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(from North & Macal 2007)

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Sometimes known as Agent-based Computational Economics (ACE) models.

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6a) Vacuous	Spacey/networked (distance matters)

(from Miller & Scott 2007)

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But see the papers at
http://www.agsm.edu.au/bobm/teaching/SimSS/ABversusSD.html

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The agents are then permitted to interact directly with one another. A macrostructure emerges from these interactions.

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Another example of an agent that won $2,000,000 in a challenge by the U.S. Department of Defense in October 2005 ...

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Agents and agency

Wooldridge & Jennings (1995) would give computer agents these properties:

- **autonomy: no others control their actions and internal state,**

- **social ability: can interact and communicate with other agents**

- **reactive: they perceive their environment and respond**

- **pro-active: they initiate goal-directed actions**

- **(intentionality: metaphors of beliefs, decisions, motives, and even emotions)**

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Further agent features:

plus (Epstein 1999):

- **heterogeneity: not "representative" but may differ**

- **local interactions: in a defined space**

- **boundedly rational (Simon): information, memory, computational capacity**

- **non-equilibrium dynamics: large-scale transitions, tipping phenomena (Gladwell 2000)**

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3. *Social models.*
   Agents, knowing about interrelationships between other agents, can develop a "social model", or a topology of their environment: who's who. etc.

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4.   *Knowledge representation.*
     Agents need a representation of beliefs: e.g. predicate logic, semantic (hierarchical) networks, Bayesian (probabilistic) networks.

> [Sebastian] Thrun [leader of the winning team in the 2005 DARPA Grand Challenge] had a Zen-like revelation: "A key prerequisite of true intelligence is knowledge of one's own ignorance," he thought. Given the inherent unpredictability of the world, robots, like humans, will always make mistakes. So Thrun pioneered what's known as probabilistic robotics. He programs his machines to adjust their responses to incoming data based on the probability that the data are correct. — Pacella (2005).

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    Emergent features? Significant in modelling agents? Or epiphenomenal?

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4.   *Policy*: a set of rules that determines how they will act, given their state and history

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Since then, five approaches:

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2. **Object Orientation**

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Ignore 3., 4. last lecture, 5. too new.

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1. Production Systems

Contain:

1. a set of rules (a condition + an action),

2. a working memory, and

3. a rule interpreter (is the condition satisfied? if so, act)

No prespecified order of rules: contingent.

The agent's designer specifies how to break ties among rules.

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2. Object Orientation

In "object-oriented" programming languages:

- "objects" are program structures containing data + procedures for operating on those data;

- the data are stored in "slots" inside the object;

- the procedures are called "methods";

- objects created from templates called "classes";

- classes are ranked in a hierarchy, with subordinate classes more specialised.

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e.g. Modelling pedestrian flow.

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OO computer languages: C++, Lisp, Java. etc.

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Results: "Stanley," Stanford's robotic Volkswagen Touareg beat the field, completing the 132-mile race with a winning time of 6 hours 53 minutes 58 seconds (an average speed of 19.1 mph). Four other vehicles successfully completed the race. All but one of the 23 finalists in the 2005 race surpassed the 7.36 mile distance completed by the best vehicle in the 2004 race.

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Grand Challenge Rules

- **The vehicle must travel autonomously on the ground in under ten hours.**

- **The vehicle must stay within the course boundaries as defined by a data file provided by DARPA.**

- **The vehicle may use GPS and other public signals.**

- **No control commands may be sent to the vehicle while en route.**

- **The vehicle must not intentionally touch any other competing vehicle.**

- **An autonomous service station is permitted at a checkpoint area approximately halfway between start and finish.**

The Stanford team won the first prize of US $2,000,000 in 2005, with "Stanley."

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The DARPA Urban Challenge 2007

In November 2007, Carnegie Mellon's robot, "Boss," pipped Stanford's "Junior," to win $2,000,000. Stanford won the $1,000,000 second prize.

"Unlike the 2005 desert race, not only had entrants to keep to the tarmac and obey the rules of the road, they had also to avoid colliding with a number of other cars being steered round the base by stunt drivers.

The desert vehicles relied on radar, laser range-finders and speedy, cleverly programmed computers to avoid meddlesome objects while racing from point to point.

The urban robots used similar technology to accomplish much more difficult tasks.

In effect, they were taking the examination to receive a driving licence by demonstrating the ability to park in narrow spaces, slow down and indicate appropriately at junctions, and so on—as well, of course, as avoiding collisions." — *The Economist*, Nov 1, 2007.

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Modelling the Environment

Definition of the environment depends on what is being modelled.

For individuals:

- **move in a space, or on a network;**
- **use sensors to perceive the environment, including other agents;**
- **perhaps be able to affect the environment directly;**
- **perhaps receive and send signals in the environment.**

For computer agents, the order of agents running can be crucial ("concurrency"). Sometimes, buffering their signals is sufficient.

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Crowds: pp.202

http://cress.soc.surrey.ac.uk/s4ss/code/NetLogo/crowds.html

As well as NetLogo, ABM platforms include: Swarm, RePast, and Mason: horses for courses.

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Railsback & Grimm discuss and develop several NetLogo models:

Their forthcoming book (*A Course in Individual- and Agent-Based Modeling*) will provide guidelines to build NetLogo models.

They also introduce the "ODD" framework for building agent-based models.

The models they build and modify including the following:

- **Flocking (Chapter 7)**

- **Investment decisions (Chapters 9, 10, 11, 14)**

- **Telemarketing (Chapters 12, 13)**

see: `http://www.railsback-grimm-abm-book.com/index.html` **as well as their *Individual-Based Modeling and Ecology,* (Princeton U.P., 2005).**

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  - **the empirical and testable manifestations of CAS in economic phenomena.**

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Markose and the *EJ* Feature on CAS:

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    - — **herding,**
    - — **stock-market crashes and extreme events such as October 1987.**

- **need the "adaptive or emergent methods" of ACE simulation**

< >

Moreover ...

Axtell (2005) argues that:

- **the decentralised market as a whole can be seen as a collective computing device**

- **the parallel distributed agent-based models of k-lateral exchange $\rightarrow$ the specific level of complexity (polynomial) in calculations of equilibrium prices and allocations.**

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Simon's Bounded Rationality

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Agent-based models, following Simon (1982), also assume Bounded Rationality. Indeed, in the absence of Turing machine (universal calculator), it is difficult not to.

But Epstein (2006) reflects:
"One wonders how the core concerns and history of economics would have developed if, instead of being inspired by continuum physics … blissfully unconcerned as it is with effective computability — it had been founded on Turing. Finitistic issues of computability, learnability, attainment of equilibrium (rather than mere existence), problem complexity, and undecidability, would then have been central from the start. Their foundational importance is only now being recognized."

< >

Epstein on the virtues of boundedly rational agents ...

"As Duncan Foley summarizes:

>   `The theory of computability and computational complexity suggest that there are two inherent limitations to the rational choice paradigm.

>   One limitation stems from the possibility that the agent's problem is in fact undecidable, so that no computational procedure exists which for all inputs will give her the needed answer in finite time.

>   A second limitation is posed by computational complexity in that even if her problem is decidable, the computational cost of solving it may in many situations be so large as to overwhelm any possible gains, from the optimal choice of action.' (See Albin 1998, 46)."

ABM → Generative Explanation:

Generative explanation (Epstein 2006):

"If you haven't grown it, you haven't explained its emergence."

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— Generative sufficiency is a necessary but not sufficient condition for explanation. Each realisation is a strict deduction.

See also Miller & Page (2007) pp. 86–87.

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See also Miller & Page (2007) pp. 86–87.

Grüne-Yanoff (2006) argues to distinguish *functional explanations* (easier for simulators) from *causal explanations* (much less achievable for social scientists).

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Josh Epstein (2006): does AB simulation lack beauty?

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Epstein compares different schools of classical music: German v. French.

Truth (from agent-based modelling) can be beautiful too.

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Formalisation of Agent-Based Models

Epstein (2006): every agent model is a computer program.

∴ **Turing computable**

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But for every Turing machine, ∃ a unique corresponding and equivalent

                    partial recursive function.

Such functions might be extremely complex and difficult to interpret, but they exist.

Hence: "recursive" or "effectively computable" or "constructive" or "generative" (after Chomsky) social science.

< >

Validation of Agent-Based Models

Moss & Edmonds (2005): for AB models at least two stages of empirical validation.

1. **the micro-validation of the behaviour of the individual agents in the model, by reference to data on individual behaviour.**

2.

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1. **the micro-validation of the behaviour of the individual agents in the model, by reference to data on individual behaviour.**

2. **macrovalidation of the model's aggregate or emergent behaviour when individual agents interact, by reference to aggregate time series.**

    with the emergence of novel behaviour, possible surprise and possible highly non-standard behaviour, it's difficult to verify using standard statistical methods.

    ∴ only qualitative validation judgments might be possible.

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Four Levels of Validation (Axtell & Epstein 1994)

Level 0: Qualitatively similar at the micro level of individuals (agents)

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**Level 3: Quantitative agreement at the micro level
eg. agents behave exactly the same.**

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Level 3: Quantitative agreement at the micro level eg. agents behave exactly the same.

Recently, I addressed Level 2, with a new moment, the SSM.

http://www.agsm.edu.au/bobm/papers/marks-talk-sydagpr-3.pdf

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Simulation and Necessity?

Mathematical "model A" comprises the conjunction $(a_1 \wedge a_2 \wedge a_3 \cdots \wedge a_n)$, where $\wedge$ means "AND", and the a_i denote the elements (equations, parameters, initial conditions, etc) that constitute the model.

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Sufficiency: If model A exhibits the desired target behaviour B, then model A is sufficient to obtain exhibited behaviour B:

$$A \Rightarrow B$$

Thus, any model that exhibits the desired behaviour is sufficient, and demonstrates one conjunction of elements (or one model) under which the behaviour can be simulated.

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But if there are several such models, how can we choose among them? And what is the set of all such conjunctions (models)?

< >

Necessity

Necessity: **Only those models A belonging to the set of necessary models $\mathcal{N}$ exhibit target behaviour B.**

That is, $(A \in \mathcal{N}) \Rightarrow B$, and $(D \notin \mathcal{N}) \nRightarrow B$.

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Necessity

Necessity: **Only those models *A* belonging to the set of necessary models $\mathcal{N}$ exhibit target behaviour *B*.**

That is, $(A \in \mathcal{N}) \Rightarrow B$, and $(D \notin \mathcal{N}) \nRightarrow B$.

A difficult challenge: determine the set of necessary models, $\mathcal{N}$.

Since each model $A = (a_1 \wedge a_2 \wedge a_3 \cdots \wedge a_n)$, searching for the set $\mathcal{N}$ of necessary models means searching in a high-dimensional space, with no guarantee of continuity, and a possible large number of non-linear interactions among elements.

Lack of Necessity Means ...

For instance, if $D \not\Rightarrow B$, it does not mean that all elements a_i of model D are invalid or wrong, only their conjunction, that is, model D.

It might be only a single element that precludes model D exhibiting behaviour B.

But determining whether this is so and which is the offending element is a costly exercise, in general, for the simulator.

Without clear knowledge of the boundaries of the set $\mathcal{N}$ of necessary models, it is difficult to generalise from simulations.

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Only when the set $\mathcal{N}$ of necessary models is known to be small (such as in the case of DNA structure by the time Watson & Crick were searching for it) is it relatively easy to use simulation to derive necessity.

< >

Formalisation of Validation

Let set P be the possible range of observed outputs of the real-world system.

Let set M be the exhibited outputs of the model in any week.

Let set $S \subset P$ be the specific, historical output of the real-world system in any week.

Let set Q be the intersection, if any, between the set M and the set S, $Q \equiv M \cap S$.

We can characterise the model output in several cases. (Mankin et al. 1977).

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Five Cases for Validation

    a.

< >

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a.   no intersection between *M* and *S* (i.e., *Q* = ∅), then the model is *useless.*

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< >

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a.   no intersection between *M* and *S* (i.e., *Q* = ∅), then the model is *useless.*

b.   intersection *Q* is not null, then the model is *useful,* to some degree: will correctly exhibit some real-world system behaviours, will not exhibit other behaviours, and will exhibit some behaviours that do not historically occur. Both incomplete and inaccurate.

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c.   If M is a proper subset of S (i.e., $M \subset S$), then all the model's behaviours are correct (match historical behaviours), but the model doesn't exhibit all behaviour that historically occurs: accurate but *incomplete.*

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c. If M is a proper subset of S (i.e., $M \subset S$), then all the model's behaviours are correct (match historical behaviours), but the model doesn't exhibit all behaviour that historically occurs: accurate but *incomplete.*

d. If S is a proper subset of M (i.e., $S \subset M$), then all historical behaviour is exhibited, but will exhibit some behaviours that do not historically occur: complete but *inaccurate.*

e. If the set M is equivalent to the set S (i.e., $M \Leftrightarrow S$), then (in your dreams!) the model is complete and accurate.
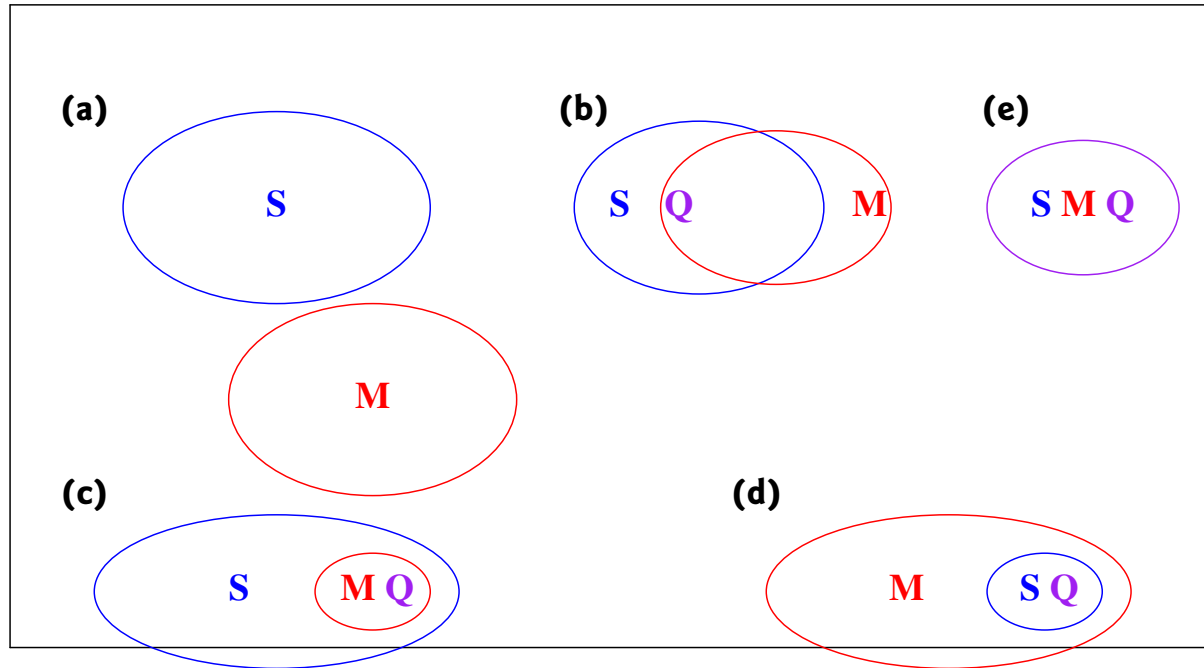
< >

Or Graphically ...

Figure 2: Validity relationships (after Haefner (2005)).

a. **useless**

b. **useful, but incomplete and inaccurate**

c. **accurate but incomplete**

d. **complete but inaccurate ← possibly the best to aim for**

e. **complete and accurate**

Modelling Goals

One goal: to construct and calibrate the model so that
$$M \approx Q \approx S$$
i.e., there are very few historically observed behaviours that the model does not exhibit, and there are very few exhibited behaviours that do not occur historically.

The model is close to being both complete and accurate.

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In practice, a modeller might be happier to achieve case d., where the model is complete (and hence provides sufficiency for all observed historical phenomena), but not accurate.

Case d. allows for the model to describe as-yet-unobserved historical behaviour in the future.

< >

Measures of Validity

A measure of validity which balances the Type I error of inaccuracy with the Type II error of incompleteness.

Define a metric $m()$ (a ratio scale) on the sets.

Define *inaccuracy* α as

$$\alpha \equiv 1 - \frac{m(Q)}{m(M)} \, , \tag{1}$$

and *incompleteness* γ as

$$\gamma \equiv 1 - \frac{m(Q)}{m(S)} \, . \tag{2}$$

Continued ...

A measure of degree of validation V: a weighted average of inaccuracy α and incompleteness γ:

$$V \equiv w(1 - \alpha) + (1 - w)(1 - \gamma) \qquad (3)$$

$$\therefore V = w\,\frac{m(Q)}{m(M)} + (1 - w)\,\frac{m(Q)}{m(S)}$$

$$\therefore V = m(Q)\left(\frac{w}{m(M)} + \frac{1 - w}{m(S)}\right) \qquad (4)$$

The value of the weight w, $0 \le w \le 1$, reflects the tradeoff between accuracy and completeness.

< >

Trade-offs

Possible to *reduce incompleteness* by generalising the model and so expanding the domain of set M until S is a proper subset of M, as in case d.

Or by narrowing the scope of the historical behaviour to be modelled, so reducing the domain of S.

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Trade-offs

Possible to *reduce incompleteness* by generalising the model and so expanding the domain of set *M* until *S* is a proper subset of *M*, as in case d.

Or by narrowing the scope of the historical behaviour to be modelled, so reducing the domain of *S*.

Also be possible to *reduce inaccuracy* by restricting the model through use of narrower assumptions and so contracting the domain of *M*.

If *M* is sufficiently small to be a proper subset of *S*, as in case c., then the model will never exhibit anhistorical behaviour.

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If *M* is sufficiently small to be a proper subset of *S*, as in case c., then the model will never exhibit anhistorical behaviour.

But not guaranteed to maintain a non-null intersection *Q*, and it is possible that the process results in case a., with no intersection.

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Look in the Right Place

Reminiscent of the economist looking for his lost car keys under the street light (M), instead of near the car where he dropped them in the dark (S).

Advocates of simulated solutions, such as Judd (2006), have argued that it is better to "have an approximate answer to the right question, than an exact answer to the wrong question," to quote Tukey (1962).

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