## A Taxonomy of Decisions

|  | certainty | uncertainty |
| :---: | :---: | :---: |
| attribute $\begin{gathered}\text { single } \\ \\ \\ \text { multipl }\end{gathered}$ | $\begin{gathered} \text { FA } \\ \text { CBA } \end{gathered}$ | Decision <br> Analysis |
|  | Multi- <br> Attribute Decisions | ? |

decisions

Decision Analysis: decision making under uncertainty

## Decision Analysis - Introduction

## 1. Basic Concepts

A technique for helping make decisions, and avoiding pitfalls.
We discuss:
$>$ Formulating the issue.
$>$ Identifying the alternative actions.
$>$ Valuing the possible outcomes.
(Not merely in monetary terms.)
$>$ Encoding uncertainty.
$\rightarrow$ probabilities
$>$ Certainty Equivalent (C.E.).
$>$ The Value of Perfect Information. (VPI)
$>$ The value of imperfect information.

## Beginning Principles:

The best you can do is to integrate in a logical manner:
$>$ What you can do.
$>$ What you know. (Such as probabilities, and values)
$>$ What you want or value. (Such as preferred outcomes)

2. The Simplest Decision - Case 1

The simplest decision under uncertainty calling a coin toss: you win $\$ 10$ or nothing.

Highlights some concepts which are useful in more complex decisions.
Let's start with a volunteer ( ) and ask some questions:

1. Would you pay $\mathbf{\$ 2}$ for a ticket to play the game?
2. What's the minimum you'd sell the ticket for?
3. What's the maximum you'd pay for perfect information about the toss (from a clairvoyant)?
4. And for imperfect information?

Everyone write down your answers to Questions 2 and 3.

## The Coin Toss

$\square$
$\qquad$ c.
values perfect information at \$ $\qquad$
values imperfect information at < \$

Consistency Check

1. You sell the Ticket to the Lottery for your Certain Equivalent, or minimum selling price $\$ X$. You walk away with $\$ X$ for certain.
2. You buy Perfect Information about the coin toss for a maximum of $\$ Y$. You then correctly call the toss and win the $\$ 10$. You walk away with $\$ 10-\$ Y$ for certain.
3. So, to be consistent: $\$ X=\$ 10-Y$

Consistency Check

$$
\begin{gathered}
\text { Minimum selling price } \\
\text { (The Certainty Equivalent) } \\
\boldsymbol{+} \\
\text { Value of Perfect Information } \\
= \\
\text { Maximum Payoff }
\end{gathered}
$$

But why?
So the Value of Imperfect Information must be less than the Maximum Payoff minus
Minimum selling price (The Certainty Equivalent)

Calling the Toss

## Concepts:

$>$ Uncertainty and probability
$>$ Profit lotteries
$>$ Decisions as allocations of resources
$>$ Sunk cost - irretrievable allocations of resources
$>$ Certainty Equivalent - value of the lottery
> Information and probability
$>$ Value of information
$>$ Consistency in decision making

## Concepts (cont.)

$>$ Decisions versus outcomes
$\rangle$ What is meant by a good decision?
$>$ Individual decisions, corporate decisions
$>$ Decision trees:


## Insights?

1. The three elements of a decision:

- actions: here call "Heads" or "Tails".
- events are Nature's possible moves: here Heads or Tails.
- outcomes: here either \$10 for a correct call or nothing.

2. Her attitude to risk: the minimum she was prepared to sell the ticket for.
3. Her value of information: limited by the Value of Perfect Information, a function of the probabilities and payoffs.
4. The Decision Analysis Process.

Stage 1
Decision analysis is a three-stage, quality process. But if at any step in the process the decision becomes obvious, you should stop and make the decision.

1. Structuring: Frame the Right Problem
$>$ Clarify the decision.
$>$ Raise and sort issues.
> Generate creative alternatives.
$>$ Model the problem.

The Decision Analysis Process - Stages 2 and 3.
2. Evaluation: Use Logical Thinking
$>$ Discover what is important.
> Apply an appropriate risk attitude.
$>$ Determine the value of new information.
3. Agreement: Have Commitment to Action
> Check for refinement.
$>$ Agree on course of action.
$>$ Implement course of action.
Decision analysis is a normative process.

## 4. Evaluation - Making Difficult Decisions:

How many decisions with complete certainty have you ever made?
Does a good decision always guarantee a good outcome?
(Does Tiger Woods always win?)

## Decisions with Certainty:



## Decisions with Uncertainty:

implies a Decision Node
O implies a Chance Node
A decision: an irrevocable allocation of resources.

## 5. Evaluation - A Second Example:

$>$ You have the opportunity to win $\$ 100$ if you correctly call the roll of a die as even or odd.
$>$ The opportunity is not costless - you must pay $\$ 35$ for the opportunity.
$>$ You will call the die roll odd or even. There is only one chance to invest.
$>$ Would you accept this opportunity?

## How would you evaluate this opportunity?

Typical answers are:
$>$ I can afford to lose \$35
$\rangle$ I could really use $\$ 100$
$>$ I would toss a coin
$>$ I need to ask my partner or spouse
$>$ I don't gamble
$>$ My internal rate of return is ...

Do you think $\$ 35$ is a good deal for this opportunity?
Yes / No ?
How did you evaluate this opportunity?

Is this a good or bad decision?

If you were able to negotiate, what price would you pay for this opportunity?

We need to think logically about the decisions we make.
> Should I take this opportunity?
$>$ What is a good decision?
$>$ What would someone else do?
e.g., my brother, etc.
$>$ Can I afford to lose the $\$ 35 ?$
$>$ What do I think are my chances of a good outcome?
Decision trees help us structure decisions in a logical manner.

Probability is a state of mind, not things.
$>$ The Bayesian approach allows us to assign probabilities in once-off situations.
What is the value to you of a single toss of a coin: \$100 if heads, nothing if tails?

Define the expected return from the single toss to be the average return of a hypothetical series of many tosses: $\$ 100 \times 1 / 2+\$ 0 \times 1 / 2=$ $\$ 50$. Treat unique events as if they were played over many times.
$>$ All prior experience must be used in assessing probabilities. (Coins are almost always fair; it's warm enough to go to the beach most weekends in March in Sydney.)

Values plus probabilities.
> Decision making requires the assessment of values as well as probabilities.

Would you pay as much as $\$ 50$ to play in the once-off coin toss?
Few people would; most people would pay a premium to reduce their risk: they are risk averse, and would sell their lottery ticket at something less than $\$ 50$; the lowest selling price is their Certainty Equivalent (C.E.).

The risk premium equals the expected return less the Certainty Equivalent, when selling.

Risk aversion can be defined and measured using utility theory.

The utility of a lottery ...
$>$ Decisions can only be made when a criterion is established for choosing among alternatives.
The utility of a lottery is its expected utility.
(by the definition of utility)
$>$ The implications of the present for the future must be considered. What discount rate to use?
> Must distinguish between a good decision and a good outcome.
Prudent decision-making doesn't guarantee the desired outcome invariably, but should improve the odds.

The Value of Perfect Information?
> Often we can, at a cost, reduce our uncertainty about Nature's future events (using market research, forecasting, statistical analysis). There must be a limit to what we should spend in these endeavours-how much is it?
The Value of Perfect Information. (VPI)
$>$ The value of imperfect information is less.
Often we can, at a cost, buy more certainty about the future (pay an insurance premium, buy a hedge against future outcomes).
What is a fair price to pay?

## 6. Using Decision Trees to Evaluate Decisions

A decision tree is a flow diagram that shows the logical structure of a decision problem. It is a visual aid to lay out all the elements of a decision. It contains four elements:
$>$ Decision nodes, $\square$, which indicate all possible courses of action open to the decision maker;
$>$ Chance nodes, 0 , which show the intervening uncertain events and all their possible outcomes; i.e., Nature plays
$>$ Probabilities for each possible outcome of a chance event; and
$>$ Payoffs, which summarize the consequences of each possible combination of choice and chance.

The decision tree for this opportunity (\$100 on calling a roll):
The decision is whether or not to invest $\$ 35$ for the opportunity to receive $\$ 100$ or $\$ 0$ as the outcome on the call of a die roll as odd or even.


What else is needed to evaluate this opportunity?

The tree is missing the probability assessments for a good and a bad outcome.

The tree does not yet incorporate the investor's judgement of the probability of success and its complement, the probability of failure or loss.

What information would help with this assessment?
$>$ The number of sides on the die
$>$ Any known bias the die might have
$>$ Who gets to roll the die


## 7. Evaluation — Opportunities and Outcomes

An important distinction is that between opportunities and outcomes.
Opportunities are the sum of their possible outcomes. This is important because
you can only choose your opportunities - not your outcomes.


How do we evaluate the opportunity?
First, decide on the decision criterion. This can be any measure that allows the decision maker to evaluate deals in a quantitative manner.

Expected Monetary Value (EMV) provides the means to evaluate risky decisions consistently.

EMV is the probability-weighted average:

Example: Calling the roll of the die.


| Probability | Outcome |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 0.50 | $\times$ | $\$ 100$ |  | $\$ 50$ |
| 0.50 | $\times$ | $\$ 0$ |  | $\$ 0$ |
|  |  |  | $\$ 50$ |  |
| Investment | $=$ | $-\$ 35$ |  |  |
| Expected Net Profit | $=$ | $\$ 15$ |  |  |

You have decided to take the opportunity.
You believe the probability of success of failure are equal, or 50/50.
You have paid the $\$ 35$ investment, now sunk.
Now what does the decision tree look like?


How has the opportunity changed?

Beware the sunk-cost fallacy.
Before deciding to pursue the investment, it is appropriate and important to include the costs to enter the deal (the price of admission).

But don't include what you've already paid to get into an investment: that decision has already been made and the resources allocated, usually irreversibly.

Let bygones be bygones.
Evaluate future decisions for what they are worth.
Your selling price and the value you place on the investment opportunity should not depend on sunk costs.

The Certainty Equivalent of a lottery.


> a deal or opportunity
C.E.: the minimum you'd sell the ticket for.
its
Certain Equivalent

## 8. Evaluation - What is Your Risk Attitude?

The difference between expected value (EMV) and the Certainty Equivalent (C.E.) is your risk premium.


Risk profiles
If you would pay more than EMV for a deal, then you are risk seeking.
If you would pay up to the EMV for a deal, then you are risk neutral.
If you would not pay EMV for a deal, then you are risk averse.

The event is set ...
The die has been rolled and the event is set.
You don't know the outcome of the roll, so the outcome of the opportunity has not been determined.

What would it be worth to have perfect knowledge about the roll of the die?

What is the value of information?

## 9. Evaluation - The Value of Perfect Information

Using the concept of a clairvoyant, who knows all things past, present, and future, we can structure a new deal:


Consistency check:

Minimum selling price?

Value of perfect information?

Are these consistent?
Ask: What would I walk away with (\$) in both cases?

Calculating the Value of Perfect Information:

Value of the deal with perfect information \$100

- Your minimum selling price (CE)
\$ $\qquad$
= Value of perfect information (VPI)
\$ $\qquad$
Calculating the Value of Perfect Information (VPI) is not difficult, but finding a clairvoyant, or source of perfect information, will be.
$\therefore$ Use the VPI as a guideline for spending time, effort, and money on gathering new information before making a decision.

Sources of imperfect information:
While there are no real clairvoyants (alas), we can find new sources of information which is imperfect:
$>$ Experiments
$>$ Experts
$>$ Models
$>$ Trial runs
$>$ Market tests
$>$ Forecasts

We must distinguish between good decisions and good outcomes.
$>$ Decisions are what we can affect.
$>$ A good decision balances the probabilities of good and bad outcomes in accordance with our risk attitudes.
$>$ Outcomes are what we get.
$>$ A good outcome is one we like.

## 10. Summary of Evaluation

$>$ A decision is an irrevocable allocation of resources.
$>$ Probabilities, representing expert judgement, are based on experience, beliefs, knowledge, and data.
$>$ The value of a deal depends on the decision maker's risk attitude.
$>$ The maximum value of gathering more information can be determined (using the Value of Perfect Information) before obtaining the actual information, in this framework.

