# Game Theory

An introduction to the concepts of dominant strategies, Nash equilibrium and strategic commitment

# Introduction

- The theory of games is a theory of economic behaviour in multi-person decision contexts.
- As such, it has applications to strategic behaviour, industrial organisation, macroeconomic policy, environmental economics, international economics, and the internal organisation of firms -- to name but a few.

# Some History

- The tools of game theory are now common-place within economics. They were originally developed by John von Neumann and Oscar Morgenstern in their 1944 book, *The Theory of Games and Economic Behavior*.
- Thomas Schelling in his 1956 book *The Strategy of Conflict* was the first to apply game theory to many contexts in social sciences.
- The theory has developed to a high degree of mathematical sophistication. The importance of this development was signified by the award of the 1994 Nobel Prize to three game theorists:

# The Elements of a Game

- We will begin by considering *strategic form* (sometimes called *normal form*) games. Such *non-cooperative games* have three elements:
  - a list of participants, or *players*
  - for each player, a list of *strategies*
  - for each array of strategies, one for each player, a list of *payoffs* that the players receive.

# The importance of details

- **“Players”** can be considered as individual agents;
- **“Strategies”** are a little more complex than actions. A strategy specifies what a player will do contingent on any eventuality. Such eventualities include not only the resolution of uncertainty (i.e., Nature’s choices), but the strategy choices of other players in the game;
- **“Payoffs”** are the objective functions of agents. They are usually stated in terms of expected utility

# A Familiar Game in Strategic Form

- There are two players: child A and child B
- Each player has three strategies:
  - {"Rock", "Paper", "Scissors"}
- The game has nine (3x3) outcomes, i.e., possible payoff configurations. These can be conveniently represented by the following table (*payoff matrix*):

# Rock- Paper- Scissors

| | | Child B | | |
|---|---|---|---|---|
| | | Rock | Paper | Scissors |
| Child A | Rock | 0, 0 | -1, 1 | 1, -1 |
| | Paper | 1, -1 | 0, 0 | -1, 1 |
| | Scissors | -1, 1 | 1, -1 | 0, 0 |

# The Prisoners' Dilemma

| | | Tchaikovsky | |
|---|---|---|---|
| | | Hold Out | Confess |
| Conductor | Hold Out | -3, -3 | -25, -1 |
| | Confess | -1, -25 | -10, -10 |

# Prisoners' Dilemma

- In this example, if both the conductor and Tchaikovsky colluded they would both minimise their imprisonment by holding out.
- *What would happen if Tchaikovsky and the Conductor could have talked to each other beforehand?*

# Applying Game Theory

- *Game Theory* and the Prisoners' Dilemma Metaphor
  1) Tragedy of the Commons
     (failed *cooperation*)
  2) Competition in Oligopoly
     (failed *collusion*)
- The solution in the Prisoners' Dilemma is based on the concept of a dominant strategy

# Dominant Strategies

- Having specified the elements of a game, we now turn to predicting what rational agents will do ...
- A weak requirement of rational behaviour is that players only use dominant strategies.
- Strategy is said to be (*strictly*) *dominated* if there exists another strategy that results in the same or better payoff level for every possible strategy choice of other players (with better for at least one such choice).

# *Example* (from Dixit & Nalebuff): Indiana Jones in the climax of the movie *Indiana Jones and the Last Crusade*.

Indiana Jones, his father, and the Nazis have all converged at the site of the Holy Grail. The two Joneses refuse to help the Nazis reach the last step. So the Nazis shoot Indiana's dad. Only the healing power of the Holy Grail can save the senior Dr. Jones from his mortal wound. Suitably motivated, Indiana leads the way to the Holy Grail. But there is one final challenge. He must choose between literally scores of chalices, only one of which is the cup of Christ. While the right cup brings eternal life, the wrong choice is fatal. The Nazi leader impatiently chooses a beautiful gold chalice, drinks the holy water, and dies from the sudden death that follows from the wrong choice. Indiana picks a wooden chalice, the cup of a carpenter. Exclaiming "There's only one way to find out" he dips the chalice into the font and drinks what he hopes is the cup of life. Upon discovering that he has chosen wisely, Indiana brings the cup to his father and the water heals the mortal wound.

# Indy Goofed!

- Although this scene adds excitement, it is somewhat embarrassing that such a distinguished professor as Dr. Indiana Jones would overlook his dominant strategy.
- He should have given the water to his father without testing it first.
  - If Indiana has chosen the right cup, his father is still saved.
  - If Indiana has chosen the wrong cup, then his father dies but Indiana is spared.
- Testing the cup before giving it to his father doesn’t help, since if Indiana has made the wrong choice, there is no second chance -- Indiana dies from the water and his father dies from the wound.

# Iterated Removal of Dominated Strategies

- By sequentially removing the dominant strategies of players from consideration, one can predict the outcome of a game.
- For example,

| | | Player 2 | | |
|---|---|---|---|---|
| | | Left | Middle | Right |
| Player 1 | Up | 1, 0 | 1, 2 | 0, 1 |
| | Down | 0, 3 | 0, 1 | 2, 0 |

- Observe that neither “Up” nor “Down” is stricly dominanted for Player 1, but “Right” is dominated by “Middle” for Player 2.
- Therefore, Player 2 will never play “Right” and it can be removed from consideration. The game then becomes:

| | | Player 2 | |
|---|---|---|---|
| | | Left | Middle |
| Player 1 | Up | 1, 0 | 1, 2 |
| | Down | 0, 3 | 0, 1 |

# And finally … the last iteration

- In this reduced game, “Down” is strictly dominated by “Up” for Player 1. Therefore, it too can be removed:

| | Left | Middle |
|---|---|---|
| Up | 1, 0 | 1, 2 |

# Nash Equilibrium

- If a game is not dominance solvable (i.e., eliminating dominated strategies does not reduce the game to a single outcome), then the notion of a Nash equilibrium can yield a more precise prediction.
- An outcome (i.e., set of strategy choices for each player) is a **Nash equilibrium**, if each player -- holding the choices of other players as constant -- cannot do better by changing their own choice. Unilateral deviations are unprofitable.

# Dominance Solvability Implies Nash Equilibrium

- In the previous game, Player 1 using “Up”and Player 2 using “Middle” is a pure Nash equilibrium. It coincides with the dominance solution of that game.
- This corresponds to the notion of a *pure* Nash equilibrium -- each player chooses one strategy. This is in contrast to the concept of a *mixed* strategy that involves players choosing several strategies randomly. The “Rock-Paper-Sissors” game has one mixed strategy Nash equilibrium.

# Cooperation and Coordination

- In 1776, Adam Smith wrote …

“It is not from the benevolence of the butcher, the brewer or the baker that we expect our dinner, but from their regard to their own self interest.... [Every individual] intends only his own security, only his own gain. And he is in this led by an invisible hand to promote an end which was no part of his intention. By pursuing his own interest, he frequently promotes that of society more effectually than when he really intends to promote it.”

# Problems of Decentralisation

- Game theory allows us to think about the interaction among selfish, rational individuals.
- It, therefore, can show us how large the span of the invisible hand actually is.
- Market failures can be understood in game theoretical contexts by looking at some familiar classes of games: the Prisoners' Dilemma and Coordination Games.

# Many Person Games

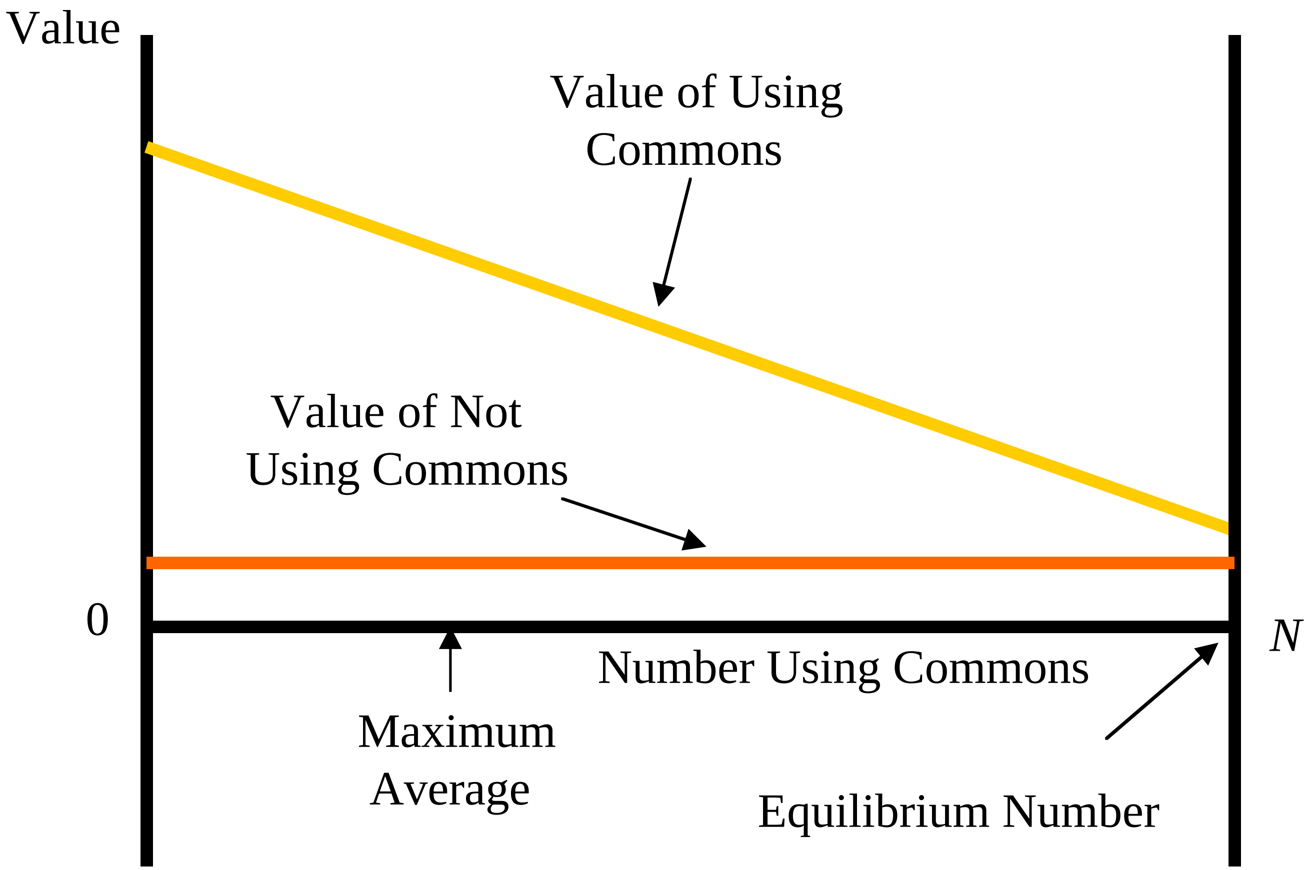
- Here is another example of a *multi-person Prisoner’s Dilemma*.
- The situation here is sometimes referred to as the *Tragedy of the Commons*.
- This can also be used to study:
  - traffic congestion
  - advertising competition among oligopolists
  - Bell curves and over-study

# Tragedy of the Commons

- Garrett Harding wrote ...

“Picture a pasture open to all. It is expected that each herdsman will try to keep as many cattle as possible on this commons. ... Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit, in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons.”

# Multi-Person Prisoners' Dilemma

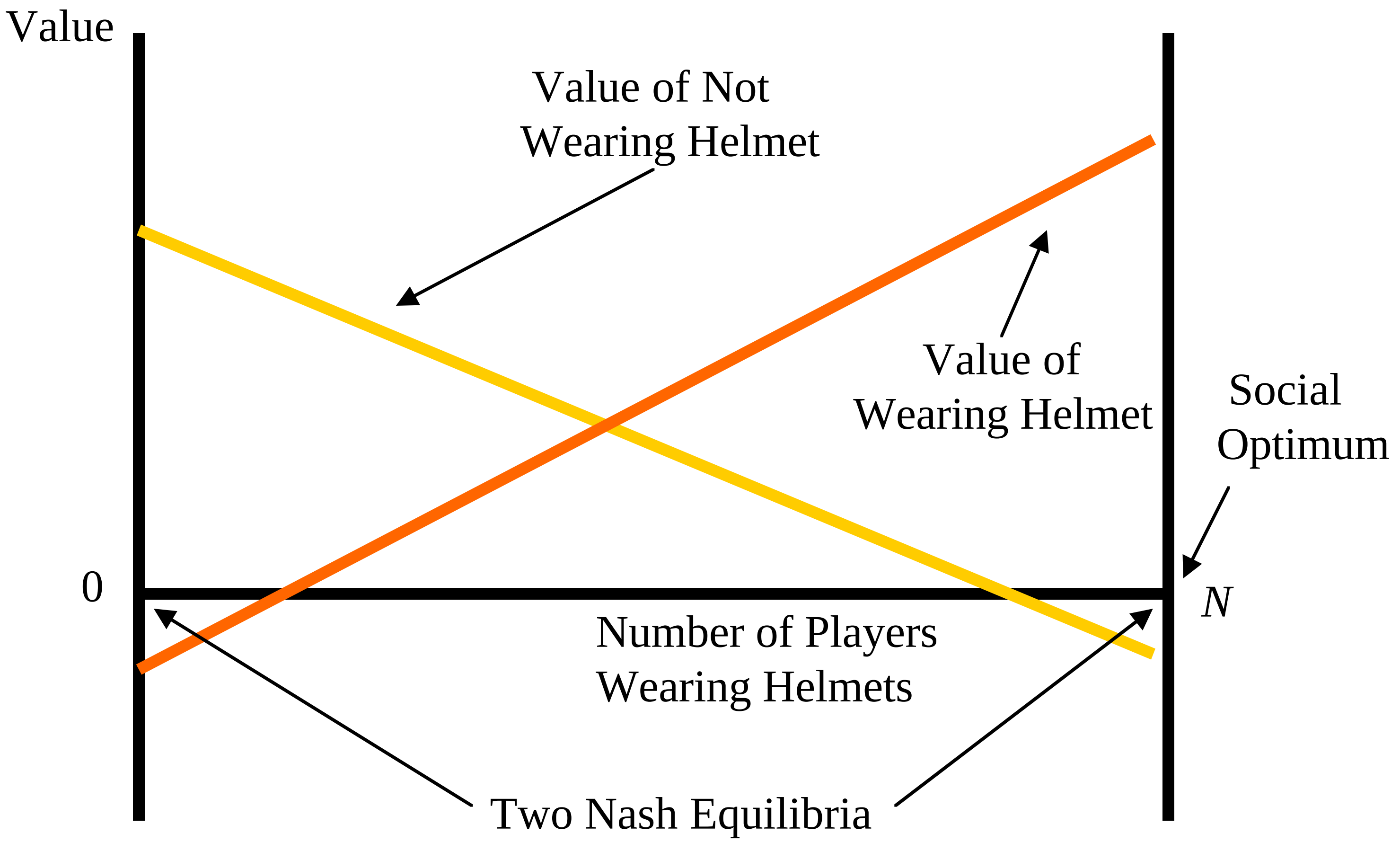


# Interdependent Preferences

- In 1969, a famous hockey player (Teddy Green) took a hockey stick to the brain. *Newsweek* wrote:

“Players will not adopt helmets by individual choice for several reasons. Chicago star Bobby Hull cites the simplest factor: ‘Vanity.’ But many players honestly believe that helmets will cut their efficiency and put them at a disadvantage, and others feel the ridicule of opponents. The use of helmets will spread only through fear caused by injuries such as Green’s -- or through a rule making them mandatory.... One player summed up the feelings of many: ‘It’s foolish not to wear a helmet. But I don’t -- because the other guy’s don’t. I know its silly, but most of the players feel the same way. If the league made us do it, though, we’d all

# Coordination Failure

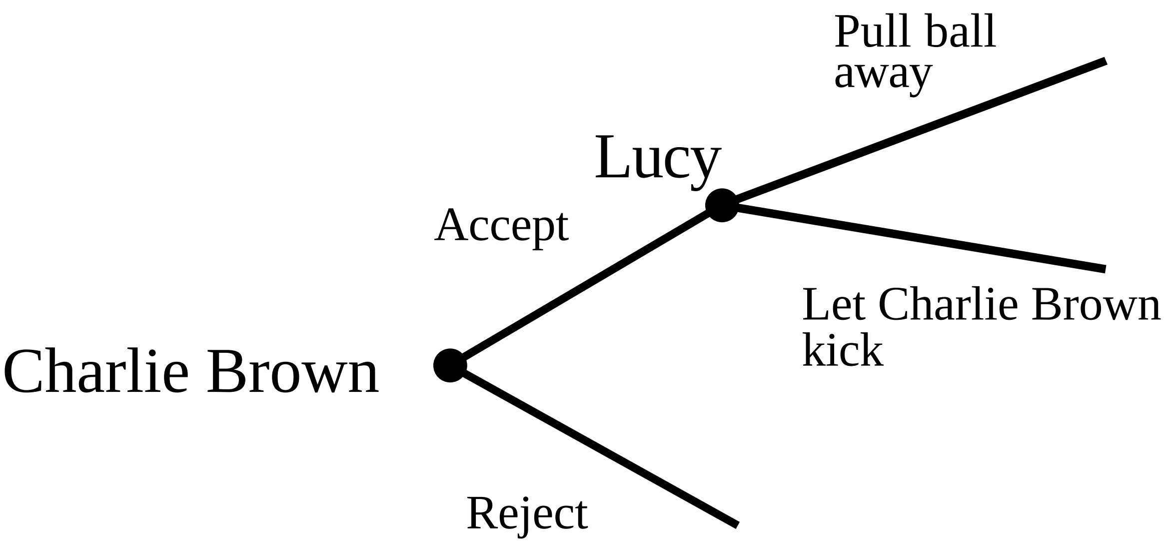


# Other Examples of Coordination Failure

- Driving on the left-hand side of the road
- Daylight saving
- Restaurant choice
- Mass revolution in Eastern Europe
- Washington Baby-Sitting Co-op
- Stock market crashes and speculative bubbles
- The Economics of QWERTY

# Game Trees

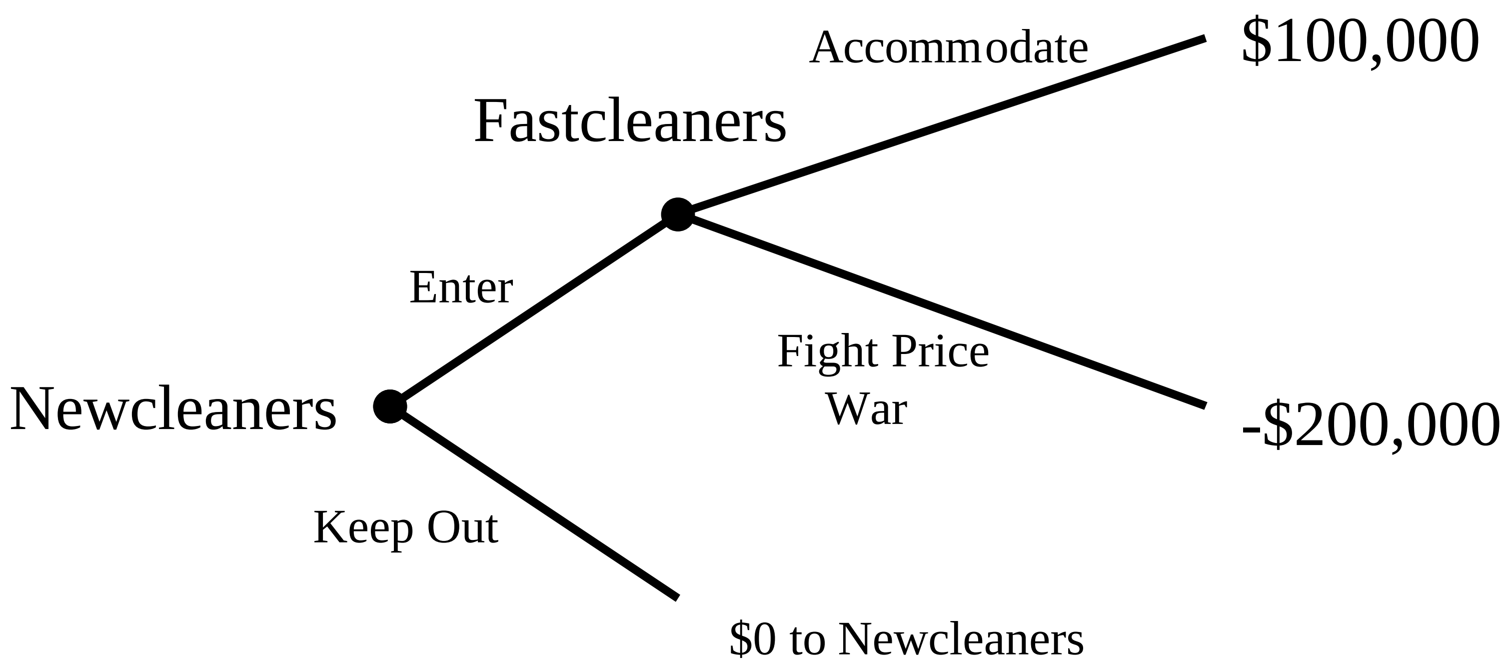
- A game tree has chance nodes (open circles) and choice nodes (black circles) for each player.



# An Entry Game

- Vacuum cleaner market currently has one incumbent (Fastcleaners)
- Potential entrant (Newcleaners). It is deciding whether to enter the market or not.
- If enters, Fastcleaners has 2 choices:
  - Accommodate: accept a lower market share
  - Price war

# Newcleaner’s Payoffs



Accommodate

$100,000

Fastcleaners

Enter

Fight Price War

Newcleaners

-$200,000

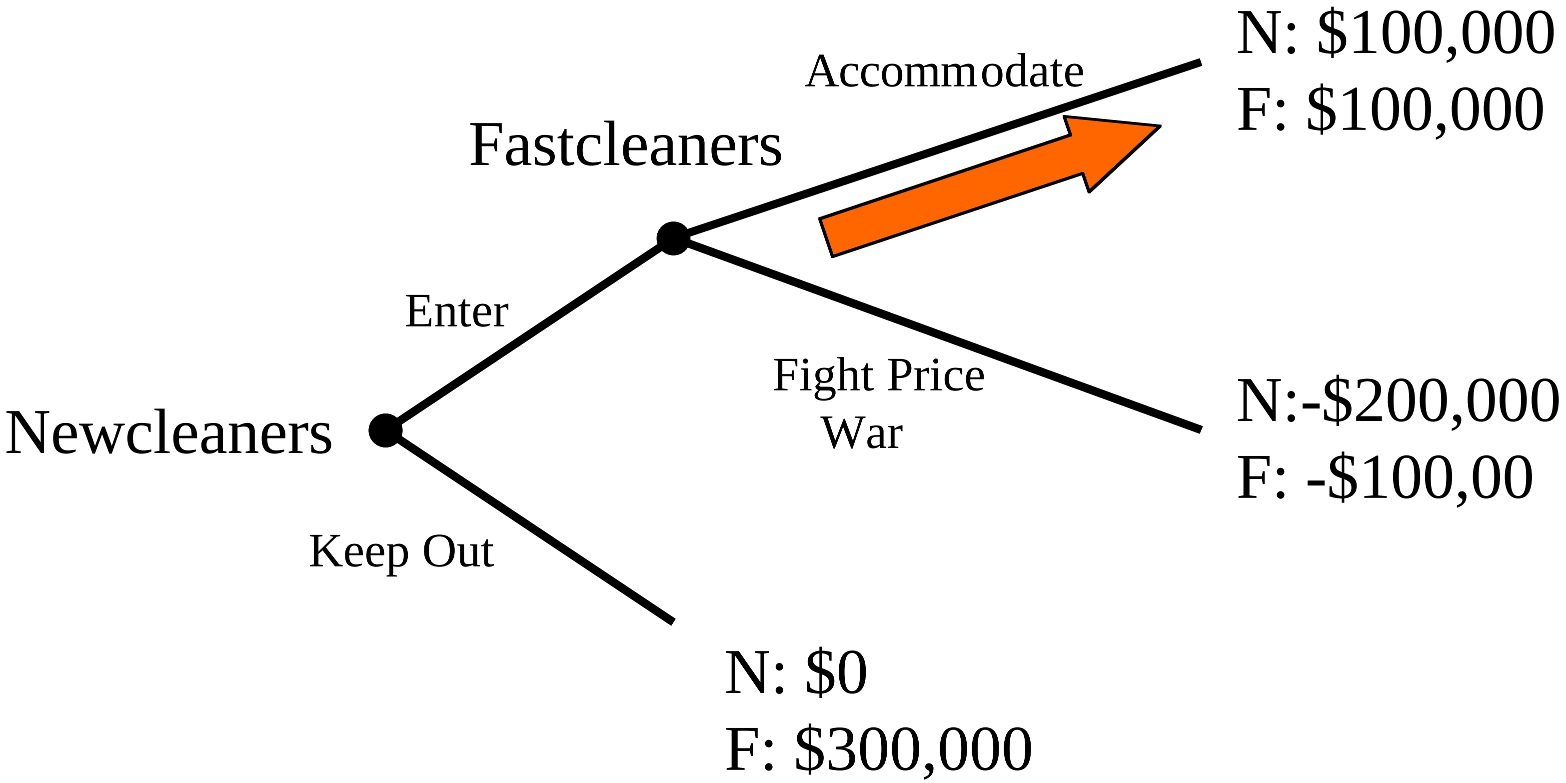
Keep Out

$0 to Newcleaners

# What should it do?

- Newcleaner needs to forecast Fastcleaner’s response
- How does it do this?
  - Put themselves in Fastcleaner’s shoes
  - Work out Fastcleaner’s payoffs

# Work Backwards



Accommodate

N: $100,000
F: $100,000

Fastcleaners

Enter

Fight Price War

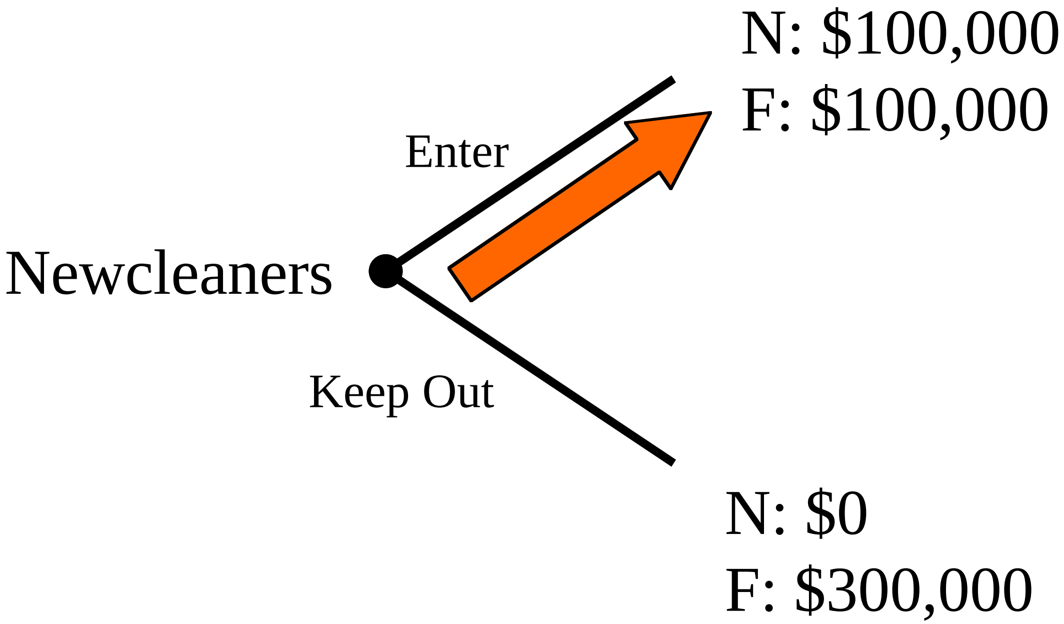
Newcleaners

N:-$200,000
F: -$100,00

Keep Out

N: $0
F: $300,000

# Reduced Game



Newcleaners

Enter

Keep Out

N: $100,000
F: $100,000

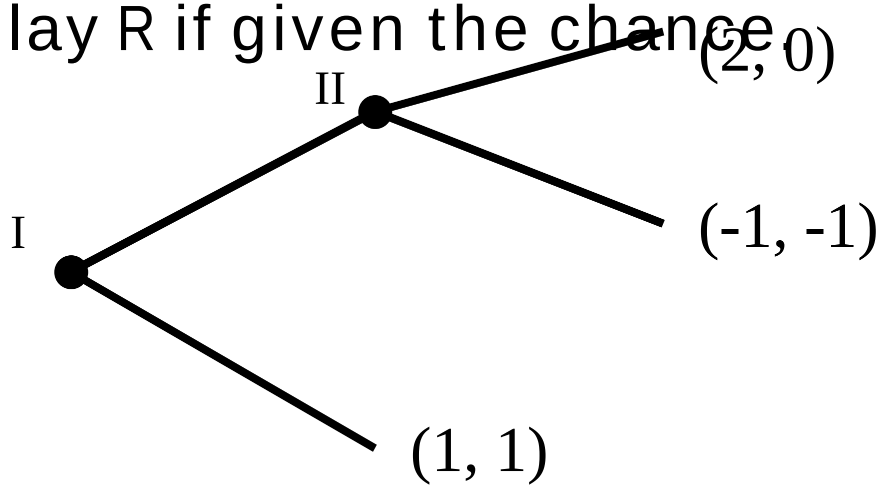
N: $0
F: $300,000

# Backwards Induction

- The predicted outcome of extensive form games can be found by solving the game by *backwards induction* (a concept very similar to iterated dominance).
- In the entry game, this solution is found by finding Fastcleaners' best response first to entry. If Newcleaners enters, Fastcleaners would rather accommodate entry than fight a price war. So entry is worthwhile.
- Fastcleaner has a *first-mover advantage*.

# Subgame Perfect Equilibrium

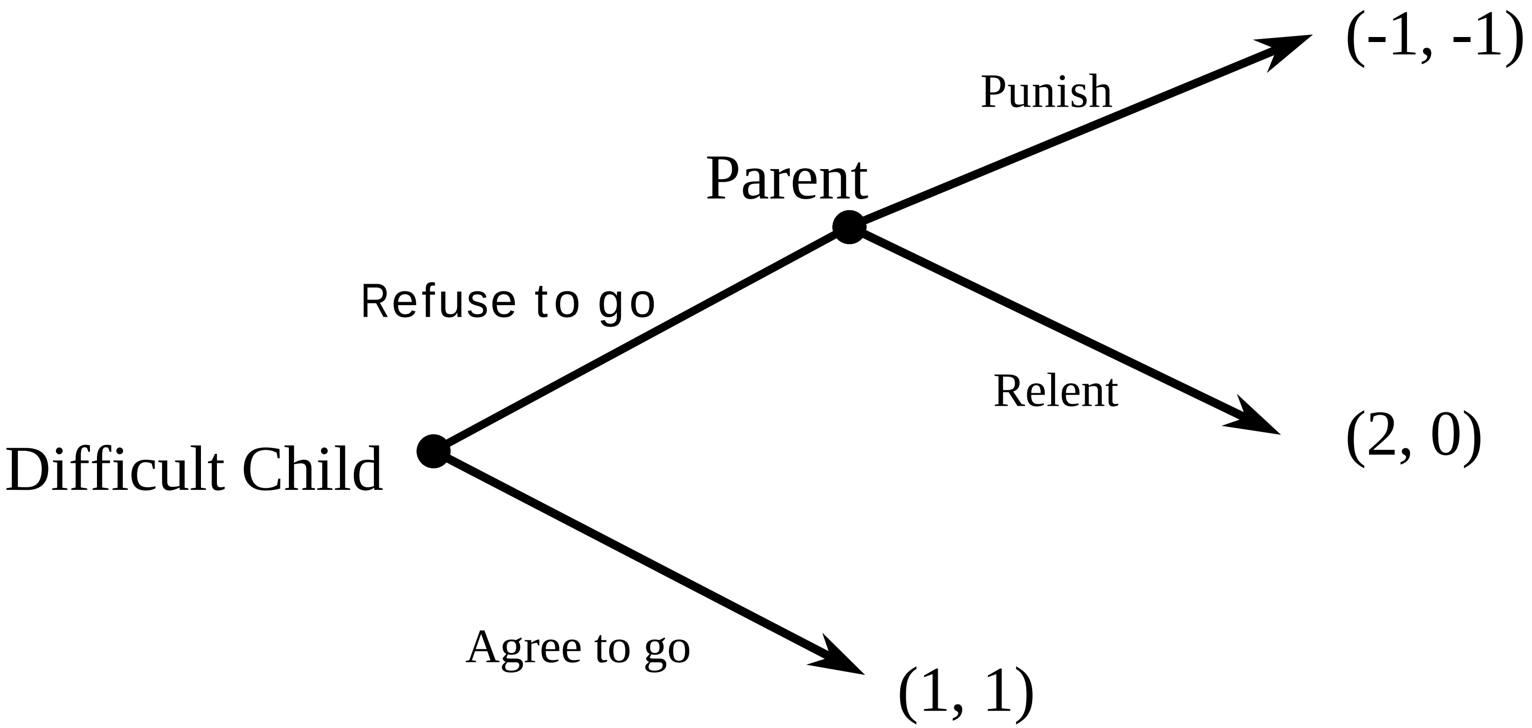
- The following game has two types of Pure Nash equilibria:
  - (i) Player I plays L and Player II says they will play L if given the chance.
  - (ii) Player I plays R and Player II says they will play R if given the chance.



# Credibility

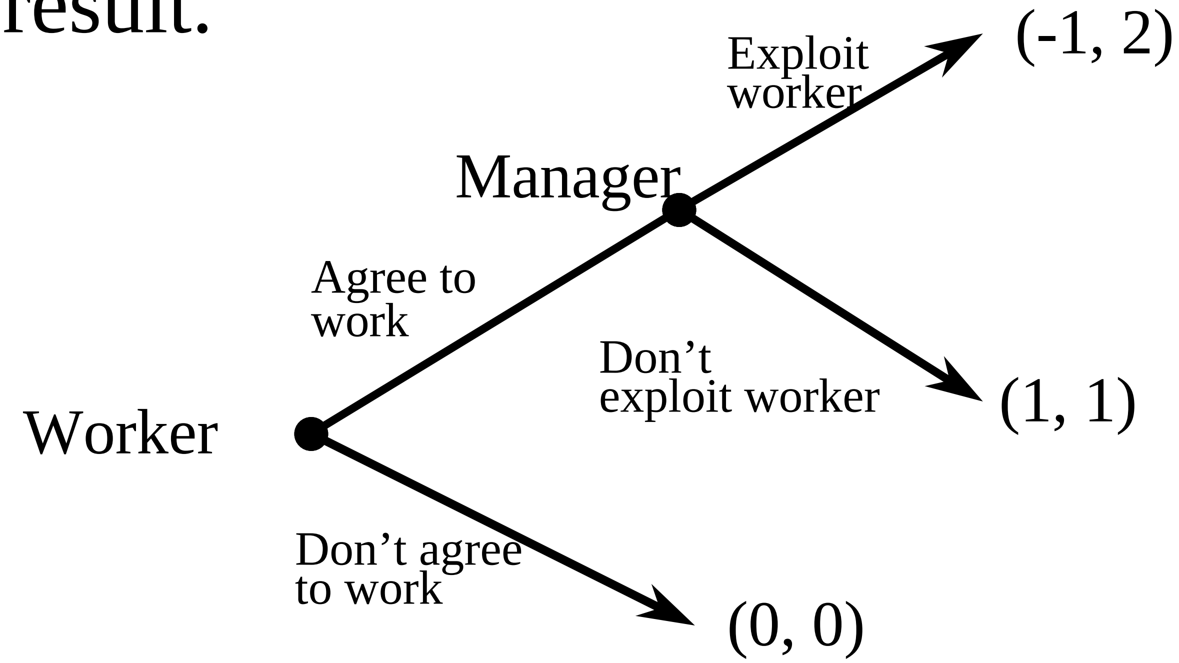
- Problem: I knows that if II gets to choose it is best for II to choose R. In that case, the equilibrium in (I) is supported by a *threat* by II that is not **credible**. II cannot *commit* to choose L if I chooses R. Therefore, Nash equilibrium is implausible.
- *Alternative solution concept*: **subgame perfection**
- Strategies are said to be subgame perfect if a Nash equilibrium is being played in every subgame.
- Subgame perfection is an example of *an equilibrium refinement*, offering an improved prediction.

# Controlling Rotten Kids: Making Credible Threats



# Importance of Commitment

- The difficulty to commit to an action can explain why some inefficient outcomes can result.



# Employment Contracts

- This game has a subgame perfect (and Nash equilibrium) outcome where the Worker does not agree to work and both receive a payoff of 0.
- It would be a Pareto improvement if the manager could commit not to exploit the worker. Without such a commitment device, an inefficient equilibrium prevails.
- This is similar to the Prisoners' Dilemma where the problem is that both parties need to find some commitment mechanism.

# Commitment Mechanisms

- establishing a reputation
- writing an enforceable contract
- cutting off communication
- burn bridges behind you
- leaving the outcome to chance
- moving in small steps
- develop credibility through teamwork
- employ mandated negotiating agents