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Players can devise strategic moves to manipulate the order of play to their advantage; see Lecture 14.

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Mother Nature may reveal her hand, too (chance nodes, with uncertainties).
Can trace different paths from the initial node to final payoffs at a terminating node.

## Bocing v. Airbus

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With peace, each firm will make a profit of $\$ \mathbf{3 0 0} \mathrm{~m}$. With a price war, each will lose $\$ 100 \mathrm{~m}$.

## A Game Tree



## A Game Tree



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Q: How should Airbus respond?

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Q: How should Airbus respond?
$\therefore$ What should Boeing do?

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6. For each player, the collection of best decisions at each decision node of that player $\rightarrow$ best strategies of that player.

## The Capacity Game Revisited

In lecture 2 the two firms Alpha and Beta simultaneously made the capacity decision:

Beta

|  | DNE | Small | Large |
| ---: | :---: | :---: | :---: |
| DNE | $\$ 18, \$ 18$ | $\$ 15, \$ 20$ | $\$ 9, \$ 18$ |
| Alpha Small | $\$ 20, \$ 15$ | $\$ 16, \$ 16$ | $\$ 8, \$ 12$ |
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N.E at (Small, Small).

Q: What if Alpha moved first?

The game tree, and first-mover advantage.
If Alpha preempts Beta, then use the game tree:


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Commitment v. flexibility?

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What will Mortimer offer \$X?
What would you offer? (Write it down.)

## The Ultimatum Game



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Most offer a 50:50 split, and almost all accept.

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Most offer a 50:50 split, and almost all accept. Most reject less than $\mathbf{\$ 2 5}$ offered, and some even $\mathbf{\$ 4 0}$.

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Most offer a 50:50 split, and almost all accept. Most reject less than $\mathbf{\$ 2 5}$ offered, and some even $\mathbf{\$ 4 0}$. A fairness (equal) focal point.

## The Centipede Game



What would you do: as A? as B?

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Perhaps players care not only about \$ and $\ddagger$, but also about fairness or reputation.
$\therefore$ Don't assume that the other player (whether an acquaintance or anonymous or new) has your values.

## THREE CLASSROOM INTERACTIONS <br> I. Auctioning a Ten-Dollar Note

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 I. Auctioning a Ten-Dollar NoteRules:
$\geqslant$ First bid: 20¢
$\geqslant$ Lowest step in bidding: 20¢ (or multiples of 20¢)
$>$ The auction lasts until the clock starts ringing.
$\geqslant$ The highest bidder pays bid to auctioneer and gets \$ 10 in return.
$\geqslant$ The second-highest bidder also pays her bid to auctioneer, but gets nothing.

## The Ten-Dollar Auction

Write down the situation as seen by
I. the high bidder, and
2. the second highest bidder.

## The Ten-Dollar Auction

Write down the situation as seen by
I. the high bidder, and
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What happened?
Escalation and entrapment
Examples?
(See O'Neal's article in the Readings.)

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$>$ Or: You needn't play at all.


## Schelling's Game



Note: the game costs \$4 to join.

## Schelling's Game

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What happened?
$>$ numbers $\mathcal{E}$ payoffs.
$\geqslant$ previous years?
Dilemma: $\left\{\begin{array}{l}\text { coöperate for the common good or } \\ \text { defect for oneself }\end{array}\right.$
Public/private information

## Schelling's $n$-person Game

## Examples?

- price
- tax avoidance
- individual negotiation
- coal exports
- market development
- others?
(See Schelling in the Package.)


## III. The Ice-Cream Sellers

## (See Marks in the Web page.)


$>$ Demonstration
$>$ Payoff matrix
$>$ Incentives for movement?
$>$ Examples?

## Modelling the ice-cream sellers.

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Each sunbather buys one ice-cream, from the closer seller.

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Since each has two choices for its location, there are $\mathbf{2 \times}$ 2 = 4 possibilities.

## The Sellers' Payoff Matrix



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## The Sellers' Payoff Matrix



## The Sellers' Payoff Matrix



The payoff matrix (You, Other).

## The Sellers' Payoff Matrix



The payoff matrix (You, Other).
A non-cooperative, zero-sum game, with a dominant strategy, or dominant move.

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This simple model: a tendency to avoid extremes, especially with barriers to entry for new players.

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A twist: What if the centre is too far for some bathers (at the ends of the beach) to walk?

Then the tendency for the sellers to offer the same product (at the centre) is reduced, and they might differentiate their products.

## Seven issues addressed in Game Theory:

I. What does it mean to choose strategies "rationally" when outcomes depend on the strategies chosen by others and when information is incomplete?
2. In "games" that allow mutual gain (or mutual loss) is it "rational" to cooperate to realise the mutual gain (or to avoid the mutual loss) or is it "rational" to act aggressively in seeking individual gain regardless of mutual gain or loss?
3. If the answers to 2 . are "sometimes," then in what circumstances is aggression rational and in what circumstances is cooperation rational?
4. In particular, do continuing relationships differ from one-off encounters (one-night stands?) in this issue?
5. Can moral rules of cooperation emerge spontaneously from the interactions of rational egoists?
6. How well does actual human behaviour correspond to "rational" behaviour in these cases?
7. If it differs, then how? Are people more cooperative than would be "rational?" More aggressive? Both?

## Cooperative and Non-cooperative Games

## Question I:

A wholesaler wants to merge with any one of four retailers who jointly occupy a city block. If the merger goes through, the wholesaler and the retailer will make a combined profit of $\$ 10$ million.
The retailers have an alternative: they can band together and sell to a real estate company, making a joint profit of $\$ 10$ million that way.

Can the outcome be predicted?
If the wholesaler joins a retailer, how should they divide the $\$ 10$ million?

## Question 2:

An inventor and either of two competing manufacturers can make $\$ 10$ million using the inventor's patent and the manufacturer's factory.

If the inventor and one of the manufacturers should manage to get together, how should they share their profit?

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- agreements binding on all players, and
- means of transferring payoffs between players.
(See Dixit \& Skeath, Chapter I7.)
But for SGTM: Non-Cooperative Game Theory only


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Non-cooperative game theory:
no binding agreements, and which strategies will players choose?

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I.

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8. Extensive-form game tree for sequential games; rollback, (information sets - later).

