing”

- set minimum levels ("satisfy") of all attributes but one (the "target" attribute)
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2. “Satisficing”

- set minimum levels (“satisfy”) of all attributes but one (the “target” attribute)
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→ iterative solution
  
  if min levels too \(\uparrow\) \(high\)
  \(\downarrow\) \(low\)

So: useful, often used, attributes explicit
3. Lexicographic Ordering
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How to:

>
3. Lexicographic Ordering

How to:

➢ rank attributes;
3. Lexicographic Ordering

How to:

- rank attributes;
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How to:

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➢ Examine the table on the next page, where countries’ performances at the Atlanta Olympics are tabulated lexicographically.

This means there is no trade-off between numbers of Silver medals and numbers of Golds, so that Denmark (4 G, 1 S, 1 B) is ranked nineteenth, while Great Britain (1 G, 8 S, 5 B) is ranked thirty-sixth.
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- Or we could rank by total number of medals, which means equal trade-offs between Gold and Silver and Bronze.
- Or we could weight the medals, say, Gold = 3, Silver = 2, Bronze = 1, which still allows a trade-off, but not an equal trade-off.
### Lexicographically Ranked by Gold, Silver, Bronze Medals (Atlanta)

<table>
<thead>
<tr>
<th>Country</th>
<th>Gold</th>
<th>Silver</th>
<th>Bronze</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>44</td>
<td>32</td>
<td>25</td>
<td>101</td>
</tr>
<tr>
<td>Russia</td>
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<td>21</td>
<td>16</td>
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<td>Germany</td>
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<td>China</td>
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<td>22</td>
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<td>France</td>
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<td>Australia</td>
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<td>Cuba</td>
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<td>8</td>
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<td>25</td>
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<td>Ukraine</td>
<td>9</td>
<td>2</td>
<td>12</td>
<td>23</td>
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<tr>
<td>South Korea</td>
<td>7</td>
<td>15</td>
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<td>3</td>
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<td>11</td>
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4. Reducing Search

e.g. which building to choose, given the two main uses for the building of Athletics and Crafts?
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<tbody>
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<td>A</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
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<td>5</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

So we see that:

D,B dominate C,A,E
B: 1,2   D: 2,1
4. Reducing Search

e.g. which building to choose, given the two main uses for the building of Athletics and Crafts?

<table>
<thead>
<tr>
<th>Building</th>
<th>Athletics</th>
<th>Crafts</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

So we see that:

D, B *dominate* C, A, E

B: 1, 2  D: 2, 1

![Diagram showing increasing preference with buildings A, B, C, D, E on a graph.]
5. Even Swaps, or Pricing Out

[see the Hammond *HBR* reading in the Package.]
5. Even Swaps, or Pricing Out

[see the Hammond *HBR* reading in the Package.]

e.g. which of five jobs to choose, given the five attributes of each job?

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Leisure Time</th>
<th>Working conditions</th>
<th>Co-workers</th>
<th>Where</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Freda has ranked the jobs in terms of each attribute.

E $\preceq$ A
E $\preceq$ C
D $\preceq$ B

$\therefore$ Freda’s comparison is reduced to D, E
Even Swaps (cont.)

Spell out the measures of each attribute:

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Leisure Time</th>
<th>Working conditions</th>
<th>Co-workers</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>$90k$</td>
<td>8 days</td>
<td>$W_D$</td>
<td>$C_D$</td>
<td>$L_D$</td>
</tr>
<tr>
<td>$E$</td>
<td>$100k$</td>
<td>5 days</td>
<td>$W_E$</td>
<td>$C_E$</td>
<td>$L_E$</td>
</tr>
</tbody>
</table>

Q:
Even Swaps (cont.)

Spell out the measures of each attribute:

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Leisure Time</th>
<th>Working conditions</th>
<th>Co-workers</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>$90k</td>
<td>8 days</td>
<td>W_D</td>
<td>C_D</td>
<td>L_D</td>
</tr>
<tr>
<td>E</td>
<td>$100k</td>
<td>5 days</td>
<td>W_E</td>
<td>C_E</td>
<td>L_E</td>
</tr>
</tbody>
</table>

Q: How much of $100K would Freda be prepared to give up to get 3 additional leisure days/year?

A:
Even Swaps (cont.)

Spell out the measures of each attribute:

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Leisure Time</th>
<th>Working conditions</th>
<th>Co-workers</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>$90k</td>
<td>8 days</td>
<td>W_D</td>
<td>C_D</td>
<td>L_D</td>
</tr>
<tr>
<td>E</td>
<td>$100k</td>
<td>5 days</td>
<td>W_E</td>
<td>C_E</td>
<td>L_E</td>
</tr>
</tbody>
</table>

Q: How much of $100K would Freda be prepared to give up to get 3 additional leisure days/year?

A: $25K → E'

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Leisure Time</th>
<th>Working conditions</th>
<th>Co-workers</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>90k</td>
<td>8</td>
<td>W_D</td>
<td>C_D</td>
<td>L_D</td>
</tr>
<tr>
<td>E'</td>
<td>75k</td>
<td>8</td>
<td>W_E</td>
<td>C_E</td>
<td>L_E</td>
</tr>
</tbody>
</table>

from above W_E (1st) > W_D (2nd)
**Even Swaps (cont.)**

Spell out the measures of each attribute:

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Leisure Time</th>
<th>Working conditions</th>
<th>Co-workers</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>$90k</td>
<td>8 days</td>
<td>W&lt;sub&gt;D&lt;/sub&gt;</td>
<td>C&lt;sub&gt;D&lt;/sub&gt;</td>
<td>L&lt;sub&gt;D&lt;/sub&gt;</td>
</tr>
<tr>
<td>E</td>
<td>$100k</td>
<td>5 days</td>
<td>W&lt;sub&gt;E&lt;/sub&gt;</td>
<td>C&lt;sub&gt;E&lt;/sub&gt;</td>
<td>L&lt;sub&gt;E&lt;/sub&gt;</td>
</tr>
</tbody>
</table>

Q: How much of $100K would Freda be prepared to give up to get 3 additional leisure days/year?

A: $25K → E'

\[
\begin{array}{c|c|c|c|c|c|c|}
D & 90k & 8 & W<sub>D</sub> & C<sub>D</sub> & L<sub>D</sub> \\
E' & 75k & 8 & W<sub>E</sub> & C<sub>E</sub> & L<sub>E</sub> \\
\end{array}
\]

from above W<sub>E</sub> (1st) > W<sub>D</sub> (2nd)

Q: How much of $90k would Freda be prepared to give up to get W<sub>E</sub>?

A:
**Even Swaps (cont.)**

Spell out the measures of each attribute:

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Leisure Time</th>
<th>Working conditions</th>
<th>Co-workers</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>$90k</td>
<td>8 days</td>
<td>$D</td>
<td>C_D</td>
<td>L_D</td>
</tr>
<tr>
<td>E</td>
<td>$100k</td>
<td>5 days</td>
<td>$E</td>
<td>C_E</td>
<td>L_E</td>
</tr>
</tbody>
</table>

Q: How much of $100K would Freda be prepared to give up to get 3 additional leisure days/year?
A: $25K → $E′

<table>
<thead>
<tr>
<th>Job</th>
<th>Salary</th>
<th>Leisure Time</th>
<th>Working conditions</th>
<th>Co-workers</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>90k</td>
<td>8 days</td>
<td>$D</td>
<td>C_D</td>
<td>L_D</td>
</tr>
<tr>
<td>$E′</td>
<td>75k</td>
<td>8 days</td>
<td>$E</td>
<td>C_E</td>
<td>L_E</td>
</tr>
</tbody>
</table>

from above $W_E$ (1st) > $W_D$ (2nd)

Q: How much of $90k would Freda be prepared to give up to get $W_E$?
A: $10k → $D′

“pricing out”
**Even Swaps (cont.)**

<table>
<thead>
<tr>
<th></th>
<th>$80k</th>
<th>8</th>
<th>W_E</th>
<th>C_D</th>
<th>L_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>D'</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E'</td>
<td>$75k</td>
<td>8</td>
<td>W_E</td>
<td>C_E</td>
<td>L_E</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$80k</th>
<th>8</th>
<th>W_E</th>
<th>C_D</th>
<th>L_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>D'</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E''</td>
<td>$70k</td>
<td>8</td>
<td>W_E</td>
<td>C_D</td>
<td>L_E</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$72.5k</th>
<th>8</th>
<th>W_E</th>
<th>C_D</th>
<th>L_E</th>
</tr>
</thead>
<tbody>
<tr>
<td>D''</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E''</td>
<td>$70k</td>
<td>8</td>
<td>W_E</td>
<td>C_D</td>
<td>L_E</td>
</tr>
</tbody>
</table>

i.e. all attributes “priced out” by Freda, whose choice is job \( D \)

\[
\begin{align*}
D' & \sim D'' - ? \\
E' & \sim B'' - ? \\
D & \sim D' - ? \\
E & \sim B' - ? \\
E'' & \sim D'' \\
\therefore & \sim E \sim D
\end{align*}
\]

\[D \sim D'' \triangleleft E'' \sim E \Rightarrow D \triangleleft E\]
6. Additive Value Models

e.g.
6. Additive Value Models

e.g. three projects: A, B, & C
three attributes:

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Project A</th>
<th>Project B</th>
<th>Project C</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>$20m</td>
<td>$15m</td>
<td>$25m</td>
</tr>
<tr>
<td>Time to Completion</td>
<td>8y</td>
<td>5y</td>
<td>12y</td>
</tr>
<tr>
<td>Impact</td>
<td>200k</td>
<td>300k</td>
<td>100K</td>
</tr>
</tbody>
</table>

Net Present Value $PV$ $\oplus$ the more, the better
Time to Completion $T$ $\ominus$ the less, the better
Impact $I$ $\oplus$
## 6. Additive Value Models

e.g. three projects: A, B, & C

<table>
<thead>
<tr>
<th>Attribute</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Present Value</td>
<td>$20m</td>
<td>$15m</td>
<td>$25m</td>
</tr>
<tr>
<td>Time to Completion</td>
<td>8y</td>
<td>5y</td>
<td>12y</td>
</tr>
<tr>
<td>Impact</td>
<td>200k</td>
<td>300k</td>
<td>100K</td>
</tr>
</tbody>
</table>

### Independence

- If the trade-off between \( PV \) & \( T \) is independent of the level of \( I \)
- & if the trade off between \( T, I \) is independent of the level of \( PV \)

then \( PV \) & \( I \) are independent of \( T \).

i.e. *Preference Independence of \( PV, T, I \)*
Value Function

\[ V(\text{project } j) = \sum_{i}^{\text{attributes}} w_i [v_{ij}(x_{ij})] \]
Value Function

\[ V(\text{project } j) = \sum_{i} w_i [v_{ij}(x_{ij})] \]

where \( x_{ij} \) is the level of attribute \( i \) in project \( j \)

>
Value Function

\[ V(\text{project } j) = \sum_{i}^{\text{attributes}} w_i[v_{ij}(x_{ij})] \]

where \( x_{ij} \) is the level of attribute \( i \) in project \( j \)

where \( v_{ij}(\cdot) \) is a “relative value preference of attribute \( i \) for project \( j \)”

\( v_{ij} \in [0, 1] \)
Value Function

\[ V(\text{project } j) = \sum_{i}^{\text{attributes}} w_i [v_{ij}(x_{ij})] \]

- where \( x_{ij} \) is the level of attribute \( i \) in project \( j \)
- where \( v_{ij}(.) \) is a “relative value preference of attribute \( i \) for project \( j \)”
  \( v_{ij} \in [0, 1] \)
- where \( w_i \) are attribute weights, \( \sum w_i = 1 \)

Project \( j \) → score \( V_j \) & can compare projects: \( V_j \) to obtain ranking

<table>
<thead>
<tr>
<th>e.g.</th>
<th>( w_i )</th>
<th>( A ) ( v_{i1} ) ( j=1 )</th>
<th>( B ) ( v_{i2} ) ( j=2 )</th>
<th>( C ) ( v_{i3} ) ( j=3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>0.9</td>
<td>$20m 0.5</td>
<td>$15m 0</td>
<td>$25m 1</td>
</tr>
<tr>
<td>( T )</td>
<td>0.06</td>
<td>8y 0.6</td>
<td>5y 1</td>
<td>12y 0 (−ve)</td>
</tr>
<tr>
<td>( I )</td>
<td>0.04</td>
<td>200k 0.8</td>
<td>300k 1</td>
<td>100k 0</td>
</tr>
</tbody>
</table>

E.g. \( x_{23} \) = level of attribute \( T \) in Project 3 = 12.
\[ \sum w_i = 1, \ w_i \geq 0 \] attribute weights
Value Function

\[ V(\text{project } j) = \sum \limits_i^\text{attributes} w_i [v_{ij}(x_{ij})] \]

- where \( x_{ij} \) is the level of attribute \( i \) in project \( j \)
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  \( v_{ij} \in [0, 1] \)
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<tr>
<td>( \text{NPV} )</td>
<td>0.9</td>
<td>$20m$ 0.5</td>
<td>$15m$ 0</td>
<td>$25m$ 1</td>
</tr>
<tr>
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<td>100k 0</td>
</tr>
</tbody>
</table>

- e.g. \( x_{23} \) = level of attribute \( T \) in Project 3 = 12.
  \( \sum w_i = 1, \ w_i \geq 0 \) attribute weights

Project A: \( V_A = 0.9 \times 0.5 + 0.06 \times 0.6 + 0.04 \times 0.8 = 0.518 \)

Project B: \( V_B = 0.9 \times 0 + 0.06 \times 1 + 0.04 \times 0 = 0.1 \)
### Alternatives

<table>
<thead>
<tr>
<th></th>
<th>Job A</th>
<th>Job B</th>
<th>Job C</th>
<th>Job D</th>
<th>Job E</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly salary</td>
<td>$2000</td>
<td>$2400</td>
<td>$1800</td>
<td>$1900</td>
<td>$2200</td>
</tr>
<tr>
<td>Flexibility</td>
<td>mod</td>
<td>low</td>
<td>high</td>
<td>mod</td>
<td>none</td>
</tr>
<tr>
<td>Business skills</td>
<td>computer</td>
<td>people man.</td>
<td>operations</td>
<td>org.</td>
<td>time man.</td>
</tr>
<tr>
<td>Development</td>
<td>computer</td>
<td>computer</td>
<td></td>
<td>multitasking</td>
<td></td>
</tr>
<tr>
<td>Annual leave</td>
<td>14</td>
<td>12</td>
<td>10</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>Benefits</td>
<td>health, dental retirement</td>
<td>health, dental retirement</td>
<td>health retirement</td>
<td>health</td>
<td>health, dental</td>
</tr>
<tr>
<td>Employment</td>
<td>great</td>
<td>good</td>
<td>good</td>
<td>great</td>
<td>boring</td>
</tr>
<tr>
<td>Location</td>
<td>Syd</td>
<td>Melb</td>
<td>Syd</td>
<td>Bris</td>
<td>Perth</td>
</tr>
</tbody>
</table>

This table provides a comparison of various job options, including their weekly salary, flexibility, business skills, development potential, annual leave, benefits, employment outlook, and location.
Landsburg

1.
Landsburg

1. Tax revenues are not a net benefits (when looking from society’s viewpoint) and a reduction in tax revenues is not a net cost.

2.
Landsburg

1. Tax revenues are not a net benefits (when looking from society’s viewpoint) and a reduction in tax revenues is not a net cost.
2. A cost is a cost, no matter who bears it.
3.
Landsburg

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3. A good is a good, no matter who owns it.
4.
Landsburg

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4. Voluntary consumption is a good thing.
5.
Landsburg

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2. A cost is a cost, no matter who bears it.
3. A good is a good, no matter who owns it.
4. Voluntary consumption is a good thing.
5. Don’t double count.

Only individuals matter

+ 

All individuals matter equally
(or: a $ is a $, no matter whose)
Real Options

(See Dixit & Pindyck and Bruun & Bason)

Disadvantages of NPV/DCF (especially for private firms):

1.
Real Options

(See Dixit & Pindyck and Bruun & Bason)

Disadvantages of NPV/DCF (especially for private firms):

1. positive-NPV opportunities might be bid away as firms enter (strategic rivalry)

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Real Options

(See Dixit & Pindyck and Bruun & Bason)

Disadvantages of NPV/DCF (especially for private firms):

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2. allocation of overhead costs in a multi-project setting is non-trivial
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Real Options

(See Dixit & Pindyck and Bruun & Bason)

Disadvantages of NPV/DCF (especially for private firms):

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6. 

< >
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5. interdependencies among projects: spillovers, asymmetric (skewed) outcomes, etc.
6. investments are sunk (sometimes assumed not)
7. the Winner’s Curse when choosing one of several: the estimates of future costs and benefits are not unbiased in the most attractive project (highest benefits – costs): possibility of negative NPV.
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  — timing: wait
  —
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   Can create value, but skew the return distribution: must use options techniques.

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3. *with growth options:*
   or follow-on investments, with distant and uncertain payoffs. Often, learning more about future options is most valuable.
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*Minus*: as the value of the underlying asset (the project) changes over time, so does its risk and so the correct risk premium.
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Minus: as the value of the underlying asset (the project) changes over time, so does its risk and so the correct risk premium.

Answer: the principles of risk-neutral valuation with the Black-Scholes option pricing techniques.