THE FIRM

This section of the subject analyses *the supply side* of the market. We're interested in the sale of the output of firms, covering costs and making a return above the opportunity cost of capital (the economist's definition of profit). We look at the following topics:

1. **The Firm's Choices**

   What do firms do? What *constraint* do firms face? What *choices* do firms make?

2. **Profit-Maximising Firms**

   We assume that the firm is interested in *maximising its profit*: given the cost curve and the revenue curve, what level of output does it choose? **H&H**, Ch. 6.1

3. **Three conditions for profit maximisation**.

4. **Costs**: opportunity costs; fixed cost versus variable cost; total cost versus average cost; marginal cost; short-run and long-run costs, time.

5. **Revenues**: a firm facing a downwards sloping demand curve has some *market power* to choose its price. A downwards sloping marginal revenue curve.

   Price elasticity of demand as a measure of market power.

6. **Price-Taking Firms**

   If the firm is a *price taker* in the market it sells into and the markets it buys from, *what choices does it have?* **H&H**, Ch. 6.2

7. With the cost function, we can derive the price-taking firm's **supply curve** of optimum output against output price.

   Suppose we know the firm's costs as a function of the level of output. (This implies that the firm has already decided how to produce the output—the levels of inputs etc.—so that the only decision is how much to produce.) What are the *short-run and long-run conditions* for operation. **H&H**, Ch. 6.3

8. **Firm's Production Process**

   How does the firm transform inputs to output? Using technology and management described by the *production function*. *Returns to scale* is related to the behaviour of costs. **H&H**, Ch. 11.1

9. **Cost minimisation** results in the *cost function* we used above.

10. Summary of the Section.

1. The Firm's Choices

The *firm*

- uses environmental services
- buys labour services
  - machine services
  - material & energy inputs
  - managerial skills
  - technical know-how
  - land

  "*factor inputs*" to production
- combines them to produce outputs
- sells its outputs

What are the firm's constraints?

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What are the firm's choices and goals?

The firm's choices: (e.g. a restaurant)

- what to produce
- how to produce it (technology)
- amounts (flows) of inputs
- amounts (flows) of outputs

What are the firm's preferences?

Assume that the firm is *profit-maximising*—
(it always prefers more profit to less)

where:

- Total *economic profits*
  = Total revenues – Total costs
- Total revenues = price × quantity sold
- Total cost includes
  the *opportunity costs*

  = alternative *opportunities* forgone

  = the pay necessary to induce the owners of the *factors of production* to sacrifice their next best alternatives or opportunities

  = the going market price for each factor × quantity bought (for a price taker).

(For *capital*: cost must include a *normal return* to owners
≥ the *opportunity cost*, the next best return, the next best opportunity forgone or sacrificed.)
≠ accounting cost

2. Profit-Maximising Firms

The firm's objective: we assume that *the firm maximises profits* (but read Simon's article in Package).

Profit = Total Revenues – Total Costs,

where the economist includes the normal return to capital as a cost. (An opportunity cost.) Profits are a return above "normal".

The *decision* of the firm: *at what level of output y to operate*, in order to maximise profits, given the technology, the input and output markets.

We assume that the firm knows its Total Revenues as a function of output, $TR(y)$ and its Total Cost function $TC(y)$, so that Total Profit $\pi(y)$ is also a function of output. Choose output y^* to maximize profit:

$$\max_{\text{output},y} \text{ Profit} = \text{Tot Revenue} - \text{Tot Cost}$$

$$\max_{y} \ \pi(y) = TR(y) - TC(y)$$

$\rightarrow y^*$ level of output to maximise profits

This is an unconstrained maximisation problem. Differentiating with respect to output y:

$$\frac{d\pi(y)}{dy} = \frac{dTR(y)}{dy} - \frac{dTC(y)}{dy}$$

$$= MR(y) - MC(y) = 0 \text{ at } y^*,$$

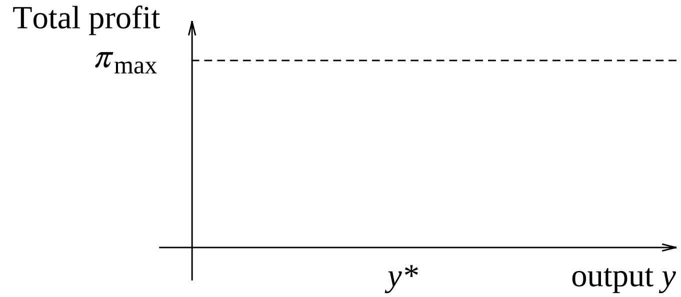
where *MR*, the Marginal Revenue, is the revenue associated with the sale of an additional unit, and *MC*, the Marginal Cost, is the cost associated with the production and sale of the additional unit.

That is, *marginal profit* equals *marginal revenue MR* less *marginal cost MC*. So when profit is a maximum, at y^*,

$$\text{Marginal Profit } \frac{d\pi(y^*)}{dy} = 0, \text{ and}$$

$$\therefore \text{ Marginal Revenue } MR = \text{ Marginal Cost } MC$$

Show TC and TR on another grpah here



Profit is maximised when marginal profit is zero (the first-order or necessary condition).

Setting $\frac{d\pi(y)}{dy} = 0$ and solving for y will result in the optimal level of output y^*.

$\therefore$ profit π is a maximum at output y^*

when $MR(y^*) = MC(y^*)$ & marginal $\pi = 0$.

3. Three Conditions

1. The First-Order, necessary condition, that

   Marginal Revenue = Marginal Cost

   at the profit-maximising level of output y^*.

   Consider: You can sell an additional unit for $2.79, and it costs you $2.50 to produce and sell that unit.

   Q: Should you sell it?

   Q: What if the most you can sell the next unit for is $2.60, while your costs have risen to $2.55 for that unit?

   Q: The next unit costs, say, $2.60 to produce and sell, but it fetches only $2.52. What then?

   So: If Marginal Revenue > Marginal Cost, then produce more,

   or if Marginal Revenue < Marginal Cost, then produce less,

   until: Marginal Revenue = Marginal Cost, and *profit is maximised*.

2. Additional condition: that profit is a *maximum*, not a minimum; that is, marginal profit is falling, or marginal cost is rising. (Second-Order, sufficient condition.)

3. And *also* that profit π is *positive*, or non-negative. $\pi \geq 0$ in the long run.

4. Costs

Profit = Sales Revenue – Costs

Four cost topics:

1. Cost Functions
   - — Total Costs *TC*
   - — Fixed *FC* and Variable Costs *VC*
   - — Average *AC* and Marginal Costs *MC*
2. Economic versus Accounting Costs
   - — Opportunity Costs
3. Short-run and Long-run Costs
   - — before and after commitment to plant size
4. Sunk Costs
   - — recoverable or not?

4.1 Total Cost Function

A price fall will stimulate sales,
but higher output will raise Total Costs.
By how much?

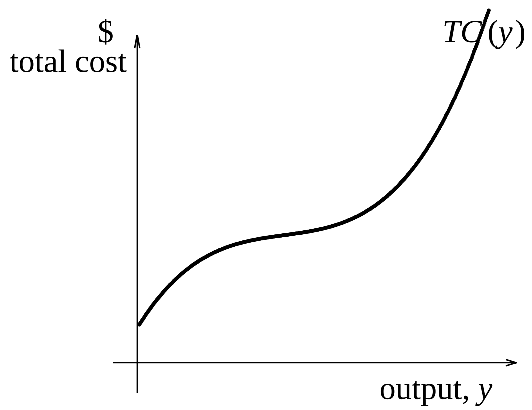
Profitable?

Total Cost Function:
for each level of output per period,
a unique level of total cost.

"unique": assume that *the firm produces at the most efficient means possible*, given its technological capabilities.

∴ "efficiency" implies: total costs always rise with output because more *input factors of production* (labour, machinery, materials) necessary.

Graphically, we show the total cost associated with any level of output:



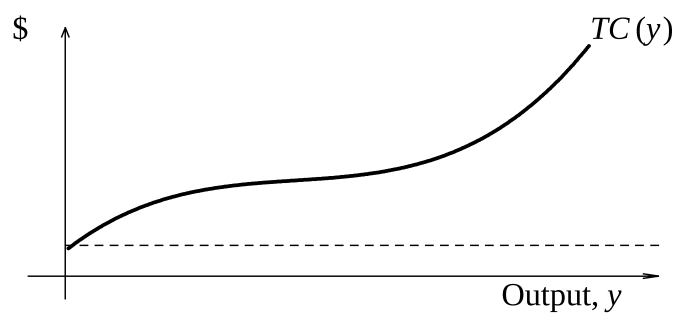
4.2 Fixed Costs and Variable Costs

Total Cost, *TC*, is made up of two parts:

1. The **Fixed Cost**, *FC*, the costs which are unrelated to a particular level of output, such as overheads, rent, telephone rental, electricity connection charge, interest payments, and
2. The **Variable Cost**, *VC*, the costs which are directly related to the level of output, *y*, and which therefore rise with output.
Example: labour costs, materials costs, energy costs.

But the distinction is not always clear:

- Fuzzy dividing line: some costs contain both fixed and variable elements, and semi-fixed costs.
- "fixed": invariable to firm's output per period but could be affected by other decisions.
- move to smaller premises (and so reduce its rent), not renew its car leases, etc. Time horizon: in the short run, many costs are fixed, but in the longer run almost all expenses are variable. The further one looks ahead, the lower the *FC* are.



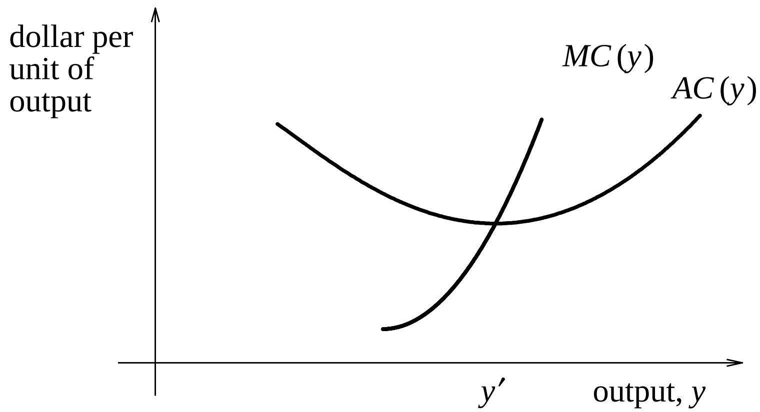
4.3 Average and Marginal Costs

Geometrically:

- the **Average Cost**, *AC is the slope of the ray through the origin* to any point on the *TC* curve corresponding to output y: $TC(y)/y$
- the slope of the Total Cost curve is at output y is the **Marginal Cost**: $\partial TC(y) / \partial y$ is $MC(y)$

The shape of the Total Cost curve is critical for the second-order (sufficient) conditions that profit is maximised and not minimised.

Usually, we use diagrams with dollars per unit of output, rather than dollars, on the vertical axis:



$$\begin{aligned} \text{Profit} &= \text{Total Revenue} - \text{Total Cost} \\ \pi(y) &= TR(y) - TC(y) \\ \therefore \text{ Average Profit} &= AR(y) - AC(y) \end{aligned}$$

Marginal Costs MC: the incremental cost of producing exactly one more unit of output.

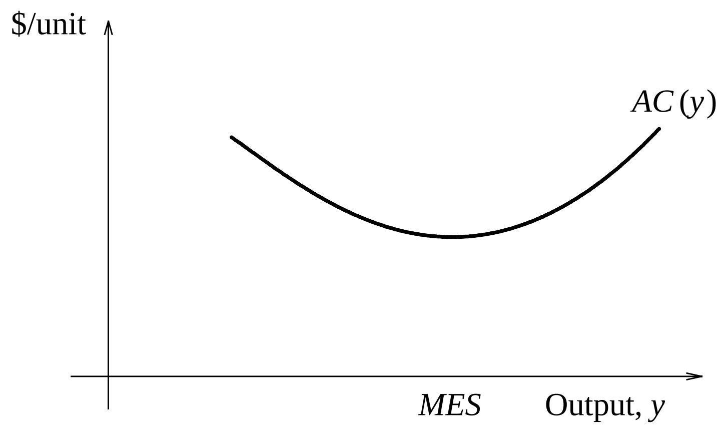
MC can vary with output level —
overtime payments? older, less reliable machinery? training temps?

Average Costs AC: how do the firm's average or per-unit costs vary with the amount of output it produces?

- Constant *AC*: *constant returns to scale*, CRTS
- Falling *AC*: *economies of scale* or increasing returns to scale, IRTS
- Rising *AC*: *diseconomies of scale* or decreasing returns to scale, DRTS

Falling Average Fixed Costs *AFC* but often rising Average Variable Costs *AVC* $\rightarrow$ U-shaped *AC* curve.

Output at lowest *AC*: the *minimum efficient scale MES*



AC is very important for size and scope of firm
AC is very important for structure of industry

Note:

1. Average Cost $AC \neq$ Marginal Cost MC:
except when Total Costs vary in direct proportion to output, (TC curve linear through the origin)
then $AC(y) = MC(y)$ = a constant, for all y

More generally,
when $MC < AC$, AC falls with output y
when $MC = AC$, AC invariant with y
when $MC > AC$, AC rises with y
(Think: average speed versus speedo reading.)

At output where AC is a minimum,
$AC = MC$
Average Cost = Marginal Cost

2. We can approximate the Marginal Cost by the incremental cost $\Delta(TC)$:

- **AC = Average Cost** $= \dfrac{TC(y)}{y}$
- **MC = Marginal Cost** $= \dfrac{\Delta(\text{Total Cost})}{\Delta(\text{Output})}$

(as Δ Output → 0) $= \dfrac{d\ TC(y)}{d\ y}$

Average Profit $= AR - AC$
= Average Revenue − Average Cost

Total Profit $\pi = y(AR - AC)$

Now,

Total Cost = **Total Fixed Cost** + **Total Variable Cost**

where **Total Fixed Cost, TFC,** is the cost when there is zero output

$$TFC = TC(y = 0)$$

= "the overheads"

and **Total Variable Cost, TVC,** is the cost associated with output = Total Cost − Total Fixed Cost

$$\therefore\ TVC(y) = TC(y) - TFC$$

Average Variable Cost $= \dfrac{TC - TFC}{y} = \dfrac{TVC}{y}$

& Marginal Cost $= \dfrac{d(TC)}{dy} = \dfrac{d\ TVC}{dy} \equiv MVC$

$$\because\ \frac{d\ TFC}{dy} = 0$$

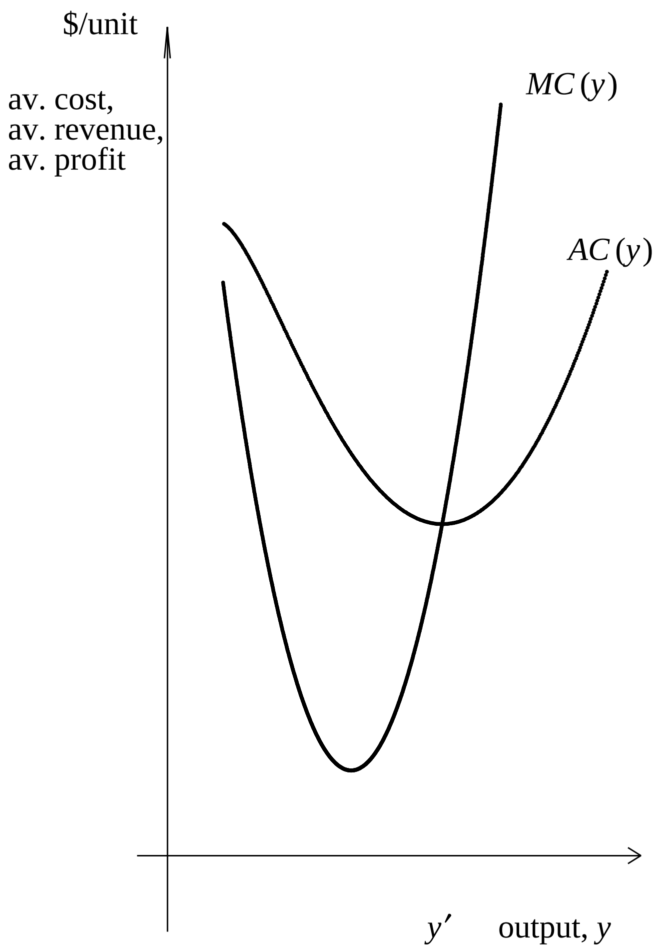
$\therefore MC \equiv$ Marginal Variable Cost

Note: Marginal Fixed Cost is zero. (Why?)

3. For a *short period*, it's possible for a firm to operate with maximum profit, π^*, less than zero ($TR < TC$), so long as $TR > TVC$; Total Revenue is greater than Total Variable Cost. (cash flow)

In the *long run*, to stay open, the firm needs *positive profit*:

$$TR \geq TC \quad (\pi^* \geq 0).$$



4.4 Economic versus Accounting Costs

Accountants use historical costs, objective and verifiable to outsiders.

Business decisions require economic costs, based on *opportunity costs*:

> the cost (or sacrifice) of using a resource is the value of the best forgone alternative use of that resource.

e.g. shareholders' funds:
could liquidate the firm for $100 million,
so forgo say 5% of $100m per year
plus 1% w&t and obsolescence per year
6% → $6 million per year
an economic cost
if firm's return on capital < $6 million a year,
then making a negative economic profit

Total costs include economic costs.

Example 1: EVA analysis:
Economic Value Added = operating profit – cost of capital × capital

Example 2: Bill asserts that he could not even "give away" (for literally zero dollars) a building that he owns and uses in his business. In economic jargon, the building has a zero opportunity cost.
Is this true?

4.5 Time Horizon & Costs

Short run: period in which the firm cannot adjust the size of its production facilities

For each plant size, is associated *SAC* curve (*Short-run Average Cost*)

SAC: annual costs of all relevant variable inputs *VC* (labour, materials, energy) plus annualised *FC* of the plant itself

$$SAC(y) = AV(y) + AFC(y)$$

Larger plant's *MES* will be higher than a smaller plant's.

AC of a smaller plant at some *y* may be lower than the $AC(y)$ of a larger plant.

Firm: choose the scale of plant to minimise *SAC* associated with planned output *y*.

If planned *y* smallish, reduce costs via
lower *FC* and lower *VC*.

LAC (*Long-run Average Cost*): minimum cost at any *y*, given the possibility of choosing the best plant for that level *y*.

LAC is the *AC* curve the firm faces before commitment.

LAC can exhibit economies of scale.
but to realise these economies,
not only large plant,
but also sufficient output

Possible: large plant with small output, and high *AC*,
but wrong to conclude: no economies of scale.

(H&H Fig. 6.5)

4.6 Sunk Costs

Sunk Costs: costs already incurred *and* which cannot be recovered.

Avoidable Costs: the opposite, could be avoided.

Decision makers should ignore sunk costs (but often don't) and consider only avoidable costs.

Example 3:

You see an advertisement for shirts on special twenty kilometres away, at prices substantially less than at your local shirt shop.

Since you "need" new shirts, and the prices advertised are substantially lower, you drive over.

But when you get there, you find that none of the shirts on special are in your size. The shop stocks your sized shirts, but at prices only slightly less than your local shop.

What should you do?

a. Should you refuse to buy any shirts because they are not cheap enough to justify the expense of the twenty-km drive?

b. Should you buy some shirts anyway?

c. Should you buy large numbers of shirts so that the total savings offset the cost of driving over?

d. What if your sized shirts are more expensive than your local shop's? Should you buy them anyway, since you might as well get something for your trip?

Answer:

a. No. Ignores sunk costs already incurred and unrecoverable.

b. Yes. You should buy some shirts anyway—you've already incurred the cost of driving over (and back): it's sunk.

c. Depends if you like them and if you think they won't go out of style or size.

d. No. Throwing good money after bad.

Irrelevance of Sunk Costs: bygones are bygones.

- Sunk costs ≠ fixed costs
  Fixed costs: the minimum necessary for producing any output at all.
  If some fixed costs are recoverable (say, by reselling equipment at purchase price, or because equipment was leased), then these costs are recoverable, and hence not sunk.
- Sunk costs important for analysing:

— rivalry among firms,

— firms' entry and exit decisions from markets, and

— firms' decisions to adopt new technology.

5. Revenues

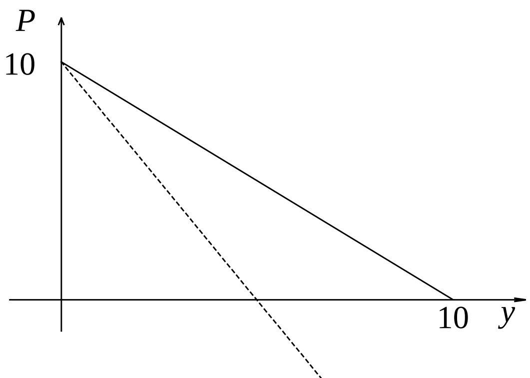
See **H&H**, Ch. 2.2, 8.1

How does a firm's Total sales Revenue *TR* depend on its pricing decision?

Now, *TR* equals the product of price *P* and quantity *y* ($TR = P \times y$), so we must examine the relationship between the changes in the price *P* and the quantity *y* sold.

$\therefore$ Need consider: the demand function and the price elasticity of demand.

e.g. Consider the demand curve $P = 10 - y$.
If the asking price is \$10/unit, then none will be sold, but if the price asked is \$6/unit, then 4 units per period will be sold, and if the price falls to zero, then 10 units per period will be sold.



What is the marginal revenue associated with this demand curve?

Total Revenue = price × quantity = $P\,y = 10\,y - y^2$.

Marginal Revenue = $\dfrac{dTR}{dy} = 10 - 2\,y$, as plotted.

With linear demand curves, the *MR* is twice as steep as the demand curve.

Average Revenue = $\dfrac{\text{Total Revenue}}{\text{output}} = 10 - y = D$.

Average Revenue is nothing other than the **Demand Curve**, for any demand curve.

- **AR** = **Average Revenue** $= \dfrac{\text{Total Revenue}}{\text{Output}}$

$$= \frac{P \times y}{y}$$
$$= P, \text{ price of output}$$
$$= \text{the Demand Curve}$$

- **MR** = **Marginal Revenue** $= \dfrac{\Delta(\text{Total Revenue})}{\Delta(\text{Output})}$

(as Δ Output → 0) $= \dfrac{d\,(P \times y)}{dy}$

$$= P + y\frac{dP}{dy}$$
$$= P\,(1 + \frac{y}{P}\,\frac{dP}{dy})$$
$$= P\,(1 + \frac{1}{\eta}),$$
$$= AR\,(1 + \frac{1}{\eta}).$$
$$< AR$$

where η is the price elasticity of demand, < 0.

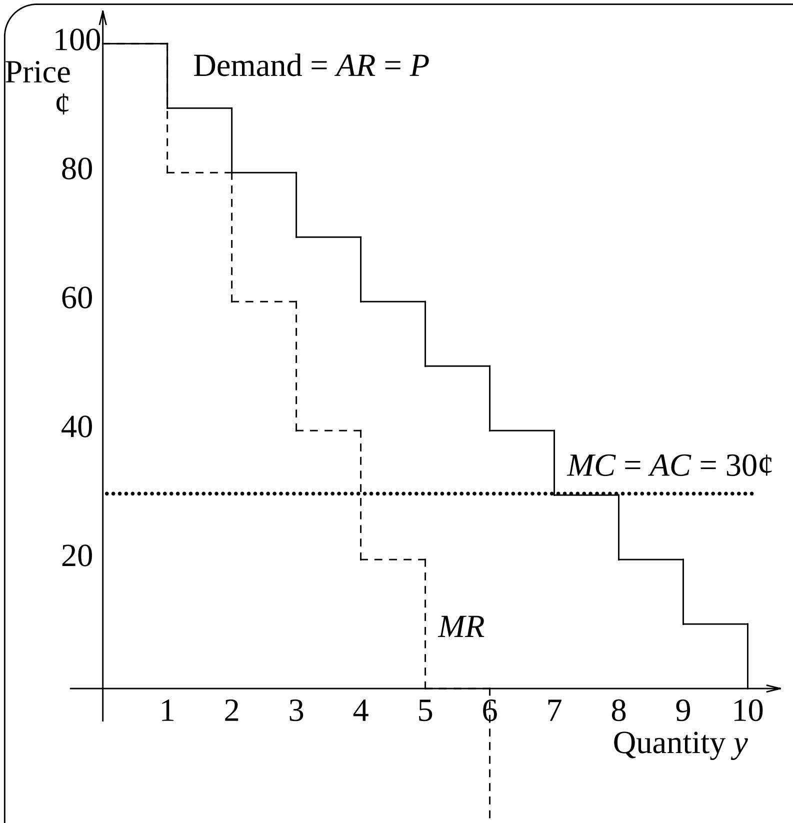
So, at any level of output *y*, the *MR* is less than the *AR* or price *P*. (Remember that the *AR* is the demand curve.)

A simple numerical example:

(1) Price	(2) Quantity Demanded	(3) Total Revenue (1) × (2)	(4) Marginal Revenue Δ (3)	(5) Profit with $AC = MC = 30¢$ $\pi = TR - TC$ (3)–[(2)×0.30]
$1.00	1	$1.00		$0.70
0.90	2	1.80	0.80	1.20
0.80	3	2.40	0.60	1.50
0.70	4	2.80	0.40	1.60
0.60	5	3.00	0.20	1.50
0.50	6	3.00	0.00	1.20
0.40	7	2.80	–0.20	0.70
0.30	8	2.40	–0.40	0.00
0.20	9	1.80	–0.60	–0.90
0.10	10	1.00	–0.80	–2.00

$MC = AC \Rightarrow$ constant cost firm
or Constant Returns to Scale CRTS

When is Revenue maximised?



$$TR = P \times y, \qquad P = P(y)$$

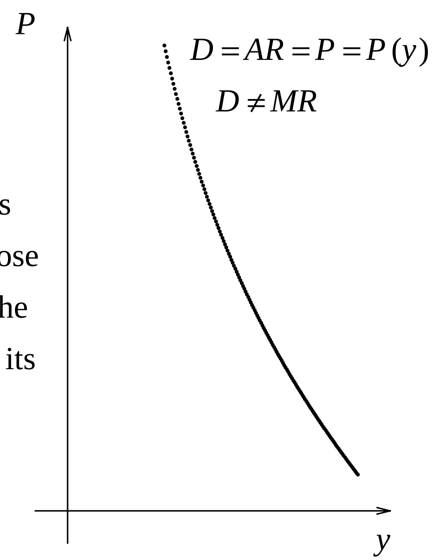
$$\Delta TR = P \times \Delta y + \Delta P \times y$$

$$MR \equiv \frac{\partial TR}{\partial y} = \frac{\partial P(y)}{\partial y} y + P$$

Profit is maximised at output y^* when $MR(y^*) = MC(y^*)$.

5.1 Market Power

If a seller faces a downward-sloping demand curve—it possesses *market power*, and can choose any combination (P, y) on the demand curve to maximize its profit π. But choose y to maximise profit π.



$$\pi = TR(y) - TC(y)$$

$$\max_y \pi \Rightarrow \frac{d\pi}{dy} = 0 = \frac{dTR}{dy} - \frac{dTC}{dy}$$

$$\therefore\ MR(y^*) = MC(y^*) \text{ for } \pi \text{ max.}$$

$$\rightarrow y^* \rightarrow P^*$$

$$\text{Marginal Revenue } MR \equiv \frac{d(P \times y)}{dy}$$

$$= P \times \left(1 + \frac{1}{\eta^P}\right)$$

$\leq P$, (i.e., market power), for any level of output,

because the price elasticity of demand η^P is negative.

6. Price-Taking Firms

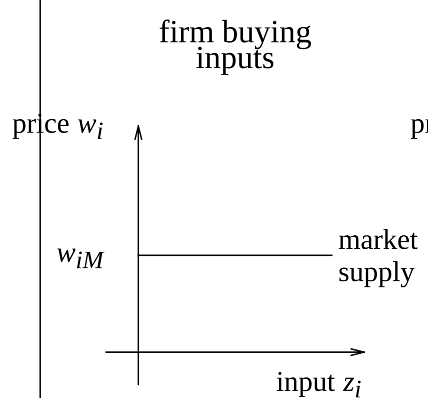
A special case: many sellers and buyers.

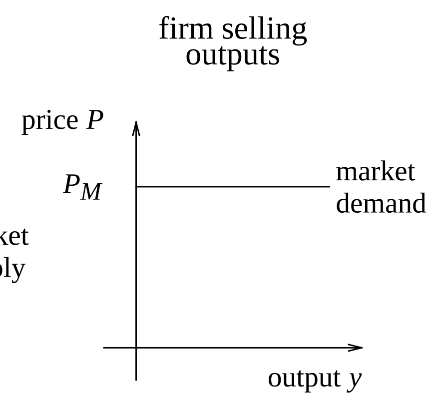
Most firms in industrial economies have some market power; only small farms or mines, selling a homogeneous, unbranded product into markets along with many other small enterprises, lack any market power. But the *benchmark* of *perfect competition* assumes no market power, so it must be understood.

First: (Assume that the firm is a *price taker*, with no market power: that is, that it faces:

- horizontal supply curves for its factor inputs
- horizontal demand curves for its outputs.)

(And assume that technology is known & fixed.)





The firm buys inputs facing a horizontal supply curve for its factor inputs: (∵ no market power)

$$\frac{\partial w_{iM}}{\partial z_i} = 0$$

and

the firm sells its output facing a horizontal demand curve for its output: ($\because$ no market power)

$$\frac{\partial P_M}{\partial y} = 0$$

since the firm is a *price taker* in both markets.

Is this a realistic model?

Is this a useful model?

Profit Maximising, Price Taking

From above, profit π is maximised at the output level y^* when Marginal Revenue equals Marginal Cost, or

$$MR(y^*) = MC(y^*)$$

But $MR(y) = \frac{dTR(y)}{dy} = \frac{d(P \times y)}{dy} = P,$

if $\frac{dP}{dy} = 0$ (for a price-taking firm P is given), then

$\therefore$ profit maximum when $P = MC(y^*)$

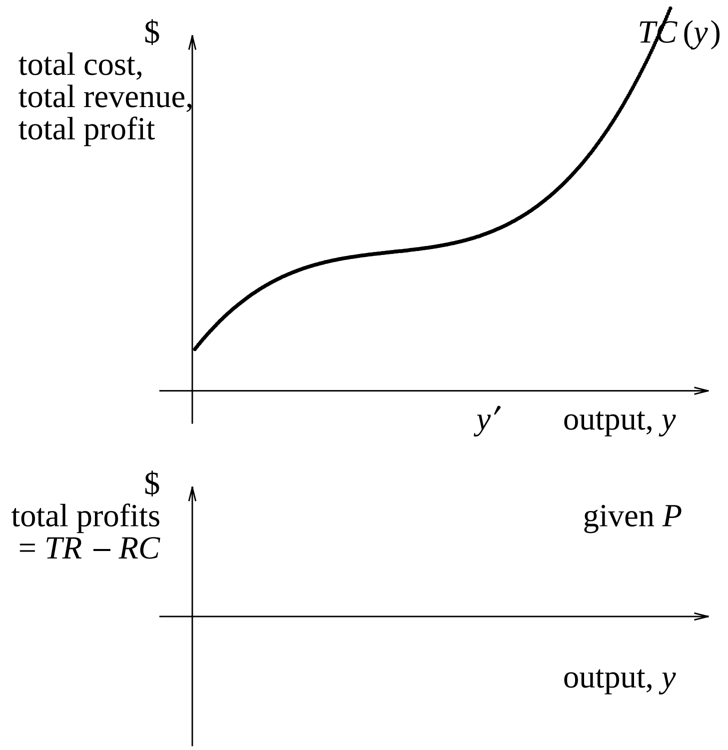
For a price-taking firm,

Marginal Revenue = Price of output
= Average Revenue

because, however much the firm offers for sale, the price remains unaltered, cet. par.

As a notational assumption: single output y from several inputs z_i (labour, capital, land, energy, materials, etc.)

Graphically:



The slope of the Total Cost curve is the Marginal Cost
$\partial TC(y) / \partial y$ is $MC(y)$

The slope of the Total Revenue curve is the Marginal Revenue
$\partial TR(y) / \partial y$ is $MR(y)$, and

for a price-taking firm $MR(y) = P$, the market-given price, because $TR = P \times y$.

Total Revenue = price $\times$ output.

By geometry, profit is maximised when the slopes of the Total Revenue and Total Cost curves are equal (the curves are parallel).

This occurs when $MR(y) = MC(y)$,
or when $P = MC(y^*)$

Profit is maximised at that level of output y^* where

price = marginal cost

Note that the shape of the Total Cost curve is critical for the second-order (sufficient) conditions that profit is maximised and not minimised.

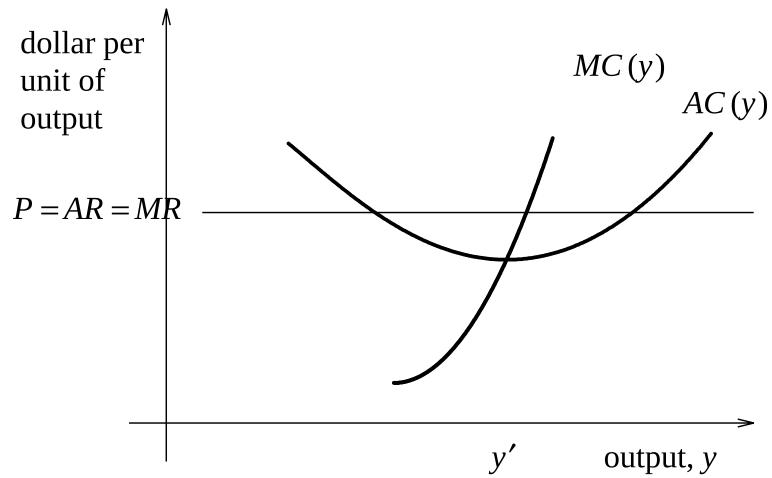
Or, for a competitive, price-taking firm, by definition,
the demand curve is horizontal ($|\eta^P| = \infty$)

$$\therefore \quad \frac{dP}{dy} = 0$$

$$\therefore \quad MR = P$$

Hence π is max. when Price = Marginal Cost (y*)
$\rightarrow y^*$

Graphically, for a price-taking firm:



Profit is maximised when $P = MC(y^*)$, so long as (1) second-order conditions hold (maximum, not minimum), and (2) profit is positive, or $AR > AC$, or $P > AC$.

This means that the firm will not produce (in the long run) at levels of output such that $MC < AC$.

The point where the maximum profit is just zero is known as the **breakeven point**, and occurs at output y', where $P = MC = AC$, the point of minimum Average Cost.

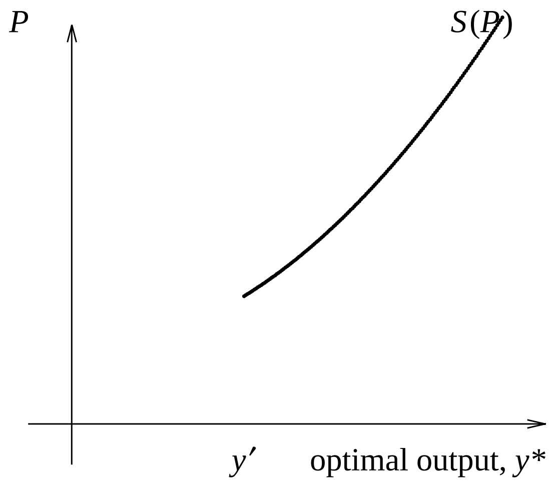
In the diagram of Total Cost and Revenue, this is the point at which a ray from the origin is tangent to the Total Cost curve.

In the short run, so long as $TR > TVC$, the firm may operate briefly. Why?

7. Supply Curve

Note that *a firm with market power* can choose the price (and quantity on its demand curve) that it chooses to operate at, and so *does not have a supply curve.* The rest of this section is for firms without market power (i.e. price-takers), facing horizontal demand curves.

Question: How does the optimal output y^* vary with P? That is, what does the *supply curve* look like?



The *supply curve*: the maximum amount of output that the (profit-maximising, price-taking) firm is willing to supply at a given price, $S(P)$; y^* as a function of the output price P.

(P' and y': *breakeven* price & quantity, $\pi(y') = 0$)

The Supply curve is the marginal cost curve above break-even y'.

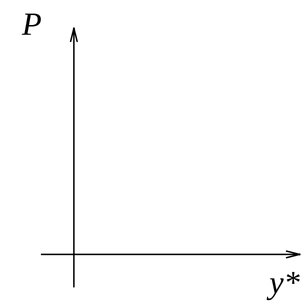
So a *profit-maximizing, price-taking firm* chooses output y to maximise its profit π:

$$\max_{\text{output } y} \quad \pi = TR - TC(y) \;\rightarrow\; y^*$$

1. First-order, necessary conditions are that *price equals marginal cost*.
2. Second-order, sufficient conditions are that marginal cost increases with output, or that Total Cost increases disproportionately with output.
3. Further, the firm will not survive unless profit is positive in the long run, and unless price is greater than Average Variable Cost in the short run.

require:	$P = MC(y^*)$	1st Order
y^*	increasing $MC(y^*)$	2nd Order
	$\pi > 0$ in long run	
	$P > AVC$ in short run	

→ output y^* as a function of price P, the *supply curve* $y^* = S(P)$



The firm's long-term supply curve is its marginal cost curve above breakeven, $\pi \geq 0$.

7.1 Short-Run Operating Condition For a Price-Taking Firm

For a *short period*, it's possible for a firm to operate with maximum profit, π^*, less than zero ($TR < TC$), so long as $TR > TVC$; Total Revenue is greater than Total Variable Cost. (cash flow)

or $P \geq AVC$ — *Short-Run* No-Shutdown Condition

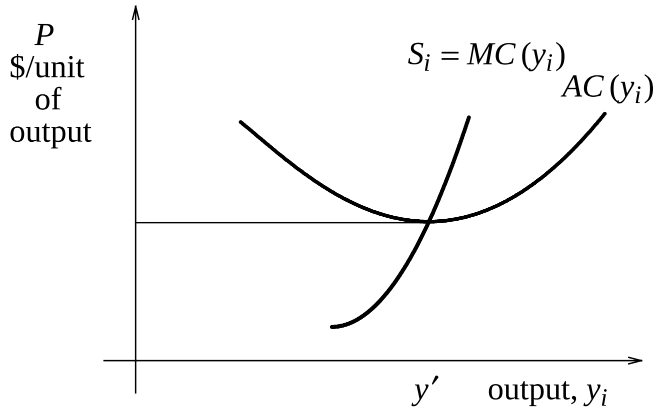
In the *long run*, to stay open, the firm needs *positive profit.*

$$TR \geq TC \qquad (\pi^* \geq 0);$$

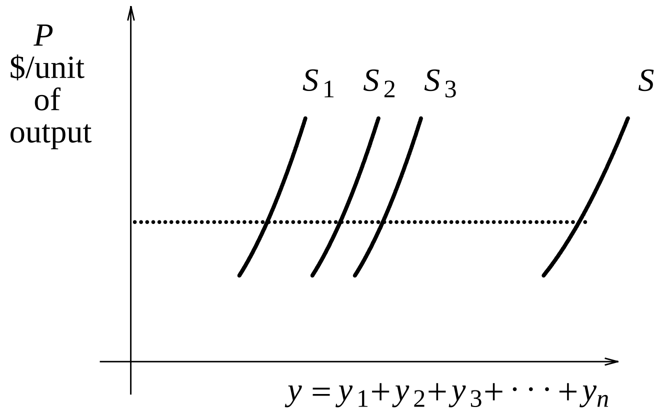
or $P \geq AC$ — *Long-Run* No-Shutdown Condition

7.2 Firm and Industry Supply Curves

The price-taking firm's *supply curve*: the firm's profit-maximising output as a function of the price P it faces, identical to its MC function, above breakeven.



The *industry supply curve* S is the horizontal sum of the supply curves $S_1, S_2, S_3, \cdots S_n$ of the n individual price-taking firms:



8. The Firm’s Production Process

H&H, Ch.11

Where does the firm’s Total Cost function $TC(y)$ come from?

A wider problem: the firm chooses *inputs* z_i as well as *output* y levels.

$$\max_{\substack{\text{output } y \\ \text{inputs } z_i}} \pi = P \times y - \sum_{i=1}^{n} w_i \times z_i$$

subject to constraints,

where P is price of output y (taken)
w_i is price of input z_i (taken).

(Still a price-taker on *all* markets.)

Constraints:

- technological — technical feasibility
- availability of inputs $\leftarrow w_i$ (supply curves)
- legal — pollution, occupational safety, etc.
- taxes
- capacity (in the short run)
- actual or potential rivals (output) $\leftarrow P$ (demand curves)
- quotas & regulations
- capital ?

8.1 The Production Function

Focus on the technological constraint, embodied in the **Production Function**, which relates the *flow* of output to the amounts of inputs: the maximum amount of output y which can be produced by a set of input quantities $\mathbf{z}$ $(= z_1, z_2, \cdots)$:

$$y \le F(z_1, z_2, \cdots, z_n)$$

The Production Function $F(\mathbf{z})$ captures the technical possibilities, and relates inputs z_i to output y.

$$\text{Total Revenue} = P \times y$$

$$\text{Total Cost} = w_1 \times z_1 + w_2 \times z_2 + \ldots = \sum_{i=1}^{n} w_i \times z_i$$

The firm’s problem:

$$\max_{\text{inputs } z_i} \pi = P \times y - \sum_{i=1}^{n} w_i \times z_i$$

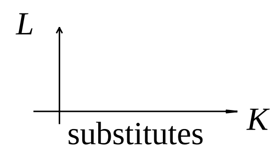
$$\text{s.t. } y \le F(z_1, \cdots, z_n)$$

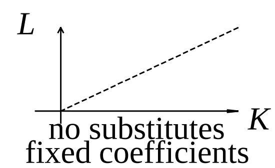
Examples: $y = F(L, K)$, L = labour, K = capital

$y = A L^{\alpha} K^{-(\alpha-1)}$ (Cobb-Douglas)

$= 212 \times L^{0.6} \times K^{0.4}$, for example

allows a trade-off between labour and capital.





The firm’s problem is *to choose inputs* z_i:

$$\max_{z_i} \quad \pi = P \times y - \sum_{i=1}^{n} w_i \times z_i$$

$$\text{s.t.} \;\; y \le F(z_1, \cdots, z_n).$$

We assume that it operates up to its technological limits, (if inequality, then it’s *inefficient*, forgoing profits): so

$$y = F(z_1, \cdots, z_n).$$

An *Efficient* Firm:

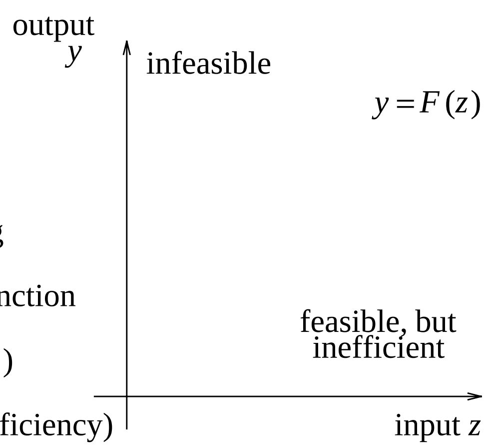
a firm is efficient

only if it is operating

on the production function

$y = F(z_1, z_2 \cdots z_n)$

(Profit maximising ⇒ efficiency)



Substitute into the profit function:
(efficient operation ⇒ equality)

$$\max_{\text{inputs } z_i} \quad \pi = P \times y - \sum_{i=1}^{n} w_i \times z_i = P \times F(z_1, \ldots, z_n) - \sum w_i \times z_i$$

Partially differentiate π with respect to input i (*not* output):

$$\frac{\partial \pi}{\partial z_i} = P \times \frac{\partial F}{\partial z_i} - w_i = 0 \quad \text{for } \pi \text{ max, for all } i,$$

$$\therefore \quad P \frac{\partial F}{\partial z_i} = P \times MP_i = w_i \quad \text{for all inputs } i \text{ (1}^{\text{st}}\text{.O.N.C.)}$$

where $MP_i \equiv \dfrac{\partial F}{\partial z_i}$ is the *Marginal Product of input i*, the additional amount of output y from one unit more of unit i (*quantity* not *revenue*), cet. par.

i.e. $MP_i = \dfrac{w_i}{P}$ Marginal Product of input i and MP_i equals the ratio of input price to output price for π max.

or $P \times MP_i = w_i$. The *value of the marginal product* i = the cost of extra unit of input i, (Why?)

Note that this is nothing more than our old friend: *MR* = *MC*, where the marginal revenue is the value of the marginal product associated with an extra unit of input *i*, and the marginal cost is the cost of that unit of input.

What is the slope of the family of iso-profit lines? That is, the slope of the lines:

$$\text{constant} = P \times y - \sum w_i \times z_i$$

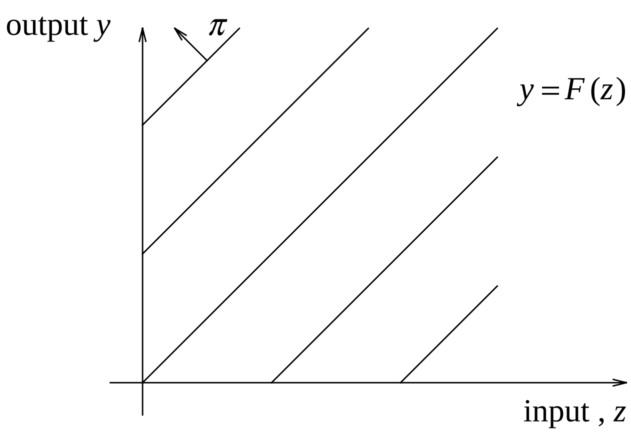
Let's consider how to deal with the firm's problem of choosing a single input z graphically. Price of output y is P; price of input z is w. Consider the iso-profit line corresponding to zero profit: it must pass through the origin, since no inputs → no output → no profit:

$$0 = P \times y - \sum w_i \times z_i$$

or

$$y = \frac{w}{P} z$$

The slope is w/P, the price w of the input divided by the price P of the output.



- As w rises or P falls, the slope is steeper, and z* and y*?
- As w falls, and/or P rises, the slope is flatter. So z* and y*?

To obtain the *constant-profit*, or *iso-profit curve*, consider a level (π^0) of profit = revenue – cost of inputs:

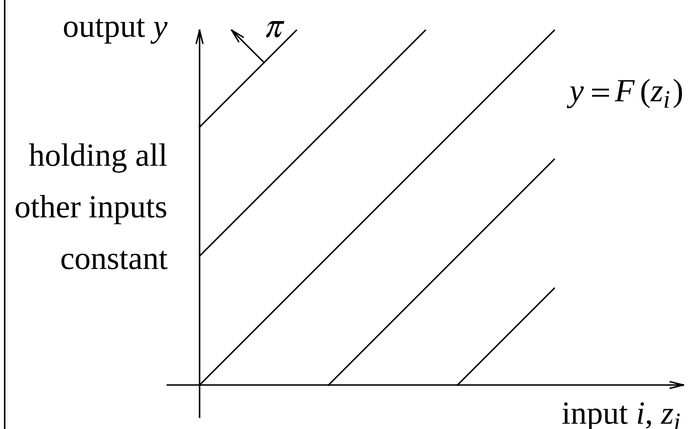
$$\pi^0 = P \times y - \sum_{i=1}^{2} w_i \times z_i$$

$$= P \times y - w_1 \times z_1 - w_2 \times z_2$$

Partially differentiating π^0 with respect to z_i, and setting $d\pi^0 = 0$:

$$d\pi^0 = 0 = P \times \left.\frac{\partial y}{\partial z_i}\right|_{\pi^0} - w_i \quad \text{for } i = 1, 2$$

$$\therefore \left.\frac{\partial y}{\partial z_i}\right|_{\pi^0} = \frac{w_i}{P} \quad \text{the slope of the iso–profit line.}$$



Profit maximization ⇒ the iso-profit line will be tangent to the production function. (First-order condition)

$$\frac{w_i}{P} = \frac{\partial F}{\partial z_i} = MP_i,$$

where MP_i is the slope of the production function. **or**: the marginal revenue associated with input i
= the marginal cost of input i.

First-order, necessary condition for Profit Maximizing:

for each input used, the value of the marginal product of the input

$$\begin{aligned} &= \text{the marginal cost of the input} \\ &= \text{its price (of input)} \\ P \times MP_i &= w_i \end{aligned}$$

Rewriting this (for $MP_i > 0$):

$$P = \frac{w_1}{MP_1} = \frac{w_2}{MP_2} = \cdots \frac{w_i}{MP_i} = \cdots \frac{w_n}{MP_n} =$$

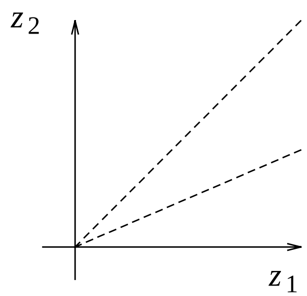
i.e. price of output = the marginal cost of producing a unit of output, for each input separately.

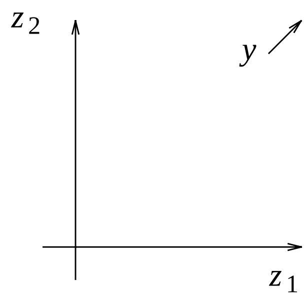
On reflection, you will see that, however we write it, it's just the old necessary condition for profit maximization:

marginal revenue = marginal cost.

Iso-quants or *contours of output*:
efficient combinations of inputs which result in equal quantity of output.

$$y = F(z_1, z_2)$$





Fixed-Proportions Technologies — Technology with Input Substitution

The slope of the iso-quant: (along $\bar{y}$)

differentiating $\bar{y} = F(z_1, z_2)$

$$d\bar{y} = 0 = \frac{\partial F}{\partial z_1} dz_1 + \frac{\partial F}{\partial z_2} dz_2$$

$$\therefore \left. \frac{dz_2}{dz_1} \right|_{\bar{y} \text{ isoquant}} = - \frac{\partial F}{\partial z_1} \bigg/ \frac{\partial F}{\partial z_2}$$

$$= - \frac{MP_1}{MP_2}, \text{ the slope of } \bar{y} < 0$$

8.2 Marginal Rate of Substitution in Production (MRSP)

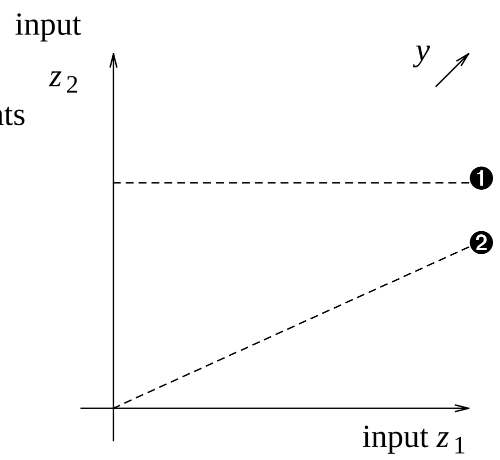
≡ the amount of input 2 to substitute for *one* unit of input 1, at constant output

$$= \frac{MP_1}{MP_2} = - \text{ slope of the isoquant} \geq 0$$

An *isoquant*: a curve showing all possible (*efficient*) combinations of inputs that are capable of producing a certain quantity of output.

In general, isoquants are convex to the origin (see later).

inputs z_1, and z_2



8.3 Returns to Inputs & Returns to Scale

There are two directions of movement:

❶ increase one input, keeping all others fixed, and

❷ increase all inputs proportionately.

❶ If we plot output y against input z_1, moving along direction ❶ we obtain a curve whose slope is

$$\frac{\partial F}{\partial z_1}, \text{ or } MP_1 > 0,$$

which in general is decreasing: $\frac{\partial^2 F}{\partial z_1^2} < 0$, $\frac{\partial MP_i}{\partial z_i} < 0$

diminishing returns to input i.

(doubling input 1 doesn't double output, cet. par.)

That is, increasing input i, ceteris paribus, leads to an increasing amount of output, but at a falling rate of increase.

❷ If we move in direction ❷, increasing all inputs proportionately, it's as though we multiplied the input vector **z** by a number $k > 1$

There are three possibilities:

1. **Constant Returns to Scale** CRTS
   if output grows proportionately

   • i.e. $y' = F(k\mathbf{z}) = kF(\mathbf{z}) = ky^o$

2. **Increasing Returns to Scale** IRTS
   if output grows **more** than proportionately

   • i.e. $y' = F(k\mathbf{z}) > kF(\mathbf{z}) = ky^o$

3. **Decreasing Returns to Scale** DRTS
   if output grows **less** than proportionately

   • i.e. $y' = F(k\mathbf{z}) < kF(\mathbf{z}) = ky^o$

In the long run, we have usually assumed CRTS. The effects of environmental constraints may result in DRTS in the future.

Note: 2^{nd} Order (*sufficient*) conditions for profit maximization are satisfied by DRTS.

>> Include Smith Fig 2.3, 2.4, 2.7 <<

9. Cost Minimization

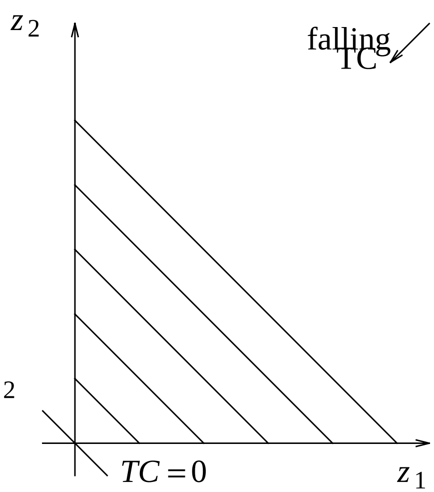
We can consider the problem of cost minimization as one of *choosing the inputs* z_i:

$$\min_{\mathbf{z}} \quad TC = \sum w_i z_i \qquad \text{subject to} \quad \bar{y}_0 = F(z_1, \cdots, z_n)$$

that is, minimize the total cost to attain a fixed level of output $\bar{y}_0$

Two inputs (z_1 and z_2), and constraint y_0 is an isoquant y_0.

Objective function, Total Cost $= w_1 z_1 + w_2 z_2$ is a straight line, whose slope can be calculated:



$$d\,TC = 0 = w_1 dz_1 + w_2 dz_2$$
$$= w_1 dz_1 + w_2 dz_2$$

$$\text{slope:} \quad \therefore \quad \left. \frac{dz_2}{dz_1} \right|_{TC_0} = -\frac{w_1}{w_2}$$

Cost minimization occurs with tangency of the iso-cost line and the isoquant: (First-order, necessary conditions)

$$\frac{w_1}{w_2} = \frac{MP_1}{MP_2} \quad \text{or} \quad \frac{w_1}{MP_1} = \frac{w_2}{MP_2} = \cdots$$

where $MP_i = \dfrac{\partial F}{\partial z_i}$ is the *marginal product* (extra output) of an extra input unit of *i*,

where w_i is the cost of the extra unit of input *i*,

$\therefore \dfrac{w_i}{MP_i}$ is the cost per unit of output (the marginal cost of output) of obtaining more output by using more of input *i*.

Example: $y = F(z_1, z_2) = \sqrt{z_1 \times z_2}$, where $F(.)$ is a production function.

$w_1 =$ \$2.00/unit input price
$w_2 =$ \$0.50/unit input price

we want | cost-minimizing inputs to produce 4 units of output |

$$\bar{y} = 4 = \sqrt{z_1 \times z_2}$$

Iso-cost curve equation:

	$w_1 \times z_1$	+	$w_2 \times z_2$	=	Total Cost
$\therefore$	$2 \times z_1$	+	$0.5 \times z_2$	=	Total Cost
$\therefore$	z_2	=	$-4 \times z_1$	+	$TC \times 2$

From the diagram,

$$z_1^* = 2$$
$$z_2^* = 8$$
$$\text{cost} = 8 = TC\,(y = 4)$$

>> Include Smith Fig 2.5, 2.6 <<

But we can also obtain this algebraically, by solving the two equations:

$$\frac{w_1}{MP_1} = \frac{w_2}{MP_2}$$

$$\text{and} \qquad F(z_1, z_2) = 4$$

So, in general:

cost minimization occurs when the iso-cost curve is tangent to the isoquant (the target)

the slope of iso-cost curve $\left.\frac{dz_2}{dz_1}\right|_{\overline{cost}} = -\frac{w_1}{w_2}$

the slope of isoquant $\left.\frac{dz_2}{dz_1}\right|_{\bar{y}} = -\frac{MP_1}{MP_2}$

$$\therefore \text{ 1st Order: } \frac{MP_1}{MP_2} = \frac{w_1}{w_2}$$

$$\text{or } \frac{w_1}{MP_1} = \frac{w_2}{MP_2} = \cdots \frac{w_i}{MP_i} \cdots$$

for all factor inputs (land, labour, capital, materials, energy, etc.).

>> Include Smith Fig 2.7 <<

9.2 Marginal Rate of (Technical) Substitution in Production (MRTS):

defined as the ratio of marginal products

$$= \frac{MP_1}{MP_2} = -\text{ slope of isoquant}$$

because it’s the rate at which z_2 can be substituted for z_1, while keeping output y constant.

The degree of substitutability between factors can vary:

in the question of the Assignment there was no possibility of substitution, except to use a different process, perhaps in combination.

The **elasticity of substitution** measures the extent to which the cost-minimising *ratio* of inputs quantities changes in response to changes in the *ratio* of prices of the inputs.

$$\frac{z_1^*}{z_2^*} \text{ compared to } \frac{w_1}{w_2}$$

Formally,

$$S \equiv \frac{d \ln \frac{z_1}{z_2}}{d \ln \frac{w_1}{w_2}}$$

$$= \frac{d\frac{z_1}{z_2}}{d\frac{w_1}{w_2}} \times \frac{\frac{w_1}{w_2}}{\frac{z_1}{z_2}}$$

(NOT FOR EXAMS)

10. Summary

1. *Cost minimization*, for given prices of output P and of inputs w_i:

   $\rightarrow$ minimum cost = $\sum w_i z_i^*$ which is a function of a target output $\bar{y}$

   $\rightarrow$ Total Cost function $C(\bar{y})$

   For different target outputs, $\bar{y}$, cost minimization will result in different Total Cost levels, and also in different amounts of cost-minimizing inputs $z_i^*(\bar{y})$

2. With Total Cost function $C(y)$, we can *maximize profit*, for given output price P

   $\pi = TR(y) - TC(y)$
   $\rightarrow$ profit-maximizing output y^* and maximum profit π^*:
   $MR(y^*) = MC(y^*)$

   subject to conditions ($\pi^* \geq 0$, increasing MC, aka DRTS)

Note: can show IRTS, DRTS, and CRTS in terms of the cost function $TC(y)$:

IRTS	aka	"decreasing costs"
DRTS	aka	"increasing costs"
CRTS	aka	"constant costs"

Package Readings:

- Wallach on the CPI
- Leibenstein on consumers (theory)
- Baumol on demand (empirical)
- Simon on decison making
- Koutsoyiannis on cost estimation
- *The Economist* and *Fortune* on increasing returns
- Boulding on agriculture
- (Thaler not in package)
- Bell on shifts in demand
- Radford on a POW micro economy
- Fels on price policy on prostitution
- *The Economist* on ski lifts and tickets
- Koch & Grupp, Nisbet & Vakil on drug economics