3. Shadow Pricing, Direct Price Effects

3.1 Shadow Pricing

[DoF Ch. 3; FP Ch. 1.6, 6; S&W Ch. 8]

The NPV formula can be written as

\[ NPV = \sum \frac{b(t)-c(t)}{(1+r)^t} \]

where \( b(t) = \sum b_i p_i \)

and \( c(t) = \sum c_j p_j \).

Now, we assume here that we have the quantities \( b_i \) benefits and \( c_j \) costs.

What of the prices \( p_i \) and \( p_j \)?
We want the true costs.

True prices reflect opportunities forgone (by suppliers, by consumers) \(\rightarrow\) shadow prices.

Obtained here by adjusting (distorted) market prices.

Consider the five possible distortions:

1. of a tax,
2. of a price change,
3. of a tax with a price change,
4. of unemployment with minimum wages,
5. of a tariff (a tax on imports).

To determine the true or shadow prices, use:

- Willingness To Pay (demand curve) for consumption
- Opportunity Costs (supply curve) for inputs

But beware whether increases or decreases.
What is a shadow price?

A *shadow price* better approximates the true opportunity cost or marginal valuation of a product or resource or service.

Five cases in which market prices are distorted, so that we must dig a little to obtain the shadow price, the true opportunity cost or valuation:

1. taxes,
2. price changes,
3. prices changes with taxes,
4. a labour market with minimum wage laws, and
5. a tariff on imports.
3.1.1 Example 1: A tax [S&W Ch. 8.3]

Q: A remote electricity-generation project pays $1/litre for its fuel oil, the costliest input to the project. The FA (financial appraisal) gives an NPV close to zero, but there is a tax on the fuel oil of 45¢/litre. What is the shadow price of fuel oil, to be used in the CBA NPV?

A: Since the tax is a transfer (paying 45¢/litre for nothing), ignore it in a CBA. The shadow price is 55¢/litre, and the CBA NPV will be positive, because of the lower opportunity cost of fuel oil at the shadow price.

A (specific) tax on a good supplied in a competitive market:

- places a wedge between the marginal cost (supply) and price (demand)
- the single (equilibrium) price $p_s^0$ can no longer represent both valuation and cost
- Suppose the good is an input into a project:
A Tax (cont.) — Infinitely elastic supply

Because of the tax, the (tax-inclusive) demand price $p^D_1$ is greater than the (tax-exclusive) supply price $p^S_0$.

The diagram shows the Tax Revenue and the Dead-Weight Loss as the tax is imposed, pushing up the effective supply, and reducing the quantity demanded, from $q_0$ to $q_1$. 
Let’s say the project results in an expansion of demand, from $D_0$ to $D_1$.

- But there is no change in $p_1^D$ with the increase in demand.
- Because of the tax, the project pays the higher, tax-inclusive price $p_1^D$.
- Is this the shadow price?
- Does $p_1^D$ reflect the opportunity cost associated with the extra quantity?
- No, in general, but it depends on the purpose of the tax (revenue or “green” tax?).
A Tax (cont.) — e.g. oil at world price plus a local excise of \( t \):

Consumers value the increase in demand (\( \Delta D \)) at the tax-inclusive price \( p^D_1 \).

Suppliers’ price is unchanged at \( p^S_1 \).

Shadow price = \( p^D_1 - t = p^S_1 \) = unchanging tax-exclusive price, \( p^S_1 \).

The tax revenue (\( t \)) is a transfer, and so changes in the tax revenue (changes in a transfer) are not changes in opportunity cost (\( \Delta \)).

This is only the case if there is no effect on existing purchasers of output, since there is no increase in price with perfectly elastic supply.
3.1.2 Example 2: Shadow prices and opportunity costs when prices change

[FP Ch. 2.2.2, 2.3.2; DoF 3.4]

Q: There is a local market for irrigation water. The going price is $50/megalitre. A new cotton farm is planned, but its size and thirst for water are such that the going price of water will rise to $60/megalitre, given its demand of 10,000 megalitres/year. At the lower price the NPV of the project is positive, but at the higher price negative. At the higher price the incumbent users cut their consumption by 1000 megalitres/year.

A: The shadow price is between $50 and $60/megalitre, say $55 (assuming linear supply and demand curves). The existing users bear a cost of $55 \times 1000 = $55,000/year for the water they can no longer afford (the displaced water). The shadow cost to the new farm is $550,000/year, which includes $55,000 to outbid the exiting users for 1000 gigalitres/year, and the opportunity cost of $495,000 to induce the increased supply of 9000 megalitres of water (the incremental water).
Prices Change (cont.)

Case 2.1: (No price change)

resource opportunity cost = total social costs for increased factor supply (2)

= P₁ • ΔG

Case 2.2: (Price change)
Case 2.2: (Price change)

resource opportunity cost = total social costs

\[ P_1 \cdot \Delta G \quad \text{for increased factor supply (2)} \]

+ value of reduced use of inputs in the rest of society as a response to higher prices (1)

\[ P_1 \cdot \Delta G < \text{Area [(1) + (2)]} < P_2 \cdot \Delta G \]

\[ \therefore P_1 < P_s < P_2 \]

→ \( P_s \) is the “effective or shadow price”:

\( P_s \cdot \Delta G \) is the resource cost = area (1) + area (2)
Prices Change (cont.)

Note: be conservative

\[
\begin{align*}
\text{if } NPV > 0 \text{ with } P_2 \sim \text{cost} & \text{ then GO} \\
\text{if } NPV < 0 \text{ with } P_1 \sim \text{cost} & \text{ then STOP} \\
& \begin{cases} \\
NPV(P_2) < 0 \quad \text{if } NPV(P_1) > 0 \\
\text{then must find } P_s \\
NPV(P_s) ?
\end{cases}
\end{align*}
\]

The point is so avoid the cost and effort of deriving a better estimate of the shadow price $P_s$ if it won’t make any difference to the decision.
Some Equations and Harberger’s Method

Area under Demand Curve

\[
\text{area} \approx \eta^P Q_D \Delta P
\]

(\(\eta^P\): price elasticity of demand)

\[\& \bar{Q}_D = \frac{Q_D + Q_1}{2}\]

Area under Supply Curve

\[
\text{area} \approx \kappa^P Q_S \Delta P
\]

(\(\kappa^P\): price elasticity of supply)

\[\& \bar{Q}_S = \frac{Q_S + Q_1}{2}\]
Case 2.2: Prices change (NFX: Not For Exam)

From above, social cost (1) + (2) = \( P_s \cdot \Delta G \)

\[ = \Delta P (\eta \tilde{Q}_D + \kappa \tilde{Q}_S) \]

\[ \therefore \ P_s = \frac{\Delta P (\eta \tilde{Q}_D + \kappa \tilde{Q}_S)}{\Delta G} \]  
shadow price

\[ \Delta P \left( \eta \left( \frac{Q_1 + Q_D}{2} \right) + \kappa \left( \frac{Q_1 + Q_S}{2} \right) \right) \]

= \frac{\Delta P \left( \eta \left( \frac{Q_1 + Q_D}{2} \right) + \kappa \left( \frac{Q_1 + Q_S}{2} \right) \right)}{\Delta G} \]  
(if \( \eta = \kappa \))

\[ \Delta P \eta \left( \frac{Q_1 + Q_D}{2} + \frac{Q_S}{2} \right) \]

= \frac{\Delta P \eta \left( \frac{Q_1 + Q_D}{2} + \frac{Q_S}{2} \right)}{\Delta G} \]

— a means of obtaining the shadow price \( P_s \) from \( Q_1, \Delta P, \eta, \kappa, Q_D, Q_S, \) and \( \Delta G \).
3.1.3 Example 3: Prices change with a tax wedge. [DoF 3.5]

Q: In the cotton-farm example, assume that the prices of $50 (before) and $60/megalitre (after) include a tax of $10/megalitre, perhaps to pay for environmental protection. What now is the shadow cost of water to the new farm?

A: We have to adjust for both the induced price increase and the tax wedge between suppliers of water and users of water.

The value of the displaced 1000 megalitres of water for the existing farmers who cannot afford to pay $60/megalitre is still $55,000/year: we use the tax-inclusive price of $55/megalitre because they evidently value this water at $50/megalitre at least, but not at $60/megalitre.

The shadow cost of the incremental water is between $40 and $50/megalitre (the tax-exclusive prices), since that’s what the suppliers of water receive to induce them to increase supply; say $45 \times 9000 \text{ megalitres/year} = $405,000/year.

∴ Total shadow cost = $460,000/year.
Price Change & Tax (cont.)

Elastic supply

Figure 1
Price Change & Tax (cont.)

In Figure 1 above:

- $p_0, q_0$ is the initial price at $A$.
- A specific tax of $p_1^D - p_1^S = t$ is imposed.
- This is perceived by consumers as a shift in supply from $S_0$ to $S_1$.

$\therefore$ buyers pay (tax-inclusive) $p_1^D$ and producers receive (tax-exclusive) $p_1^S$.

- The tax revenue is $(p_1^D - p_1^S)q_1 = tq_1$.
- Consumption falls from $q_0$ to $q_1$ (by $b$).
- The tax revenue is a transfer from consumers of the product to consumers in general (via tax receipts and government expenditure).
Price Change & Tax (cont.)

![Diagram showing supply and demand shifts](image)

Figure 2
Price Change & Tax (cont.)

In Figure 2:

- $p_1$ is the initial tax-inclusive price = $p_1^D$
- because supply $S_1$ includes the tax $t$
- The project to be evaluated shifts the demand for the input to the right from $D_1$ to $D_2$ (assume $\Delta D = 1$)
- \[ \therefore \text{the tax-inclusive price is forced up from } p_1 \text{ to } p_2 = p_2^D \]
- and production goes up in total by $1 - a$
- The price increase induces other uses of the good to release an amount $a$ which is absorbed by the project
- \[ \therefore \text{total usage of the input is } a + 1 - a = 1 \text{ in the project.} \]
- The expansion in output takes place at the tax-exclusive cost $S_1 - t$ or $p_2 - t$.
- The gain to the taxpayer is simply a transfer \[ \therefore \text{ignore it.} \]
- Existing consumers value the reduction in $a$ at the tax-inclusive price $p_2^D$ that they pay
- \[ \therefore \text{unit social cost} = a(\text{gross-of-tax price}) + (1 - a)(\text{net-of-tax price}) \]
- \[ = a p_2^D + (1 - a)(p_2^D - t) = \text{shadow price} \]

Hence shadow price = a weighted average of the tax-inclusive and tax-exclusive prices, $p_2$ and $p_2 - t$, respectively.
Price Change & Tax (cont.)

**How do we calculate the weights $a$ and $1-a$?**

$\eta_D =$ initial price elasticity of demand at $p_1$

$$\eta_D = -\frac{a/q_1}{(p_2-p_1)/p_1}$$  (using initial-point convention)

$$= \frac{\% \text{ change in quantity}}{\% \text{ change in price}}$$

Similarly:

$\kappa_S =$ supply elasticity $= \frac{(1-a)/q_1}{(p_2-p_1)/p_1}$

Hence

$$\frac{\eta_D}{\kappa_S} = -\frac{a}{1-a}$$

and $a = -\frac{\eta_D}{\kappa_S - \eta_D}$

and Shadow Price $= ap_2 + (1-a)(p_2-t)$:

- for an increased demand for the input
- also for an increase in supply of the good if the project results in more of the good
Price Change & Tax (cont.) — **Figure 3**

- Valuation of the increased supply $1 - a$ depends on $S_0$ (tax-exclusive)
- Valuation of the demand shifted to the project $a$ depends on the shaded area under $D_1$
- Figure 3 adds the tax-exclusive supply curve $S_0$ to Figure 2
- So $\Delta D \times p_s =$ the sum of the two areas
3.1.4 Example 4: Shadow pricing of labour [FP Ch. 6.4.1, Ch. 10; DoF 3.9]

- In a properly functioning labour market the shadow price of labour is simply the market wage:

\[\$/\text{unit} \quad S \]

- But the project shifts the demand for labour from \(D_1\) to \(D_2\)
- more jobs are “created” \((n_2^* - n_1^*)\)
- workers move from lower-paid to higher-paid jobs
- and there is no “involuntary” unemployment
Labour (cont.) — Suppose there is a minimum wage $w$ for labour set by the IRT. Then the employment level $n_1$ will be less than the competitive level, $n_1^*$. 

- The projects shifts out demand for labour from $D_1$ to $D_2$
- If the additional workers who receive jobs value leisure at $w_1^{SP}$, then $w_1^{SP}$ is the shadow wage
- New employed workers may have a higher value of leisure $w_2^{SP}$ than $w_1^{SP}$
  \[ \therefore \text{this higher average value } w_2^{SP} \text{ should be used} \]
- The social cost (shadow price) is lower than the market price $w$
- Why? Because there is unemployment at minimum wage $w$. 

![Graph showing the shift in demand from $D_1$ to $D_2$ and the employment levels $n_1$, $n_1^*$, and $n_2$.]
Labour (cont.)

- The change in the wage bill \(= \bar{w} \times (n_2 - n_1)\); in F.A. it was the “rectangle” (brown + green).

- The change in the social cost \(= \frac{w_{1}^{SP} + w_{2}^{SP}}{2} \times (n_2 - n_1)\).

- The opportunity cost of getting a job is less than \(\bar{w}\), which is reflected in the supply curve.
3.1.5 Example 5: **Foreign exchange** [FP Ch. 9.2; DoF 3.8]
Foreign Exchange (cont.)

In the figure:

- the vertical axis shows the real price of traded goods = the inverse of the exchange rate.
- the supply and demand for foreign currency is initially in balance at $q_1$, $e_1$.
- if the demand for imports by Australians goes up by $\Delta D$, the real price of traded goods will cost more.
- the $A$ will devalue in terms of foreign currency as the real price of imports rises (and $e$ rises).
- Australian exporters will gain more revenue in $A$ terms, to encourage additional exports.

∴ an upwards sloping supply curve $S$. 
A tariff (tax) of $t$ is now imposed on imports

- $A$ represents the equilibrium value of exports
- $B$ is the tariff-inclusive value of imports
- measured in terms of foreign currency, the value of imports and exports is equal.
Foreign Exchange (cont.) —
Project now increases our supply of foreign exchange $\Delta S$ via additional exports. Let $\Delta S = 1$.

- this situation is similar to the tax example (p.5-12)
Foreign Exchange (cont.) —

- increased exports facilitates increased imports valued at $e_1 + t$ (area base $1 - a$)
- increased new exports displaces traditional exports a valued at net-tax price of $e_1$
- shadow price = $a$ (post-tax price of traded goods) $+ (1 - a)(pre-tax price of traded goods)$
- shadow exchange rate will exceed market exchange rate, since exports are under-valued by the market exchange rate. — The Gregory Thesis or Dutch Disease.
Summary of Lecture 5

This lecture introduced the use of market prices — suitably adjusted to become shadow prices which accurately reflect the opportunity cost of the goods and services used by the project, whether produced in response to the project’s demand (incremental) or bid away from existing uses (displaced) — in CBA studies.

- How to adjust market prices for taxes (which are transfers, by and large).
- How to adjust market prices for price changes caused by the project.
- How to adjust market prices for regulated prices, such as minimum wages with unemployment among the workers the project will hire.
  Shadow wages.