Calibrating Methods for Decision Making Under Uncertainty

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An engineering approach: normative (should), not positive (how actually).

An ongoing research program.

Calibrating Methods for Decision Making Under Uncertainty

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1. Introduction

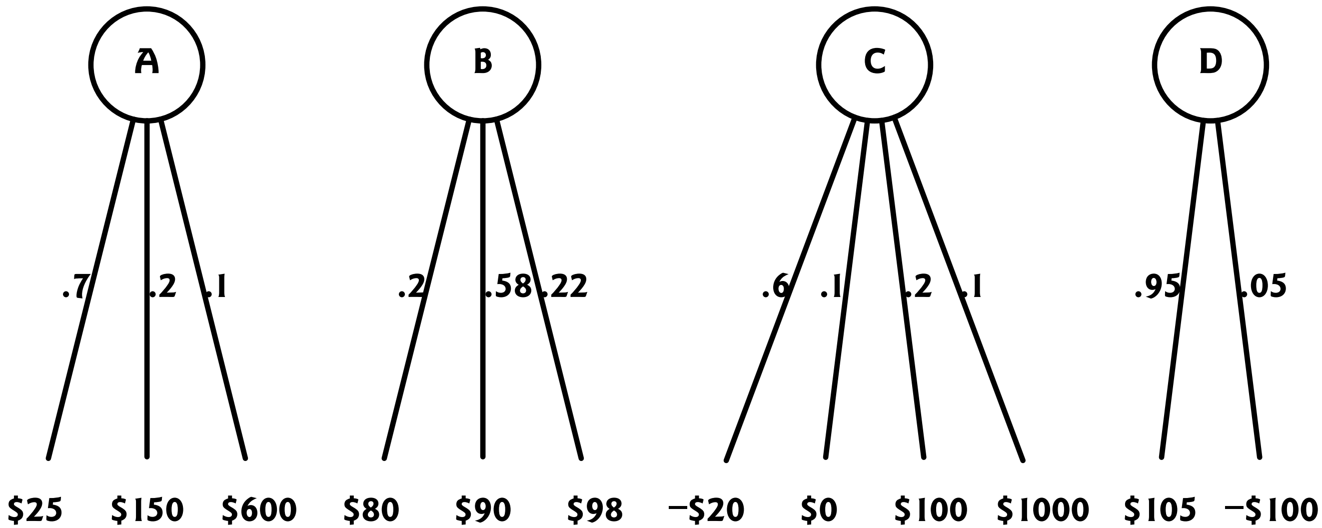
What is the best choice among lotteries (when the prizes and their probabilities are known) in a risky world?

"Best" means that the agent's average net winnings from choosing successive lotteries is highest: this is risky decision making.

By "risky" is meant that both the possible outcomes and probabilities are known.

2. Example: Four Lotteries

Figure 1:



Which is the best lottery?

The agent's choice depends on her method of choice.

But after the lottery is *realised*, which method is best?

The agent's choice varies with method:

1. *Expected Value* → C
A: 107.5 B: 89.76 C: *108.0* D: 94.75

2. *Laplace* (equal likelihoods) → C
A: 258.33 B: 89.33 C: *270* D: 2.5

3. *Max-Max* (ignore probabilities) → C
A: 600 B: 98 C: *1000* D: 105

4. *Max-Min* (ignore probabilities) → B
A: 25 B: *80* C: −20 D: −100

5. Risk-averse CARA (γ = 0.00111) → A
A: *0.098* B: 0.095 C: 0.086 D: 0.074
(see equation (1) below)

2. Simulating the Choice

Instead of four lotteries (as above), we generate eight lotteries, each with six prizes, chosen uniformly between $10 and −$10, with their probabilities chosen at random.

Then run 10,000 experiments where each time, for each of the 5 methods, a lottery is chosen, and then, using the known probabilities of that lottery's prizes, the lottery is *realised*, and the method's score is added (or subtracted, if a negative realisation) to its previous score.

The Clairvoyant is the benchmark: if the agent knew the outcome for each of the lotteries, she would choose the lottery with the highest outcome. With simulation, we can determine each lottery's realised outcome *before* the Clairvoyant chooses.
See R code at
http://www.agsm.edu.au/bobm/papers/riskmethods.r

Table 1: Simulations, the mean scores by method:

Method	Payoff ($)	% Clairvoyant	% EV
Clairvoyant	7.7880	100	
Expected Value	3.8718	49.7143	100
Laplace	3.3599	43.1425	86.7800
Max-Max	1.3917	17.8702	35.9500
Max-Min	2.4279	31.1752	62.7100
Random	0.02	0	0

The Random is zero, as it should be, given the prizes are chosen randomly.

The Expected Value dominates the 4 methods, although the Laplace method is not too bad (almost 87% of EV). But, surprisingly, the Max-Min (choosing the lottery with the highest worst possible prize) is almost twice as good (63%) as the Max-Max method (36%):
Does pessimism dominate optimism?

3.1 CARA Utility Functions

The exponential CARA utility function is

$$U(x) = 1 - e^{-\gamma x}, \tag{1}$$

where $U(0) = 0$ and $U(\infty) = 1$, and where γ is the *risk aversion coefficient*:

$$\gamma \equiv - \frac{U''(x)}{U'(x)}. \tag{2}$$

Sign of γ	Risk profile	Curvature
$\gamma = 0$	risk neutral	$U''(x) = 0$
$\gamma > 0$	risk averse	$U''(x) < 0$
$\gamma < 0$	risk preferring	$U''(x) > 0$

Table 2: Simulations of CARA, mean payoffs, varying γ

gamma γ	Payoff ($)	% Clairvoyant	% EV
-0.2000	3.4714	44.5739	89.6600
-0.1600	3.6111	46.3670	93.2669
-0.1200	3.7005	47.5160	95.5782
-0.0800	3.8196	49.0448	98.6532
-0.0400	3.8582	49.5405	99.6503
≈ 0	3.8718	49.7151	100
0.0400	3.8330	49.2163	98.9982
0.0800	3.7840	48.5873	97.7330
0.1200	3.7290	47.8818	96.3138
0.1600	3.6534	46.9111	94.3613
0.2000	3.5615	45.7301	91.9858

It is clear that the best (mean payoffs) occur with $\gamma \approx 0$: risk neutral.

3.2 CRRA Utility Functions

The Constant Elasticity of Substitution (CES) CRRA utility function:

$$U(w) = \frac{w^{1-\rho}}{1-\rho}, \quad w > 0, \tag{3}$$

where w is agent's wealth, and ρ is the Arrow-Pratt measure of relative risk aversion (RRA):

$$\rho(w) = -w \frac{U''(w)}{U'(w)} = w\gamma \tag{4}$$

This introduces wealth w into the agent's risk preferences, so that lower wealth can be associated with higher risk aversion. The coefficient γ is as in (2).

As $\rho \rightarrow 1$, (3) becomes logarithmic: $u(w) = \ln(w)$, risk averse. With $w > 0$, $\rho > 0$ is equivalent to *risk averse*, while $\rho < 0$ is equivalent to *risk preferring*; $\rho = 0$: *risk neutral*.

Table 3: Simulations of CRRA, mean payoffs, varying ρ

rho ρ	Payoff ($)	% Clairvoyant	% EV
−2.5000	3.7570	48.2408	97.0360
−2.0000	3.8114	48.9391	98.4407
−1.5000	3.8350	49.2426	99.0512
−1.0000	3.8490	49.4222	99.4124
−0.5000	3.8665	49.6475	99.8656
≈ 0	3.8718	49.7143	100
0.5000	3.8577	49.5343	99.6379
1.0000	3.8284	49.1581	98.8812
1.5000	3.8056	48.8655	98.2926
2.0000	3.7773	48.5012	97.5598
2.5000	3.7521	48.1780	96.9098

As with CARA, the simulations show that CRRA performs best when $\rho \approx 0$, the risk-neutral, Expected Value method.

3.3 The DRP function.

This function is inspired by Prospect Theory (Kahnemann & Tversky 1979):

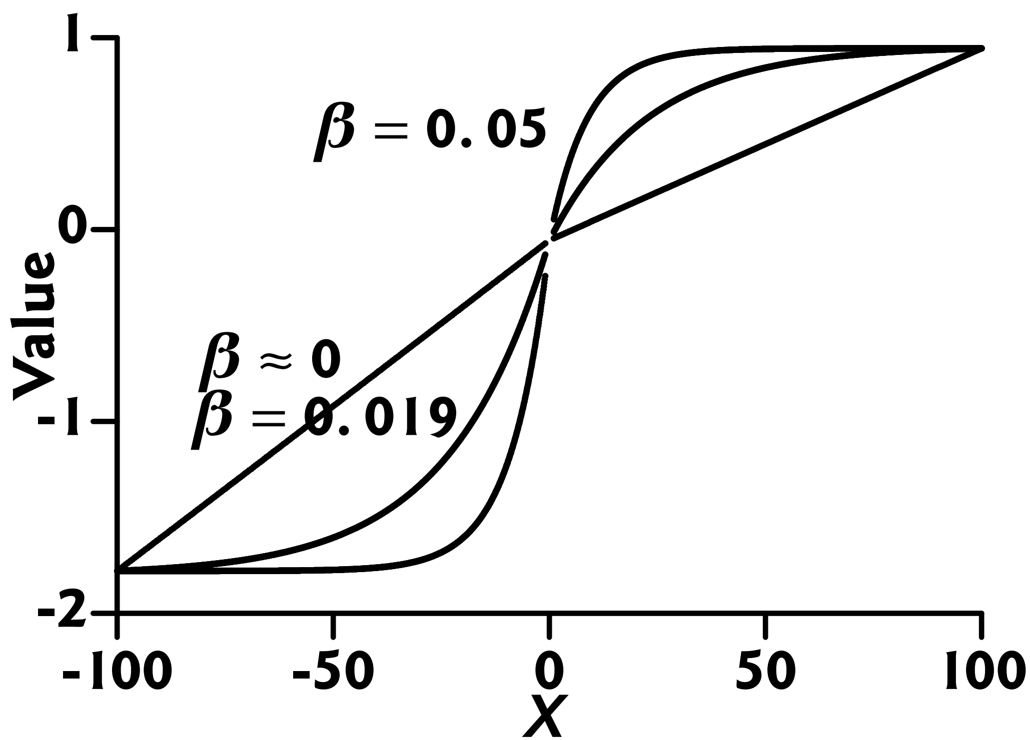
$$V = \frac{1 - e^{-\beta X}}{1 - e^{-100\beta}}, \qquad 0 \le X \le 100 \tag{5}$$

$$V = -\delta \, \frac{1 - e^{\beta X}}{1 - e^{-100\beta}}, \qquad -100 \le X \le 0. \tag{6}$$

$\beta > 0$ models the curvature of the function, and $\delta \ge 1$, the asymmetry associated with losses. The DRP function is not wealth-independent.

DRP exhibits the S-shaped asymmetric function of Prospect Theory. It exhibits risk seeking (loss aversion) when X is negative with respect to the reference point, $X = 0$, and risk aversion when X is positive.

Figure 2: Dual-Risk-Profile DRP Functions



A DRP Function (δ = 1.75).

As $\delta \to 1$ and $\beta \to 0$, the value function asymptotes to a linear, risk-neutral function.

Table 4: Simulations of DRP, % of EV, varying δ and β

beta β	δ = 1.001	δ = 1.2	δ = 1.4
0.0010	100	99.8069	99.4421
0.1000	99.5989	98.5836	98.6017
0.2000	98.2308	97.9890	97.2238
0.4000	96.9848	95.9122	95.2202

Again, the best results from the DRP functions occur when $\beta \approx 0$ and $\delta \approx 1$: this is risk-neutral, Expected Value decision making.

Discussion

Previous views: "Risk aversion is one of the most basic assumptions underlying economic behavior" (Szpiro 1997), perhaps because "a dollar that helps us avoid poverty is more valuable than a dollar that helps us become very rich" (Rabin 2000).

But is risk aversion the best risk profile? Even with bankruptcy as a possibility?

Previous researchers' answers:

- **Szpiro (1997): risk averse,**
- **Chen et al. (2008): risk averse (log utility), and**
- **DellaVigna & LiCalzi (2001) model Kahneman-Tversky agents which learn to make risk-neutral choices.**

Our answer: NO. RISK NEUTRAL IS BEST.